Economic Growth

Problems Sets

Winter Semester 2025

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Problem Set 4

Preliminary remarks: Convergence analysis with GDP data

Before answering the questions I demonstrate the basic idea using an example with fictitious data from two countries. All GDP data are per capita data. Figure **??** shows the GDP time series. Level of GDP and average growth rate of country 2 are larger than the those of country 1. The main hypothesis of Barro-type convergence analysis is that

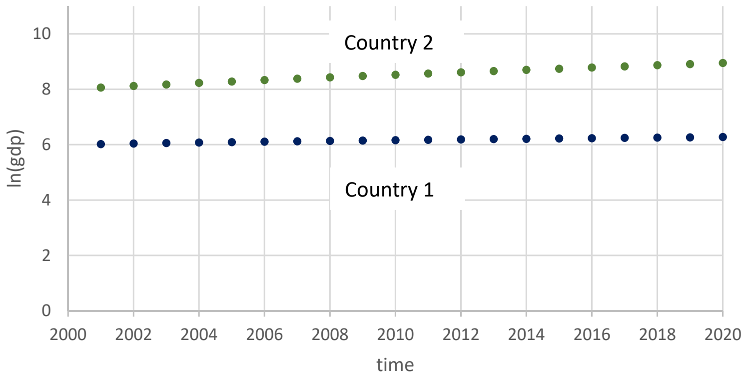

Figure 1: GDP time series

of a negative correlation between level and growth rate of GDP. Figure **??** clearly shows

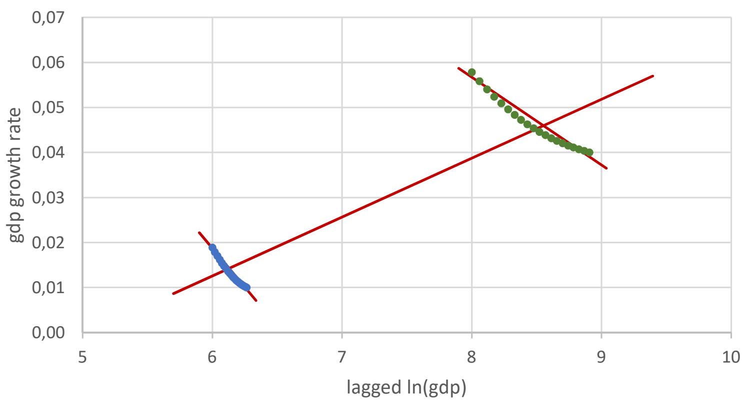

Figure 2: Country-by-country and pooled regression

a negative correlation between level and growth rate within each country. However, the regression of the pooled data shows a strong positive correlation. The $beta_1$-coefficient of the pooled data is positive. The reason is the segregation of countries of the kind mentioned above. The particular segregation of the two countries gives rise to the following consideration: If you consider the data relative to the respective country average the data would overlap. The country-by-country *beta*-convergence would merge into a picture of conditional *beta*-convergence of the whole sample. Technically speaking a fixed effect regression takes up this idea.

Figure **??** merges the data of country 1 and 2 by subtracting the country averages in levels and growth rates. In the picture we include the contry-by-country regression lines.

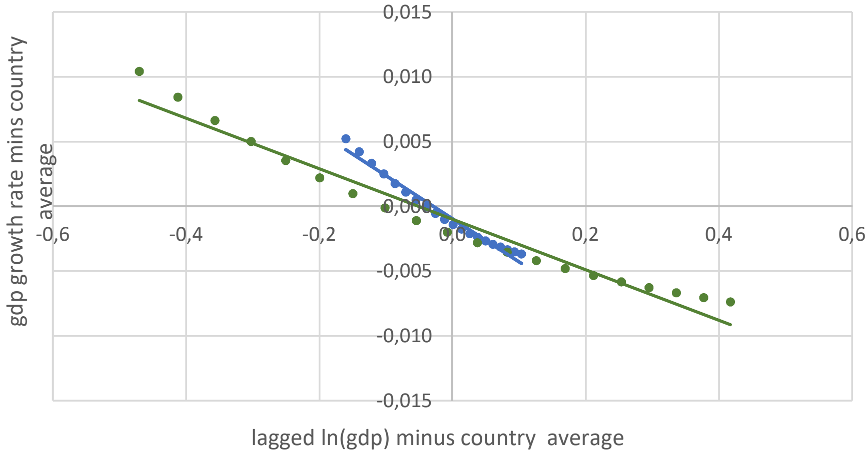

Figure 3: Country-by-country conditional regressions

Figure **??** shows the regression of the pooled conditional data.

