Universität Siegen

Fakultät III
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Exam “International Macroeconomics“
Winter Semester 2017-18
(1st Exam Period)

Solution

Available time: 60 minutes

For your attention:

1. The exam is made up of 9 pages (including this cover page). Please check and see if the exam you are holding is **complete**.

2. For your answers, use the designated spaces. Should these not suffice, use the backside of the pages. Please do not use a **pencil**.

3. Additional materials you may use for the exam: a non-programmable calculator. (Smart phones and mobile **phones** are **not** allowed!)

4. ATTENTION: The names for variables have the same meaning as in the lecture. Insofar as you also use the same symbols for the variables as we did in the lecture you will not have to define these any further.

Question	**1**	**2**	**3**	**4**	**Sum**	**Mark**
Points achievable	27	16	17	14.5	60	
Points achieved						

Problem 1:

Consider the following utility function and budget constraint:

$$U_1 = u(C_1) + \beta u(C_2), \quad \beta \in (0,1)$$

where $u(C_i) = \frac{C_i^{1-\sigma} - 1}{1-\sigma}, \quad i = 1,2$

$$Y_2 = C_2 + (1+r)(C_1 - Y_1)$$

a) Derive the indirect utility function given that the value of σ equals 1.
Hint: use L’Hôpital’s Rule [16 points]

Solution:

$u(C_i) = \ln C_i, \; i = 1,2$ for $\sigma = 1$ (1)

$\max U_1 = \ln C_1 + \beta \ln C_2$

w.r.t. $C_1 + \frac{C_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r}$

$\mathcal{L} = \ln C_1 + \beta \ln C_2 + \lambda \left(Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} \right)$ (3)

FOCs:

$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{1}{C_1} - \lambda \overset{!}{=} 0$ (1)

$\frac{\partial \mathcal{L}}{\partial C_2} = \beta \frac{1}{C_2} - \lambda \frac{1}{1+r} \overset{!}{=} 0$ (1)

$\Rightarrow C_2 = C_1 \cdot \beta (1+r)$ (1) (*)

↳ plug into budget constraint.

$\frac{C_2}{\beta(1+r)} + \frac{C_2}{1+r} = Y_1 + Y_2/(1+r)$

$C_2 (1+\beta) = (1+r)\beta \cdot Y_1 + Y_2 \beta$ (3)

$C_2 = \left(\frac{\beta}{1+\beta}\right) [(1+r) Y_1 + Y_2] \Rightarrow$ plug back into (*) (1)

$C_1 = \frac{C_2}{\beta(1+r)} = \frac{1}{(1+\beta)(1+r)} [(1+r)Y_1 + Y_2]$ (3)

$$\left\{ \begin{array}{l} C_1 = \frac{1}{(1+\beta)(1+r)} [(1+r)Y_1 + Y_2] \\ C_2 = \frac{\beta}{1+\beta} [(1+r)Y_1 + Y_2] \end{array} \right. \Rightarrow \text{plug into utility function.}$$

$$\Rightarrow U_1 = \ln \left[\frac{1}{(1+\beta)(1+r)} [(1+r)Y_1 + Y_2] \right] + \beta \ln \left[\frac{\beta}{1+\beta} [(1+r)Y_1 + Y_2] \right]$$ (3)

b) Suppose that there are two types of individuals in autarky: type A individuals have 0 unit endowment in the first period but 3 in the second while type B individuals have 3 in the first and 0 in the second. Derive their utility levels and tell which type has a higher one. [11 points]

Solution:

plug into utility function, $1+r=\frac{1}{\beta}$ in autarky (2)

utility function becomes:

$$U_1 = \ln\left[\frac{\beta}{1+\beta}\cdot\left[\frac{1}{\beta}Y_1+Y_2\right]\right]+\beta\cdot\ln\left[\frac{\beta}{1+\beta}\left(\frac{1}{\beta}\cdot Y_1+Y_2\right)\right]$$

$$U_1^A = (1+\beta)\ln\left(\frac{1}{1+\beta}Y_1^A+\frac{\beta}{1+\beta}Y_2^A\right) = (1+\beta)\cdot\ln\left(\frac{3\beta}{1+\beta}\right) \quad (3)$$

$$U_1^B = (1+\beta)\ln\left(\frac{3}{1+\beta}\right) \quad (3)$$

Compare U_1^A & U_1^B $\quad U_1^A - U_1^B = \ln\beta$ (3)

