Games and Information

Problems Sets

Summer Semester 2025

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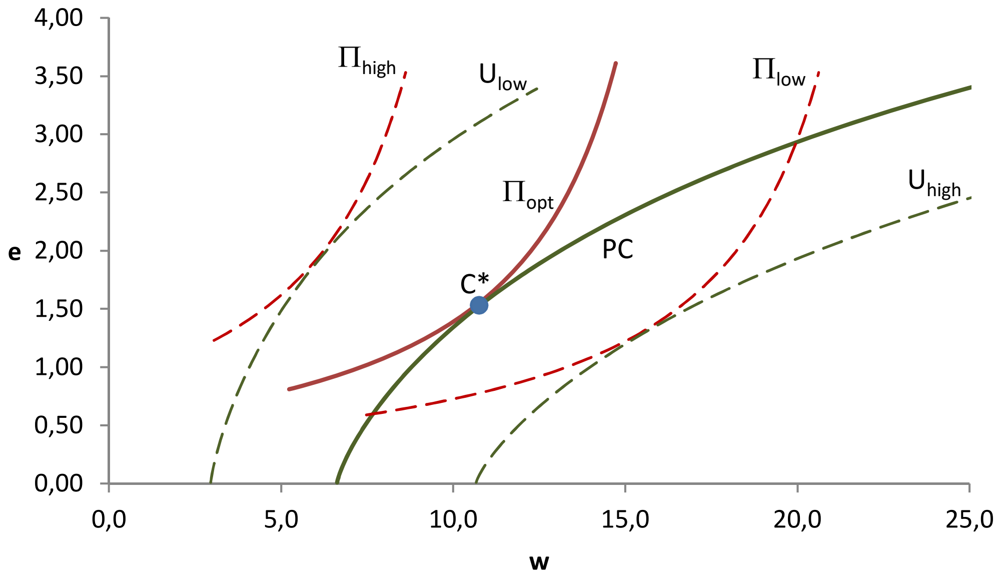
Problem Set 4

Exercise 6

The picture below shows isoquants, i.e. level curves of the agent's and the principal's objective function, for some selected levels.



(1) Recall the objective function (utility) of the agent. Pick any of the levels of utility U depicted in the graph and identify the domain (area) of contracts with utility larger than U and the one with utility smaller than U.

Solution: The picture does not refer to a particular type of agent. So, we use the generic form of utility and allow k to be equal to or larger than one.

$$U(w,e) = u(w) - kv(e)$$

At any given level of effort e, U increases with w. In other words, to the right of a level curve U is larger. Likewise you can express this in terms of a given level of w. U is larger below a level curve.

(2) Recall the objective function (profit) of the principal. Pick any of the levels of profit Π depicted in the graph and identify the domain (area) of contracts with profit larger than Π and the one with profit smaller than Π.

Solution: The function of expected revenue is

$$R(e) = \sum_i p_i(e)x_i$$

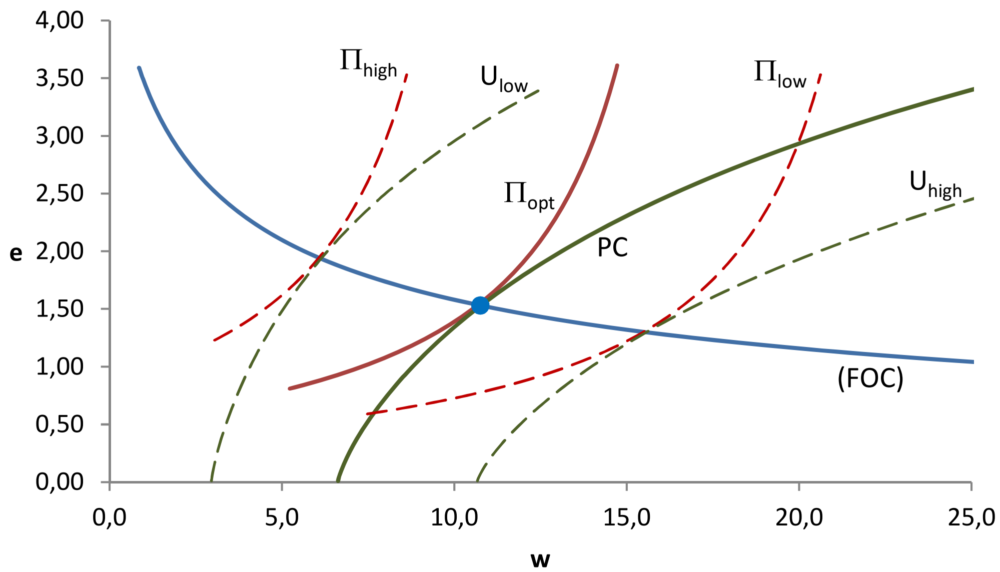
Assuming that effort rises the probability of high outcome whereas it reduces the probability of low outcome, expected revenue R will be an increasing function of e. By definition expected profit is

$$\Pi(e, w) = R(e) - w$$

To the left or above a level curve of Π expected profit is larger, whereas to the right or below a level curve it is smaller.