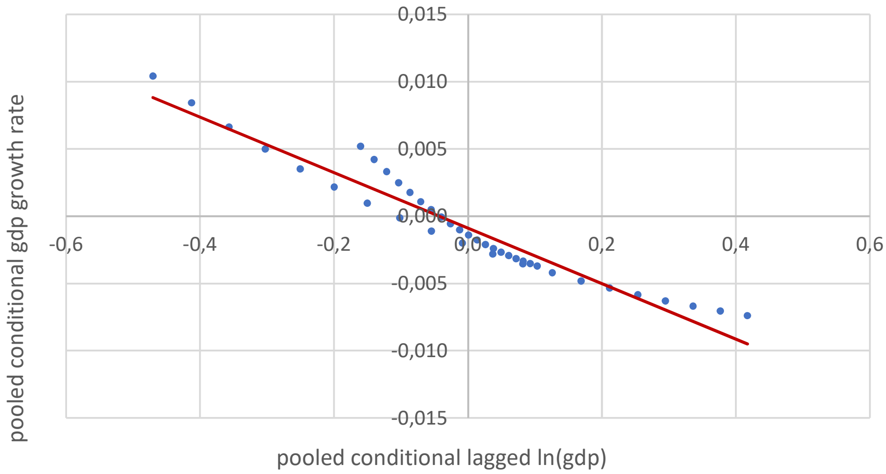

Figure 4: Pooled conditional regressions

Finally figure **??** shows the original GDP data with fixed effects regression lines. They have a common slope and country specific intercepts. I conclude these statements with

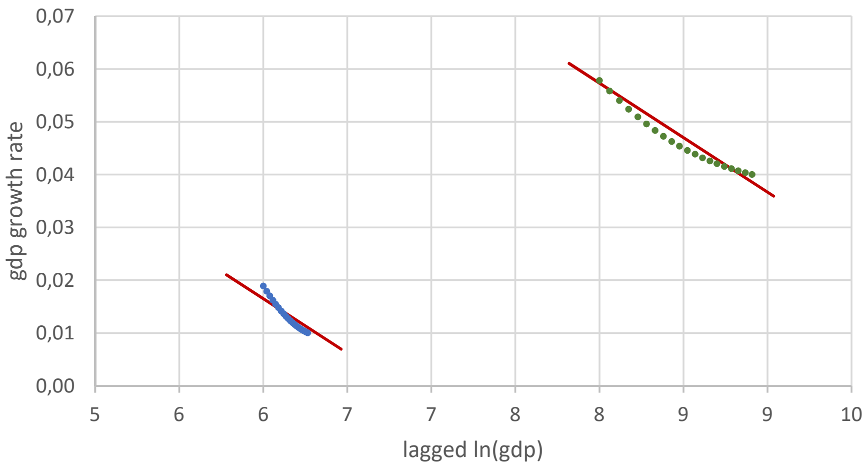

Figure 5: Pooled conditional regressions

two further notes:

- The fictitious data represent only two countries. Therefore, conditioning only makes sense country-by-country. In general it is a challenge in itself to identify groups of countries or perhaps regions with a common fixed effect.
- The fixed effect intercepts may be explained by further informations about the countries. At the same time we may assume that the mechanisms by which this data influences the economy are uniform. In the informal estimation the term $X'_{i,t}b^0$, with country specific, time varying information $X_{i,t}$ and a vector b^0 as a common (linear) mechanism represents this idea.

One can use the GDP data of the Penn World Table to analyze convergence with Barro-type regressions. Consider a data set with per capita GDP time series of a sample of countries.