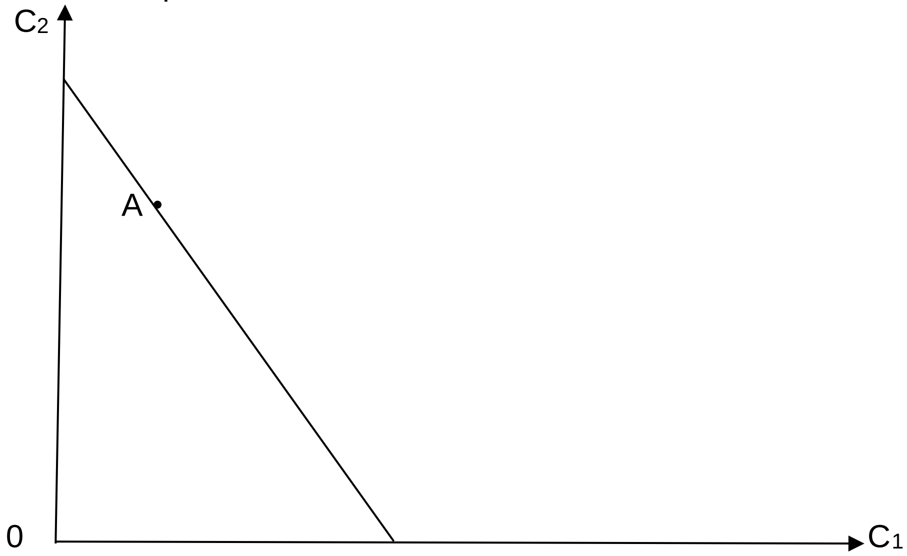
Since $\beta\in(0,1)$

thus $\ln\beta<0$ (2)

$\Rightarrow U_1^A < U_1^B$. (1)

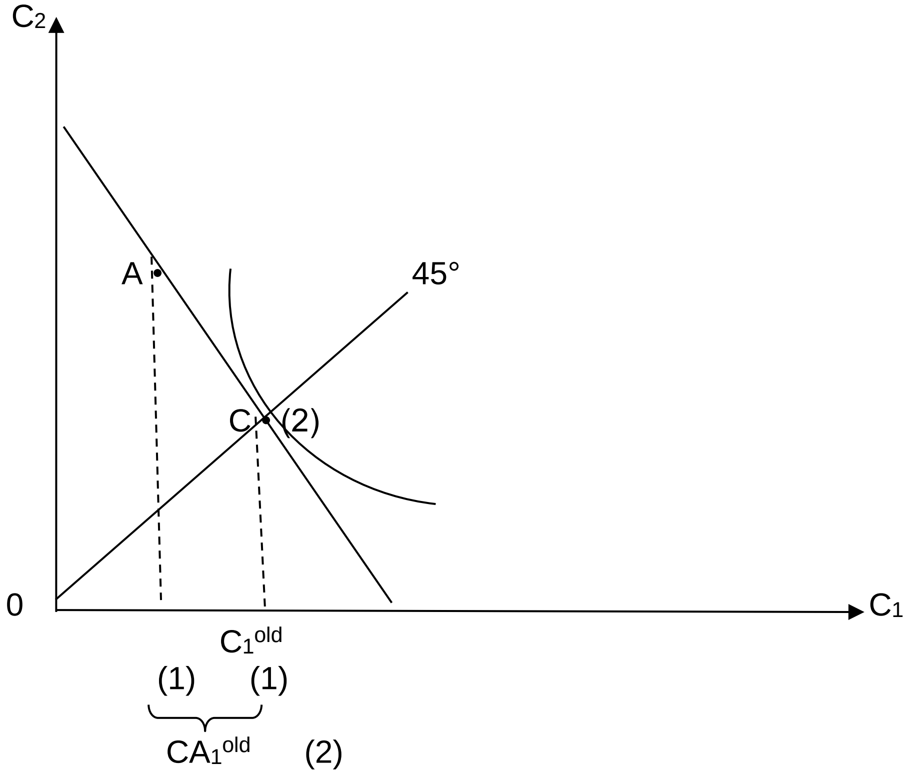
Problem 2: Representative Consumer in a Small Endowment Economy

We consider an economy with exogenous incomes where the representative consumer has a two-period horizon and wants to smooth consumption over time. There is no initial international investment position ($B_1 = 0$). In the following graph, A represents the endowment point.



a) Please illustrate the optimal consumption point (C) such that the balance on current account is negative ($CA_1 < 0$). On the horizontal axis, please indicate the following variables: income (Y_1), Consumption (C_1^{old}), current account balance (CA_1^{old}). [6 points]

