UNIVERSITÄT SIEGEN Econ MEPS WIRTSCHAFTSWISSENSCHAFTEN WIRTSCHAFTSINFORMATIK | WIRTSCHAFTSRECHT

Games and Information

Problems Sets

Summer Semester 2025

Karl-Josef Koch

Problem Set 3

Exercise 4

Recall the concept of stochastic dominance, as presented in the lecture.

(1) What is the formal definition that a distribution p^H for returns $x_1 \leq x_2 \leq \cdots \leq x_n$ stochastically dominates p^L of the first order?

(2) The distributions p^H and p^L give rise to continuous distributions $prob^H$ and $prob^L$ such that $prob^H\{x \leq \bar{x}\}$ is the probability that, according to p^H, the variable x assumes any value less than or equal to $\bar{x}$. Similarly, $prob^L\{x \leq \bar{x}\}$ is defined in relation to p^L.

Relate this definition to the statement

$$prob^H\{x \leq x_i\} < prob^L\{x \leq x_i\} \text{ for all } \; x > 0 \text{ and } x_i,\, i < n$$

Exercise 5

A very simple case of the moral hazard setting discussed in the lecture is the one with two effort levels $e^H > e^L$, and two levels of outcome $x_1 < x_2$. The outcome probabilities are p_i^H and p_i^L.

Moreover, let us consider specific utility and dis-utility functions $u(w) = \ln(w)$ and $v(e) = e^2$.

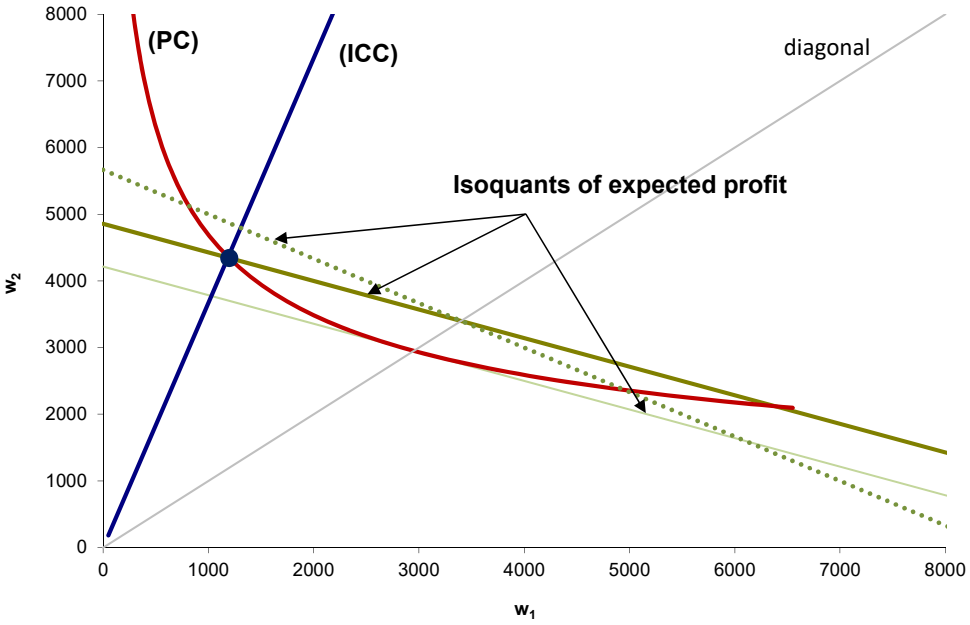
(1) Recall the condition for the principal to have higher expected revenue in case of high effort and reduce the inequality to a simple condition in terms of p_1^L and p_1^H.

(2) Assume the condition of 1. is satisfied, extend the comparison to the case of the principal's profit, and comment on the result.

(3) Write down the participation constraint (PC) for the utility and disutility functions specified above assuming that high effort is preferred by the principal.

(4) Write down the incentive compatibility constraint (ICC) for the utility and disutility functions specified above, and reduce it to a simple form.

(5) Interpret the graph below. In particular, mark the domain where the inequalities of (PC) and (ICC) are satisfied.


(PC)
(ICC)
diagonal
Isoquants of expected profit
8000
7000
6000
5000
4000
3000
2000
1000
0
0
1000
2000
3000
4000
5000
6000
7000
8000
w2
w1