Exercise 1

(1) Recall the system of structural equations for the sample with countries $i = 1, \ldots, n$ over a period of $t = 1, \ldots, T$ years

$$\ln y_{i,t} - \ln y_{i,t-1} = b^0 + b^1 \ln y_{i,t-1} + \varepsilon_{i,t}$$

Solution: On the left-hand side of the equation, you should recognize the GDP growth rate of country i in year t. The right hand side claims, that this growth rate depends linearly on the countrie's lagged GDP with an error term ε. The coefficients of the linear relationship are time independent and the same for all countries.

(2) Do you remember the OLS estimators for b^1 and b^0

$$\widehat{b^1} = \frac{cov(\ln y_{i,t} - \ln y_{i,t-1}, \ln y_{i,t-1})}{var(\ln y_{i,t-1})}$$

$$\widehat{b^0} = \overline{\ln y_{i,t} - \ln y_{i,t-1}} - \widehat{b^1}\, \overline{\ln y_{i,t-1}}$$

Apply the OLS estimation and interpret the results!

Solution: The OLS estimator for the slope b^1 is equal to the covariance between the independent and the dependent variable relative to the variance of the independent variable. (Hint: Be careful with the time lags if you apply the estimator.) I leave it to you, to choose appropriate data for an application.

(3) Do you see a reason to investigate conditional convergence?

Solution: In a sample of countries it is likely that you find no β convergence, i.e. the regression is gives no significant results. In particular β is not significantly negative. The reason may be that the countries' gdp time series converge but to different equilibria. Country specific intercepts can improve the results. They take care of country specific economic structures and at the same time the convergence mechanism (i.e. b^1) may be the same for all countries.

(4) Consider the country specific regression for conditional convergence

$$\ln y_{i,t} - \ln y_{i,t-1} = b_i^0 + b^1 \ln y_{i,t-1} + \varepsilon_{i,t}$$

Solution: The only difference is the country index of b_i^0. However this means, the ordinary OLS estimator can no longer be used. But there is a simple trick to cope with that (see below).

(5) Do you remember how to derive the country fixed effects estimators?

Solution: Form the averages country by country. (Take care of the lag structure! $\ln y_{i-}$ is the gdp series lagged by one year, i.e. running from 0 to $T-1$.)

$$\overline{\ln y_i} - \overline{\ln y_{i-}} = b_i^0 + b^1 \overline{\ln y_{i-}} + \overline{\varepsilon_i} \qquad \text{for country } i$$

Subtract the average equations from the original one and get an equation without fixed effects

$$(\ln y_{i,t} - \ln y_{i,t-1}) - (\overline{\ln y_i} - \overline{\ln y_{i-}}) = b^1 (\ln y_{i,t-1} - \overline{\ln y_{i-}}) + (\varepsilon_{i,t} - \overline{\varepsilon_i})$$

Notice that the term $\overline{\ln y_i} - \overline{\ln y_{i-}}$ reduces to $(\ln y_{i,T} - \ln y_{i,0})/T$ which is country $i's$ average growth rate over the period $1, \ldots, T$. (To be precise: it is $(T+1)/T$ times the average growth rate over the period $0, \ldots, T$.) In other words we regress the deviation of growth rates from the respective country's average growth rate on the deviation of logarithms of *gdp* from the respective country's average logarithm of *gdp*.

(6) Compute the common slope estimator $\widehat{b^1}$ from

$$(\ln y_{i,t} - \ln y_{i,t-1}) - (\ln y_{i,T} - \ln y_{i,0})/T = b^1 (\ln y_{i,t-1} - \overline{\ln y_{i-}}) + (\varepsilon_{i,t} - \overline{\varepsilon_i})$$

and the country specific intercepts $\widehat{b_i^0}$ according to

$$\widehat{b_i^0} = \overline{\ln y_{i,t} - \ln y_{i,t-1}} - \widehat{b^1}\, \overline{\ln y_{i,t-1}}$$

for each particular country i.

Solution: Whatever your choice of data is, the application of the estimators is straight forward.

(7) Compare your results with the unconditional estimators.

Solution: You will probably find an improvement only if you group countries according to economically reasonable criteria.