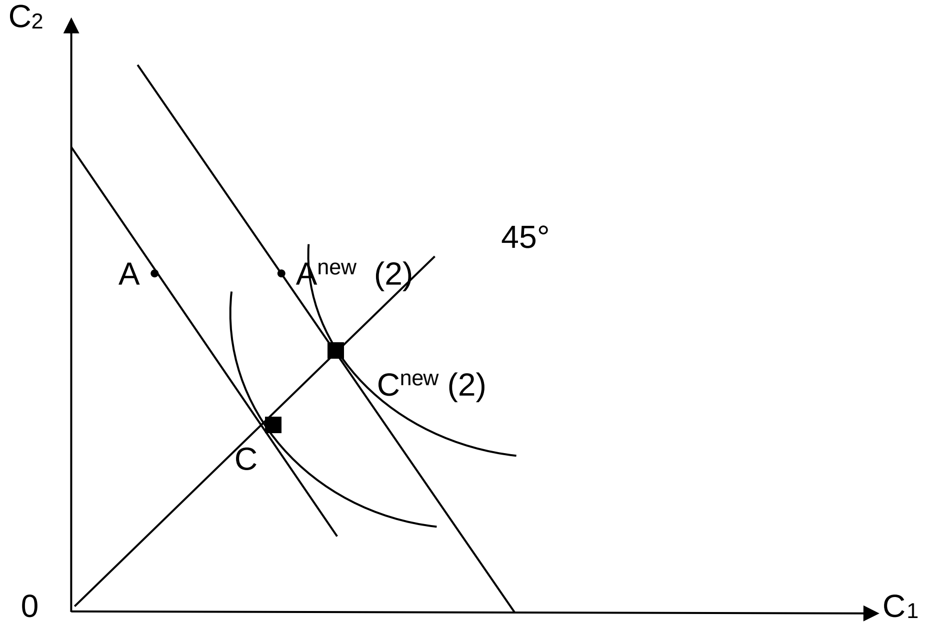
Solution:



b) Now, assume that income of the first period (Y_1) increases.

b_1 In the above graph, please indicate the new endowment point as A^{new} and the new consumption point as C^{new}. [4 points]

Solution:



b_2 Does the current account of the first period improve or deteriorate? Please explain. [3.5 points]

Solution:

improve because the increase of C_1 is lower than the increase of Y_1
(1) (1) (1) (0.5)

b_3 Does the international investment position at the end of the first period (B_2) improve or deteriorate? Please explain. [2.5 points]

Solution:

- improve (1)

- because of $B_1 = 0$, we have $B_2 = CA_1$
(0.5) (1)

(or: change of B_2 equals change of CA_1)
(1)

Problem 3: Two-Country Model

We consider a two-period model of two large open economies (Home (H) and Foreign (F)) that are linked by a perfect international capital market. Lifetime utility of the representative consumers are given by

$$U_1^H = \mathrm{u}(C_1^H) + \beta^H \mathrm{u}(C_2^H) \quad , \quad U_1^F = \mathrm{u}(C_1^F) + \beta^F \mathrm{u}(C_2^F) \quad .$$

Both countries face an exogenous time path of incomes where $Y_1^H = Y_1^F$, $Y_2^H = Y_2^F$. We assume that the net international investment positions are zero both at the beginning of the first period and at the end of the second.

a) Explain why, in such a setting, the interest rate r is not an exogenous variable. [3 points]

Solution:

“large” → economic changes/actions of each country change the interest rate

b) We assume that $\beta^F = z\,\beta^H, \quad z > 1$

b1 Please interpret this assumption. [2 points]

Solution: F has a lower time preference

(or: F appreciates future consumption more than H does)

b2 In autarky, which country would have a lower interest rate? Please derive your answer <u>verbally</u>. [6 points]

Solution:

- F would have a lower rate (2)
- reason: rate equals inverse of discount factor which is higher in F (4)

  or:
  - in autarky, saving of the RC must be zero in each period (1)
  - because of its lower time preference, saving in period 1 tends to be higher in F (2)

- in order to push saving down (or: to compensate for the lower time preference), the rate of interest must be lower (1)

b_3 With international economic relations between H and F, will F be a net debtor or a net creditor at the end of period 1? Please derive your answer verbally.

[6 points]

Solution:

- F has lower consumption than H in period 1 (2)

  → F has current account surplus in period 1 (2)

  → F is net creditor in period 1 (2)

- Or: in the common financial market, interest rate will be higher than in autarky for F

  → F will increase its saving to a positive level and export the excess saving

  → F will be net creditor in period 1