Economic Growth

Problems Sets

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Problem Set 3

Exercise 1

GDP data as provided by the Penn World Table can easily be presented graphically. Beyond that one can derive secondary data like yearly growth rates, average growth rates, and the like to characterize the growth performance of countries.

The exercises below look very technical. But the purpose rather is deepen the understanding of average growth rate concepts. Make sure that you understand what average growth rate means even if you have problems with the calculations.

Exercise 1 Consider the Penn World Table time series data of Bangladesh.

(1) Extract the time series of expenditure-side real GDP and of population size, and compute the corresponding per capita GDP data.

Solution: The excel-file `pwt1001_Bangladesh.xlsx` is available for download together with this pdf. It contains the required selection of data, some spread sheet calculations and the key figures as answers to the questions below.

(2) Display the per capita data graphically in decimal and in logarithmic scale. Add a linear regession line to both pictures. (Use the tools of your program and save on extra effort for explicit own calculations would require.)

Solution: This is done in the excel file. You find copies of the graphs at the bottom of this pdf. The data are displayed as a solid dark blue curves.

(3) Compute the average growth rate of GDP per capita between 1959 and 2019 according to the continuous time model.

Solution: In the continuous time model average growth rates of a time dependent variable y_t are computed according to the formula

$$\bar{\gamma}_{t_1-t_2} = \frac{\ln(y_{t_2}) - \ln(y_{t_1})}{t_2 - t_1}$$

In Bangladesh the average growth rate between 1959 and 2019 turned out to be $\bar{\gamma} = 1.81\%$.

(4) Add the fictional performance with constant growth rates fitting the original date at the beginning and the end of the time period in decimal and in logarithmic scale.

Solution:

- In decimal scale the formula for exponential growth with initial value y_0 and growth rate $\bar{\gamma}$ is
$$y(t) = y_0\, e^{\bar{\gamma}\, t}$$
If $\bar{\gamma}$ is computed correctly the function will match the data at the end of the period considered.
- In logarithmic scale the function above transforms into
$$\ln(y(t)) = \ln(y_0) + \bar{\gamma}\, t$$
It describes a straight line conecting the logarithms of the variable at the beginning and the end of the period considered.

(5) Explain the difference between the regression lines and the curves with constant growth rates?

Solution:

- In decimal scale the (linear) regression line is a line of form $y(t) = a + b\,t$. It minimizes the sum of squared deviations between the data and the line. Along the line the growth rate is not constant: $\dot{y}/y = b/(a + b\,t)$ which is decreasing over time.
- In logarithmic scale the (linear) regression line is a line of form $y(t) = a+b\,t$ as well, but for different parameter values. The growth rate along this line is equal to the slope of the line (cf. question 3)! Yet, it is not an average growth rate as the regression line does not match the (logarithms of) the data at the binning and the end point of the period considered.

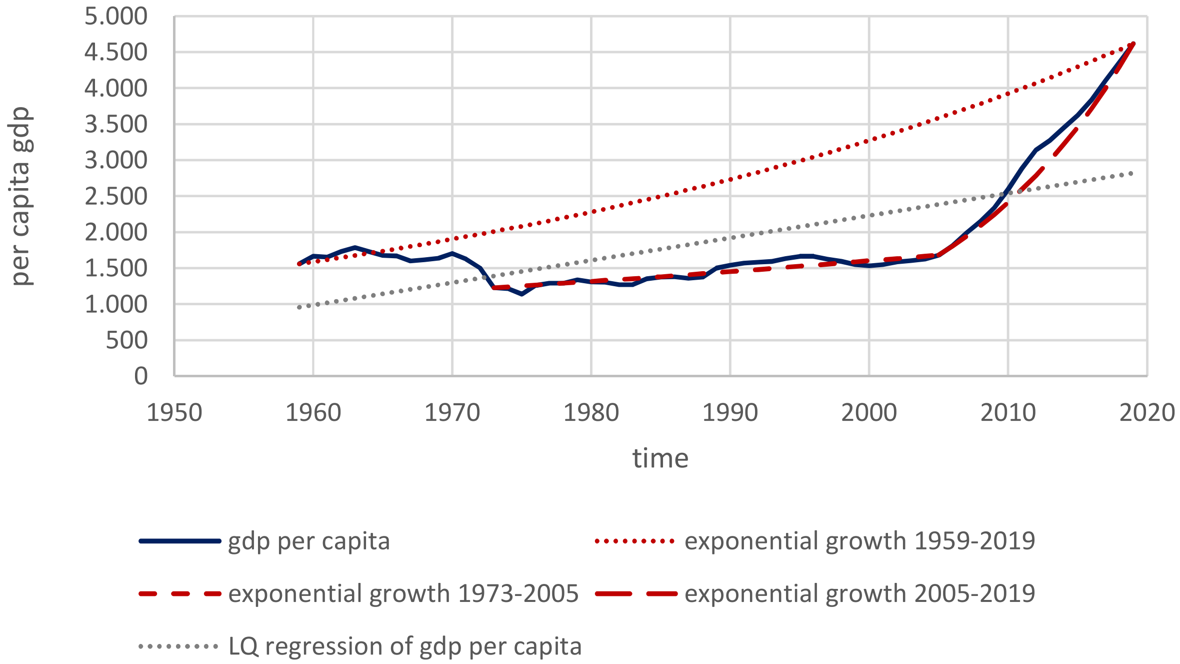
Altogether, the regression line in decimal data is of pure econometric value, whereas it corresponds to the constant growth rate model in the logarithmic scale picture. Yet, with its slope and intercept it corresponds to the constant growth rate model which minimize the average of squared deviations between the logarithms of the data and the regression line. The line is fitted to the data along the period considered. It is not fitted to the boundary points of the period.

