

Economic Growth

Problems Sets

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Barro Regressions

Structural equations

- (1) Recall the system of structural equations for a sample of countries $i = 1, \dots, n$ over a period of years $t = 1, \dots, T$.

$$\ln y_{i,t} - \ln y_{i,t-1} = b^0 + b^1 \ln y_{i,t-1} + \varepsilon_{i,t}$$

and the OLS estimators for b^1 and b^0

$$\hat{b}^1 = \frac{\text{cov}(\ln y_{i,t} - \ln y_{i,t-1}, \ln y_{i,t})}{\text{var}(\ln y_{i,t-1})}$$

$$\hat{b}^0 = \overline{\ln y_{i,t} - \ln y_{i,t-1}} - \hat{b}^1 \overline{\ln y_{i,t-1}}$$

- (2) consider the country specific regression for conditional convergence

$$\ln y_{i,t} - \ln y_{i,t-1} = b_i^0 + b^1 \ln y_{i,t-1} + \varepsilon_{i,t}$$

- (3) Derive the country fixed effects estimators

Solution: Form the averages country by country

$$\overline{\ln y_i} - \overline{\ln y_{i-}} = b_i^0 + b^1 \overline{\ln y_{i-}} + \overline{\varepsilon_i} \quad \text{for country } i$$

Subtract the average equations from the original one and get an equation without fixed effects

$$(\ln y_{i,t} - \ln y_{i,t-1}) - (\overline{\ln y_i} - \overline{\ln y_{i-}}) = b^1 (\ln y_{i,t-1} - \overline{\ln y_{i-}}) + (\varepsilon_{i,t} - \overline{\varepsilon_i})$$

Notice that the term $\overline{\ln y_i} - \overline{\ln y_{i-}}$ reduces to $(\ln y_{i,T} - \ln y_{i,1})/T$ which is country i 's average growth rate over the period $1, \dots, T$. In other words we regress the deviation of growth rates from the respective country's average growth rate on the deviation of logarithms of *gdp* from the respective country's average logarithm of *gdp*.