

Remarks on accumulation and growth I

2.1 Solow, Swan, Ramsey, Cass and Koopmans

1. The basic model with generic population growth

$$\begin{aligned}\dot{k} &= f(k) - c - (\hat{L} + \delta)k \\ \dot{c} &= \frac{1}{\theta} (f'(k) - \delta - \rho) c\end{aligned}$$

- Population growth is exogenous; the growth rate does not necessarily need to be constant.
- The differential equation for capital intensity reflects national accounting and accounts for population growth.
- The Keynes-Ramsey-rule recognizes the preference for intertemporal substitution of consumption.
- Notice that the Keynes-Ramsey-rule is independent of the population growth rate.

2. The implication of exogenous labor productivity growth

$$\begin{aligned}\dot{\tilde{k}} &= \tilde{y} - \tilde{c} - (x + n + \delta)\tilde{k} \\ \dot{\tilde{c}} &= \frac{1}{\theta} \left(\alpha \tilde{k}^{\alpha-1} - (\delta + \theta x) - \rho \right) \tilde{c}\end{aligned}$$

transforms back¹ into

$$\begin{aligned}\dot{k} &= f(k) - c - (n + \delta)k \\ \dot{c} &= \frac{1}{\theta} \left(f'(k/(E_0 e^{xt})) - (\delta + \rho) \right) c\end{aligned}$$

- Population growth is exogenous, and the growth rate is assumed to be constant. This assumption can be abandoned.
- The differential equation for capital intensity reflects national accounting and accounts for population growth.
- The Keynes-Ramsey-rule recognizes the preference for intertemporal substitution of consumption.
- Notice that the Keynes-Ramsey rule is independent of the population growth rate.
- Convergence needs to be revisited.

¹In growth rate the transformation of capital intensities is $\hat{k} = \tilde{k} + x$. In nominal changes this is $\dot{k} = \dot{\tilde{k}} \frac{k}{\tilde{k}} + xk$

2.3 Romer's model of endogenous technical progress

There is neither population growth nor depreciation.

$$\begin{aligned}\dot{k} &= A^\sigma u^{(1-\alpha)} k^\alpha \\ \dot{A} &= \gamma A(1-u)L \\ \dot{c} &= \frac{1}{\theta} \left(\alpha u^{(1-\alpha)} k^{(\alpha-1)} - \rho \right) c\end{aligned}$$

- There is a competing allocation of labor to generate output and to improve technology.
- The Keynes-Ramsey rule depends on the allocation of labor. This is how the allocation of labor endogenizes the growth rate of consumption.
- There is a constant rate of technical progress in the long run.