UNIVERSITÄT SIEGEN Econ MEPS WIRTSCHAFTSWISSENSCHAFTEN WIRTSCHAFTSINFORMATIK | WIRTSCHAFTSRECHT

Games and Information

Problems Sets

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Problem Set 1

Exercise 1

Imagine, the MEPS alumni association EAAS is planning a day hike in the Siegerland in July. You know that according to long-term statistics the probability for a sunny day (s) is 20%, the probability for a cloudy but dry day (c) is 50% and otherwise it will rain(r).

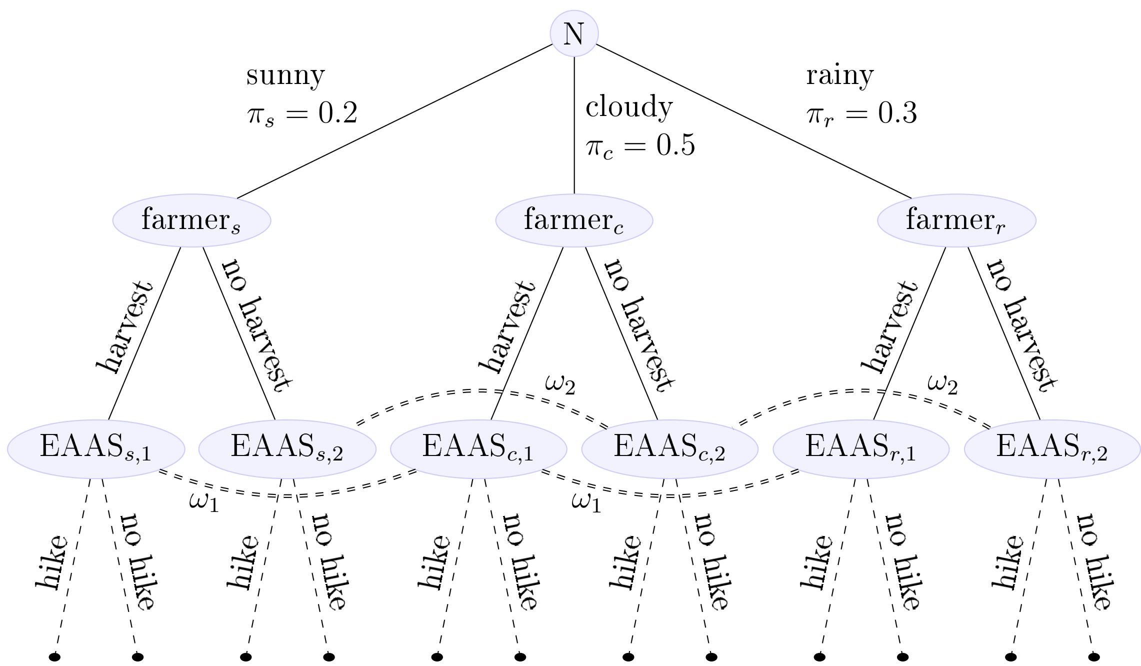
Farmers have their professional forecasting system. They receive private information about s, c or r a day ahead and all react in the same way. Early in the morning they choose between the option to harvest or not to harvest.

(1) What are the EAAS' priors π_s, π_c, and π_r?

Answer: Theay are $\pi = (\pi_s; \pi_c; \pi_r) = (0.2; 0.5; 0.3)$ using the most obvious order.

(2) Draw a game tree (without pay-offs, as we are going to investigate beliefs only)!

Answer:



(3) On the day before the trip is scheduled the weather is fine. You watch the farmers' weather channel on TV and either see the farmers harvest or not. (Remember, they all act the same way!)

(a) You know that they harvest only if it is going to rain the next day. Formalize the farmers' strategy and compute the Bayesian beliefs of player EAAS?

Answer: A simple way to denote a strategy of the farmers in this game is: $(0, 0, 1)$ where 0 stands for *no harvest* and 1 for *harvest*.

A simple way to denote beliefs for player EAAS is: $\mu_1 = (\mu_{1,s}; \mu_{1,c}; \mu_{1,r})$ in information set ω_1 and μ_2 correspondingly in ω_2.