(3) Add a rough sketch of the (FOC) to the picture.

Solution: The (FOC)-curve is added in the picture below.



(4) Convince yourself by inspection of the picture, that the first order condition of the problem identifies the contracts such that you cannot increase profits without decreasing utility, nor can you increase utility without decreasing profit. Do you know the terms in your repertoire of microeconomic concepts which correspond to the curve of the first order condition (FOC)?

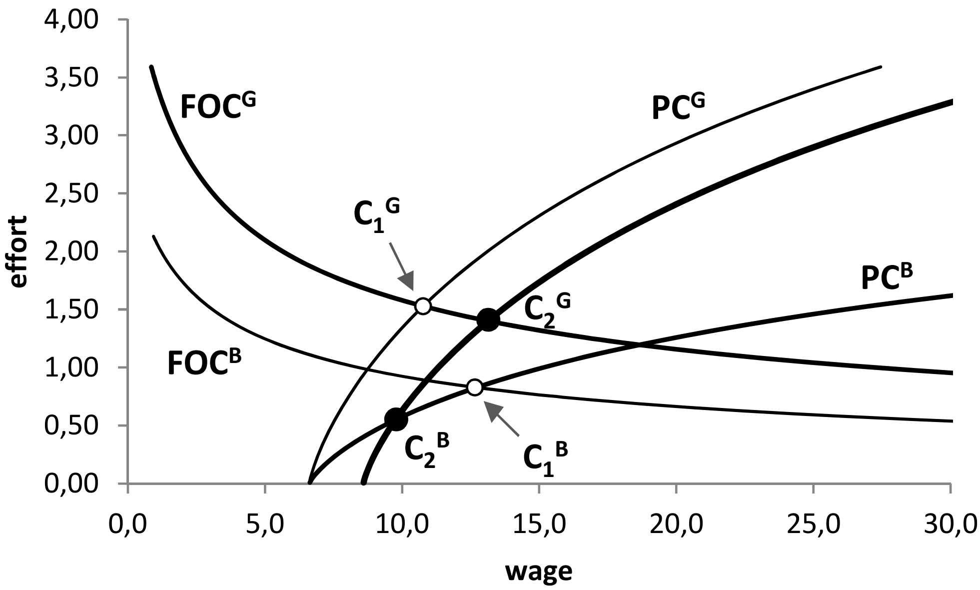
Solution: Domains of $U \geq \bar{U}$ as well as domains $\Pi \geq \bar{\Pi}$ are convex. For matching levels they touch on the (FOC)-curve and have no further points in common. In standard microeconomic terminology we call the points on the (FOC)-curve efficient or the corresponding pairs of wage and effort Pareto-efficient.

Notice that altogether this implies that along the (FOC)-curve from the upper left to the lower right or with increasing wage and decreasing effort expected profit decreases whereas expected utility increases.

Exercise 7

Consider the following picture of two menus of contracts a principal may offer to two types of agents

$$\mathcal{M}_1 = (C_1^G, C_1^B), \quad \mathcal{M}_2 = (C_2^G, C_2^B)$$



Recall, that for any given contract the expected profit of the principal does not depend on the type of agent.

(1) Does a shift from C_1^G to C_2^G increase or decrease the principal's profit or the agent's utility if the principal hires any type of agent?

Solution: The wage increases when switching from contract C_1^G to contract C_2^G while the effort decreases. Hence, the principal's expected profit decreases, and the agent's expected utility increases regardless of their type.

(2) Does a shift from C_1^B to C_2^B increase or decrease the principal's profit or the agent's utility if the principal hires a bad agent?

Solution: Imagine the curve of constant expected profit of the principal if he hires a bad agent tangent to (PC^B) at C_1^B. Clearly C_2^B yields a lower expected profit. Both contracts lie on the (PC^B)-curve. I.e. Bad agents are indifferent between these contracts.

(3) What will be the choice of agents when the menu $\mathcal{M}_1$ is offered to the agents or $\mathcal{M}_2$? Which menu is pooling which is (weakly) separating?

Solution: When $\mathcal{M}_1$ is offered, both types of agents clearly prefer C_1^B to C_1^G. C_1^B satisfies (PC^G) strictly and (PC^B) weakly. When $\mathcal{M}_2$ is offered good agents are indifferent between the two contracts of this menu willing to accept either one. Hence, they weakly prefer C_2^G to C_2^B. Bad agents strictly prefer C_2^B to C_2^G, because C_2^B satisfies (PC^B) whereas C_2^G does not.

