Games and Information

Problems Sets

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Problem Set 2

Exercise 3

Recall the *Follow the Leader* game with two states of nature as discussed in the lecture.

(1) Choose a reasonable notation for strategies and explain it.

Solution:

My favorites are

- Name moves conditional on paths which lead to some information set like $\sigma_{Jones} = (Large|Small,\ Small|Large)$.
- One may want to replace the path by explicit reference to the label of the information set like $\sigma_{Jones} = (Large|\omega_1,\ Small|\omega_2)$.
- Sometimes one may prefer a semantic description such as *Follow the Leader*, *Always Small*.

(2) Express the *Follow the Leader* Nash-equilibrium in your notation of strategy profiles.

Solution:

$((Small|A,\ Large|B);(Small|Small,\ Largel|Large))$. Obviously, the first strategy of the profile can only denote a strategy of *Smith*, the second one a strategy of *Jones*.

(3) The Perfect Bayesian equilibrium of this game is a strategy profile together with a set of Bayesian beliefs of player *Jones*. Reconfirm that the Bayesian belief of the *Follow the Leader*-strategy profile consists of zero-one probabilities only.

Solution:

The strategy of *Smith* in this equilibrium is one of the two fully informative or fully revealing strategies of *Smith*. The other one is $(Large|A,\ Small|B)$. Observing such a strategy *Jones* knows exactly what type of market they are in. Intuitively this can only mean that rational beliefs are zero-one. Bayes' formula is in line with this intuition. Whatever the move, one of the conditional probabilities *prob(a|s)* must be equal to 0, the other one equal to 1. Hence, the ratio of the Bayesian formula reduces to zero or to one. Obviously Smith needs to have enough different moves in order to show different behavior in different situations. This is the case in our game.

(4) Comment on the claim that the example with this pay-off structure is a robust version of a *Follow the Leader* game.

Solution:

Slight changes of the pay-off structure or the priors will not affect the equilibrium, i.e. neither strategies nor Bayesian beliefs will change. Beyond that, if player *Jones* makes slight mistakes in deriving beliefs, this will not affect the equilibrium strategies either.

(5) Imagine *Jones* is not sure how to use Bayes rule and tends to use intuitive beliefs instead of priors or Bayesian beliefs. Find the constraints for the intuitive beliefs that are compatible with the *Follow the Leader* equilibrium.

Solution:

At ω_1 or in other words after observing *Small* the best response of Jones is *Small* if and only if

$$\begin{aligned} \mu_{1A} \cdot 5 + \mu_{1B} \cdot 0 &\geq \mu_{1A} \cdot (-1) + \mu_{1B} \cdot 1 \\ 6 \cdot \mu_{1A} &\geq \mu_{1B} = 1 - \mu_{1A} \\ \mu_{1A} &\geq 1/7 \approx 0.14 \end{aligned}$$

At ω_2 or in other words after observing *Large* the best response of Jones is *Small* if and only if

$$\begin{aligned} \mu_{2A} \cdot 2 + \mu_{2B} \cdot (-3) &\geq \mu_{2A} \cdot 0 + \mu_{2B} \cdot 0 \\ 2 \cdot \mu_{2A} &\geq 3 \cdot \mu_{2B} = 3 - \mu_{2A} \\ \mu_{2A} &\geq 0.6 \end{aligned}$$

(6) Can you find priors (π_A, π_B) such that $\sigma_{Jones} = (Small|Small,\ Small|Large)$ and $\sigma_{Smith} = (Small|A,\ Large|B)$ form a Nash-equilibrium strategy profile on the basis of the priors π but not a Bayesian equilibrium profile?

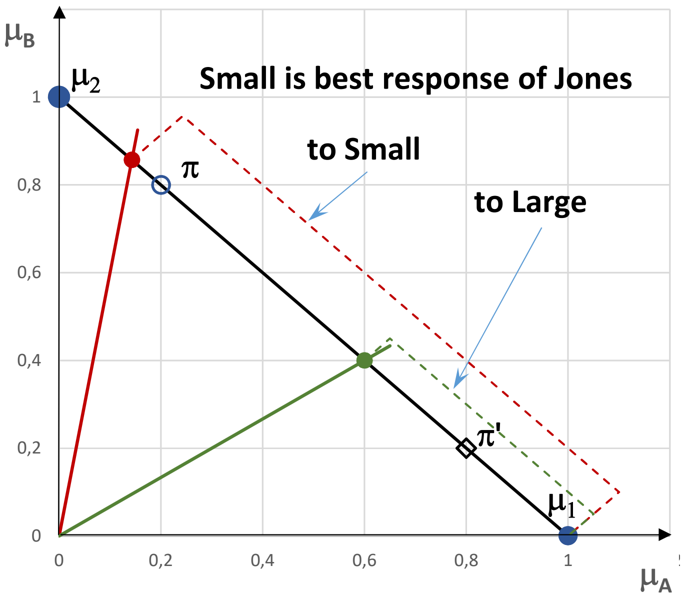
Solution:

- σ_{Smith} is a best response to σ_{Jones}! Indeed, in a type A market *Small* gives a pay-off of 5 to *Smith*, whereas *Large* gives only -1. In a type B market *Small* and *Large* give the same pay-off -1.
- Use beliefs to check for best responses and than discuss the beliefs:

  The beliefs $\mu_1 = (1, 0)$ and $\mu_2 = (0, 1)$ are the Bayesian beliefs of Jones given the strategy of Smith. Hence, with Bayesian beliefs *Follow the Leader* is the best response of Jones and not *always Small*. The priors we used in the lecture, namely $\pi = (0.2, 0.8)$ imply *Follow the Leader* as best response. The follow the leader equilibrium is Nash and Bayesian. If on the other hand the priors make A sufficiently likely, i.e. $\pi_A > 0.6$, $\sigma_{Jones} = (Small|Small,\ Small|Large)$ and $\sigma_{Smith} = (Small|A,\ Large|B)$ are mutually best responses with the priors. But not with the Bayesian beliefs!

  The Nash equilibrium with priors shows ignorance because *Jones* ignores the information revealed by *Smith's* strategy.

The graph below illustrates the situation:



We check:

- μ_A and μ_B add up to one.
- The components of the two priors π and π' add up to one.
- The red (steep) line flags the beliefs where Jones is indifferent between *Small* and *Large* at ω_1, i.e. after observing *Small*. To the right of this line *Small* is the better response to *Small*.
- The green (less steep) line flags the beliefs where Jones is indifferent between *Small* and *Large* at ω_2, i.e. after observing *Large*. To the right of this line *Small* is the better response to *Large*.
- With beliefs π *Small* is the best response of Jones to *Small* but not to *Large*.
- With beliefs π' *Small* is the best response of Jones to both, *Small* and *Large*.
- Hence, *allways Small* is *Nash* with priors π', but not *Bayesian Nash*.