Here farmers chose *harvest* only in case of r. $\mu_1 = (0; 0; 1)$. Farmers do not harvest in case c or r. We apply Bayes' rule to split the probability between the two cases: $\mu_2 = (\pi_A/(\pi_A + \pi_B); \pi_B/(\pi_A + \pi_B); 0/(\pi_A + \pi_B)) = (2/7; 5/7; 0)$

(b) You know that farmers are cautious and harvest the hay even if a cloudy but dry day is predicted. Formalize their strategy and derive player EAAS' beliefs!

Answer:

The strategy is : $(0; 1; 1)$

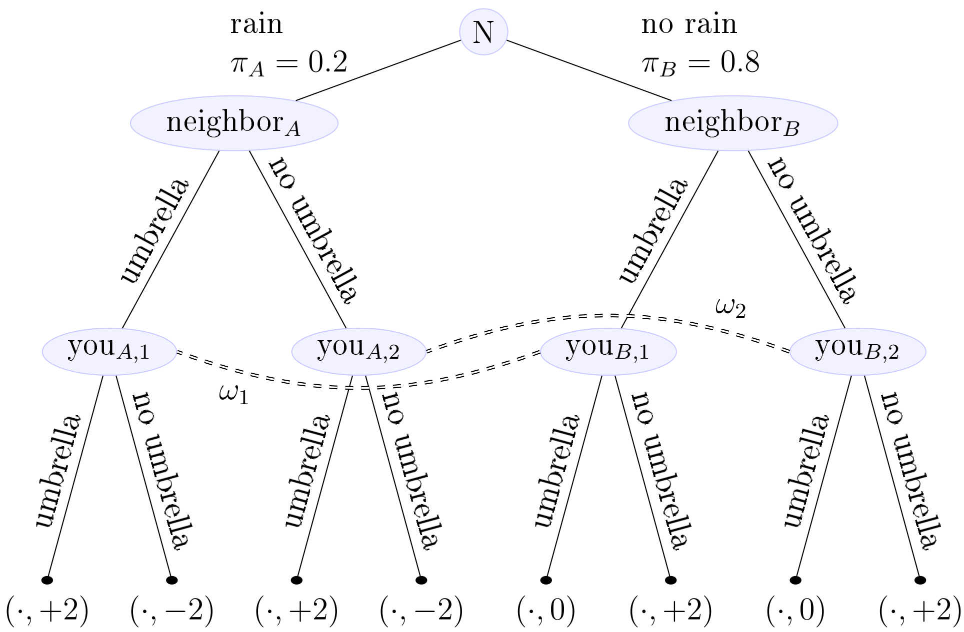
The Bayesian beliefs are: $\mu_1 = (0/(\pi_B + \pi_C); \pi_B/(\pi_B + \pi_C); \pi_C/(\pi_B + \pi_C) = (0; 5/8; 3/8)$ and $\mu_2 = (1; 0; 0)$

Exercise 2

Consider the following almost true story: You see your neighbor over the road of your house every morning when he walks to work. Sometimes he carries an umbrella, sometimes not. You have to leave right after him and each time you ask yourself whether you should take your umbrella or not.

Take a look at the game tree below and make sure that you understand how this game tree matches the story. (In the game tree we list only your pay-offs because we focus on your behavior as a reaction

The weekly long-term forecast is represented by π. The daily forecast is exact and reliable, say $\psi = (\psi_A, \psi_B)$, and $\psi = (1, 0)$ or $\psi = (0, 1)$. Try to form reasonable, i.e. possibly Bayesian beliefs about the real weather.



(1) What are your neighbor's strategies? Some of the strategies give you the chance to improve your information, some don't? Identify them and explain!

Answer: The strategies are:

(a) Always to take along the umbrella.

(b) Never to take along the umbrella.

(c) To take along the umbrella only if he knows it will rain.

(d) To take along the umbrella only if he knows it will not rain.

- Strategy (a) and (b) do not make a difference between the states of nature *rain* and *no rain*. Strategy (a) only leads to the information set ω_1 and leaves out ω_2. Strategy (b) only leads to the information set ω_2 and leaves out ω_1. We may call such strategies undifferentiated or uninformative.