Altogether, the menu $\mathcal{M}_1$ is pooling, and the menu $\mathcal{M}_2$ is weakly separating.

(4) Keep the choice of the agents in mind and check what a shift from menu $\mathcal{M}_1$ to $\mathcal{M}_2$ implies for the utility of agents and the efficiency of contracts.

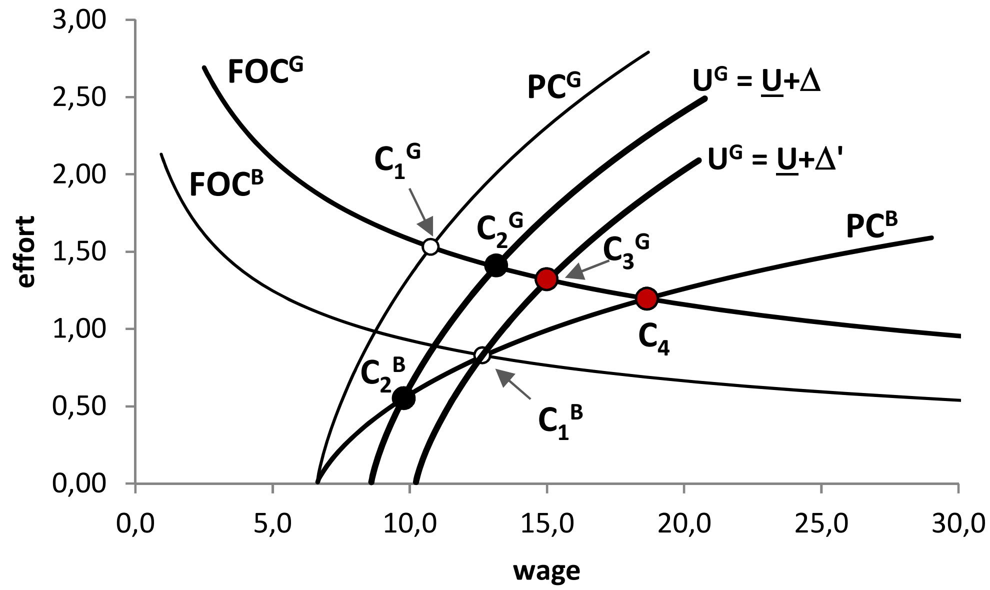
Solution: Remember that good agents choose C_1^B and not C_1^G when $\mathcal{M}_1$ is offered. When $\mathcal{M}_2$ is offered they are indifferent between C_2^G and C_2^B. We assume they choose C_2^G, the contract which is efficient for good agents. We can say they take the one that is made for them. Their expected utility still will be strictly larger than $\underline{U}$.

Bad agents do not experience any change in terms of expected utility and receive $\underline{U}$.

Exercise 8

In the following picture we add more menus of contracts. $\mathcal{M}_4$ is a "menu" that leaves the agents no choice

$$\mathcal{M}_1 = (C_1^G, C_1^B), \quad \mathcal{M}_2 = (C_2^G, C_2^B)$$
$$\mathcal{M}_3 = (C_3^G, C_1^B), \quad \mathcal{M}_4 = (C_4)$$



(1) Compare $\mathcal{M}_2$ and $\mathcal{M}_3$. Is $\mathcal{M}_3$ separating? Which contract yields higher expected profit (keeping in mind the preference of the agents).

Solution: In exercise 7 we found that in $\mathcal{M}_2$ good agents choose C_2^G whereas bad agents choose C_2^B, a weakly separating choice. Based on the same considerations as in the case of $\mathcal{M}_2$, the good agents in $\mathcal{M}_3$ choose contracts C_3^G and the bad agents C_1^B. In $\mathcal{M}_3$ good agents receive higher expected utility compared to $\mathcal{M}_2$. The expected utility of bad agents remains unchanged and still is equal to $\underline{U}$. Again, the choice in $\mathcal{M}_3$ is weakly separating.

Note: For the principal the comparison between $\mathcal{M}_2$ and $\mathcal{M}_3$ is not obvious! From $\mathcal{M}_2$ to $\mathcal{M}_3$ the expected profit from good agents decreases whereas the expected profit from bad agents increases (verify!). Which contract is better for the principal depends on the data of the problem and in particular share of good versus bad agents.

(2) Compare $\mathcal{M}_3$ and $\mathcal{M}_4$.

Solution: In $\mathcal{M}_4$ good agents receive the largest expected utility of all four menus. Bad agents are still indifferent receiving $\underline{U}$.

More advanced question: Why is $\mathcal{M}_4$ not the menu that promises the principal the greatest expected profit? Hint: You may argue on the basis of the graph above augmented by a particular curve of constant expected profit of the principal.