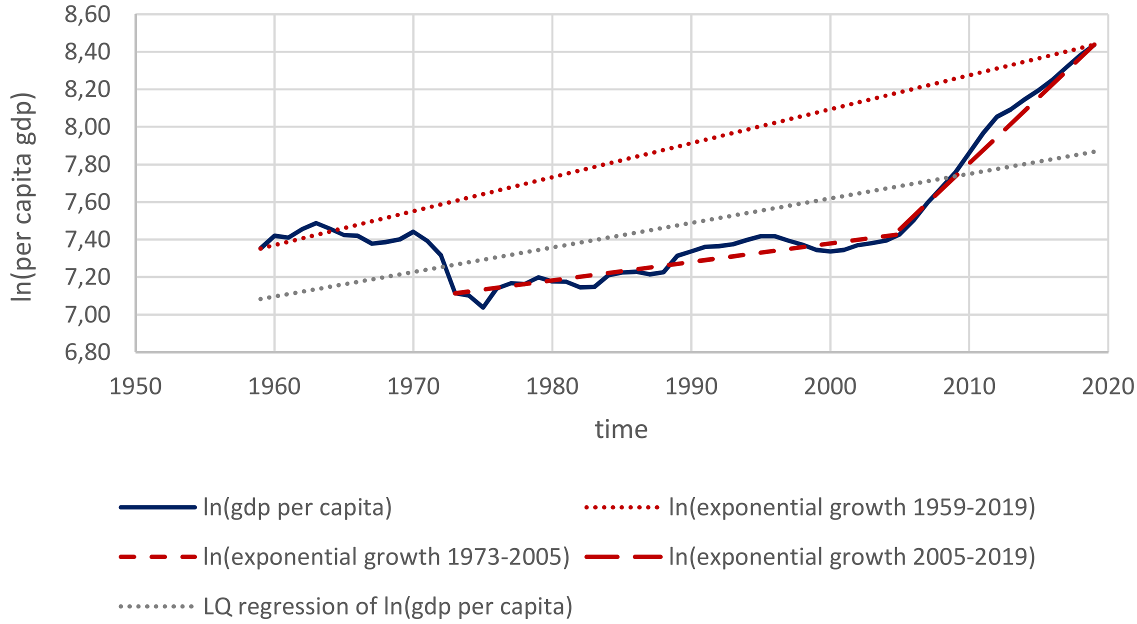
(6) Inspection of the data suggests to look at the subperiods 1973 - 2005 and 2005 - 2019. Fit the constant average growth rate curves in decimal and in logarithmic scales for these sub-periods.

Solution: Already a first look at the data suggests a structural break around the year 2005. Although we don't know the cause we can characterize its growth effect. To do so we devide the data in three periods[1]. In the first period (1959 - 1973) the data is quite erratic. We don't have a model for that. Between 1973 and 2005 growth seems to be steady with a more or less constant growth rate, but on a low level. Around 2005 the growth rate jumps on a higher level and stays on this level until 2019. For the second and the third period we can compute average growth rates and add this to the graphs in decimal as well in logarithmic scale.

The calculations are done in the excel-file. The curves are also added there.

[1] One can use econometric methods to identify the splitting points.


5.000
4.500
4.000
3.500
3.000
2.500
2.000
1.500
1.000
500
0
per capita gdp
1950
1960
1970
1980
1990
2000
2010
2020
time
gdp per capita
exponential growth 1959-2019
exponential growth 1973-2005
exponential growth 2005-2019
LQ regression of gdp per capita


8,60
8,40
8,20
8,00
7,80
7,60
7,40
7,20
7,00
6,80
ln(per capita gdp)
1950
1960
1970
1980
1990
2000
2010
2020
time
ln(gdp per capita)
ln(exponential growth 1959-2019)
ln(exponential growth 1973-2005)
ln(exponential growth 2005-2019)
LQ regression of ln(gdp per capita)

Additional remark: The average growth rate is the weigthed average of the growth rates of the consecutive sub-periods with their relative lengths as weights.