- Strategy (c) and (d) are differentiated and informative. The actions taken under these strategies perfectly reveal the state of nature to you, the observer.

(2) What are your strategies?

Answer: Your strategies consist of pairs of actions, one action you take in ω_1 and one in ω_2. The list of (feasible) strategies is: $\sigma_1 = (yes, yes)$, $\sigma_2 = (yes, no)$, $\sigma_3 = (no, yes)$, and $\sigma_4 = (no, no)$. Interpretation: You always take along your umbrella, you always follow your neighbor (i.e. you only take along your umbrella if your neighbor does so), you never follow your neighbor, and you never take along your umbrella.

(3) Consider your neighbor's strategies which improve your information. What are your beliefs based on these strategies, and what are your best responses to these strategies?

Answer: Remember that your neighbor knows the daily forecast, whereas you only know the long-term forecast.

(a) Your neighbor's strategy is always to take along the umbrella which leads to ω_1 only. The strategy is not informative, i.e. we expect no updating through Bayes' rule. The strategy yields conditional probalities for *umbrella* conditional on either state of nature equal to one, $prob(umbrella|A) = prob(umbrella|B) = 1$ and the complementary conditional probabilities are equal to zero, of course: $prob(no\ umbrella|A) = prob(no\ umbrella|B) = 0$. In Bayes' rule we have to substitute the fundamental probabilities by priors and all conditional probabilities by one.

$$\mu_{A,1} = prob(A|umbrella) = \frac{1 \cdot \pi_A}{1 \cdot \pi_A + 1 \cdot \pi_B} = \pi_A$$

and $\mu_{B,1} = \pi_B$ is calculated in the same way. I.e. there is no updating in ω_1, the beliefs are equal to the priors: $\mu_{.,1} = \pi$.

In ω_2 you cannot apply Bayes rule. Off the paths compatible with your neighbor's strategy all conditional probabilities are equal to zero! Bayes rule would end in the evaluation of zero divided by zero. But there is no need to form beliefs for this case in this exercise!

Based on your beliefs you can compute your best response to (a). Your expected pay-offs will only depend on your plan in ω_1. Hence, $E\Pi(\sigma_1) = E\Pi(\sigma_2)$ which turns out to be $0.2 \cdot 2 = 0.4$. On the other hand, $E\Pi(\sigma_3) = E\Pi(\sigma_4) = 0.2 \cdot (-2) + 0.8 \cdot 2 = 1.2$. We conclude: σ_3 and σ_4 are best responses.

(b) Your neighbor's strategy is never to take along the umbrella. The reasoning for this case will be similar to the case above. There is no basis for an updating of priors in the relevant information set ω_2, i.e. $\mu_{.2} = \pi$. But not only your beliefs will be the same. Whether your neighbor chose to take his umbrella or not is completely irrelevant for your considerations.

(c) Your neighbor's strategy is to take along the umbrella if he knows it will rain. What are your beliefs, and what is your best response to your neighbor's strategy?

Answer: The conditional probabilities $prob(umbrella|A)$ and $prob(no\ umbrella|B)$ are equal to one and the complements equal to zero. I.e. you are perfectly informed. In ω_1 your Bayesian beliefs are $\mu_{A,1} = \frac{1 \cdot \pi_A}{1 \cdot \pi_A + 0 \cdot \pi_B} = 1$ which implies $\mu_{B,1} = 0$. In ω_2 you get $\mu_{A,2} = 0$ and $\mu_{B,2} = 1$. The "reasonable" bahavior of your neighbor implies that you follow your neighbor: σ_2 is your unique best response, and this can easily be confirmed by comparing the pay-offs.

(d) Your neighbor's strategy is to take along the umbrella only if he knows it will not rain. What are your beliefs, and what is your best response to your neighbor's strategy?

Answer: Your Bayesian beliefs are $\mu_{.,1} = (0, 1)$ and $\mu_{.,2} = (1, 0)$; you never follow the somewhat strange behavior of your neighbor: σ_3 is your best response. Now you will stay dry if it rains and your neighbor will be wet.