Economic Growth

Problems Sets

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Problem Set 2

Population growth

Labor is one of the most important factors of production. So let's take a look at some simple models of population growth. Let $N(t)$ be the size of the population, which can change over time. Let time run from zero to infinity, and $N_0 = N(0)$ the initial size of N.

Exercise 1 Assume that N changes with t in the following way

$$\dot{N}(t) = \nu N(t) \quad \text{or in short notation} \quad \dot{N} = \nu N$$

(1) What is the growth rate $\hat{N}$ of N?

(2) Is $N(t) = N_0\, e^{\nu \cdot t}$ the function matching the dynamic assumption, i.e. solving the differential equation?

Exercise 2 Now assume that N changes with t in a different way

$$N(t) = N^* + (N_0 - N^*)e^{-\beta t}$$

with some positive constant β.

(1) What is the growth rate?

(2) What is the differential equation $\dot{N} = \dots$ of this process?

(3) Describe the difference between the first model and the second in words!

Exercise 3 An empirically plausible formal dynamic model of N matching recent data is the logistic model of the following form

$$N(t) = \frac{N_{max}}{1 + \frac{N_{max} - N_0}{N_0}\, e^{-\nu t}}$$

with initial value N_0 as before and a maximum value N_{max}

(1) What is the limit of $N(t)$ for $t \longrightarrow \infty$?

(2) Draw a rough sketch of $N(t)$!

(3) Show that $N(t)$ as given above solves the following differential equation

$$\dot{N} = \nu N \left(1 - \frac{N}{N_{max}}\right)$$

(4) What is the limit of $\hat{N}(t)$ for $t \longrightarrow \infty$?