

# A Dynamic Theory of Collective Persuasion\*

Athanasios Pitsoulis<sup>†</sup>

**JEL Classification** D71; D74; D78

**Keywords** Economic Models of Political Processes; Conflict; Positive Analysis of Policy-Making; Elections, Legislatures, and Voting Behavior

## Abstract

In this paper, debates and talks, aimed at persuading voters, between large groups in decision-making bodies are modeled as dynamic stochastic attrition-processes. We explore two methods of organizing interaction, plenary debate and confidential talks, which offer different ways to concentrate persuasive efforts on the opposition. Collective interaction is subject to Parliamentary Procedures, which are modeled as control structures over given sets of decision-making rights. It will be shown that the distribution of control rights has serious consequences for collective decision-making. Consent may be invariably depending on the distribution of procedural power between the groups, and may thus be engineered by exploiting decision rights. It will also be shown that collective persuasion may explain some apparent voting paradoxes.

## 1 Introduction

Democracy means government by discussion, but it is only effective if you can stop people talking.

CLEMENT ATLEE (1883-1967)

No one would deny that persuasive speech and rhetoric are important elements of deliberative democracy. Most people also agree that debates have to be regulated in a reasonable way in order to ensure a fair and orderly discussion. This is a necessary prerequisite to find a balance between the right to speak at as much length as seems desirable, and the right of parliament to make decisions. Yet, the role of institutions concerned with the regulation and organization of discussions and debates has long been neglected by economic science.

---

\* For useful comments and inspiring discussions on earlier versions of this paper I am indebted to Hans Juergen Schloesser, Ruediger Pethig, as well as my discussant Mark Schelker and several other participants of the annual meeting of the European Public Choice Society in Turku, Finland, April 20-23 2006. I would like to thank John Wilson, Ashley Piggins and other participants of the 8th INFER conference in Cork, Ireland, September 22-24 2006 for helpful suggestions. Financial support by the German Bundesbank (via the Verein fuer Socialpolitik) is gratefully acknowledged.

<sup>†</sup> University of Siegen, School of Economic Disciplines, Hoelderlinstrasse 3, 57068 Siegen, Germany. Phone +49 271 740 3146. Fax +49 271 740 2609. E-Mail: [pitsoulis@vwl.wiwi.uni-siegen.de](mailto:pitsoulis@vwl.wiwi.uni-siegen.de).

Observers of politics will without doubt concur with the thesis that democratic procedures can easily be exploited in order to manufacture consent. One could call this the ‘dark side’ of collective persuasion. Some strategies are rather unsophisticated, like abusing the right to cut short the debate and forcing a vote on a motion. But there are subtler stratagems. A majority can be manufactured by exploiting the fact that the outcome of a debate is invariably linked with *how* individuals interact. This paper’s aim is to explore these strategies and to show that the rules concerning interaction may even explain some apparent voting paradoxes, i.e. cases where “the relationship between the voting result and the voter preferences is counter-intuitive or unreasonable in some sense” (Nurmi 1998: 335).

## 2 Yokoyama’s static theory of individual persuasion revisited

A good starting-point to explore the dark side is Akira Yokoyama’s economic theory of persuasion (Yokoyama 1991). It covers persuasive activities aimed at solving conflicts through the changing of people’s preferences by the interchange of information, ideas or value statements. Yokoyama’s model is basically built around the idea that individuals have the means to deliberately create external effects aimed at manipulating values and preferences by appealing to “reason and emotion” (Yokoyama 1991: 102).<sup>1</sup>

Yokoyama throws a light on important and interesting aspects of individual persuasion. The model nicely reflects the central aspects of Buchanan’s “public reason perspective” of democracy (Buchanan 1954: 120). Yokoyama demonstrates the working of persuasion in two applications of his framework. He first discusses how individuals persuade the median voter in a collective decision-making situation concerning the supply level of a public good. In the second example Yokoyama explores persuasion as a solution for externalities concerning smoking in a society of two persons.

In this example two individuals, a smoker,  $A$ , and a non-smoker,  $B$ , are facing the problem of finding an equilibrium level for an externality  $Q$  through mutual persuasion (Yokoyama 1991: 107-13).  $A$ ’s optimal level is some  $Q_A > 0$  while  $B$ ’s optimal level is  $Q_B = 0$ . A social planner would choose a level corresponding to the equi-marginal sacrifice rule. The same outcome would be achieved in a negotiated solution with side-payments, as proposed by Coase (1960).

Yokoyama (p. 110) argues that in the absence of a social planner and side-payments the equi-marginal sacrifice rule is unlikely to be chosen, as no individual has an incentive to consider social welfare prior to its own. Behind the veil of uncertainty only the equi-total sacrifice rule would be mutually acceptable. Both individuals still have an incentive to induce variations on the optimal externality-

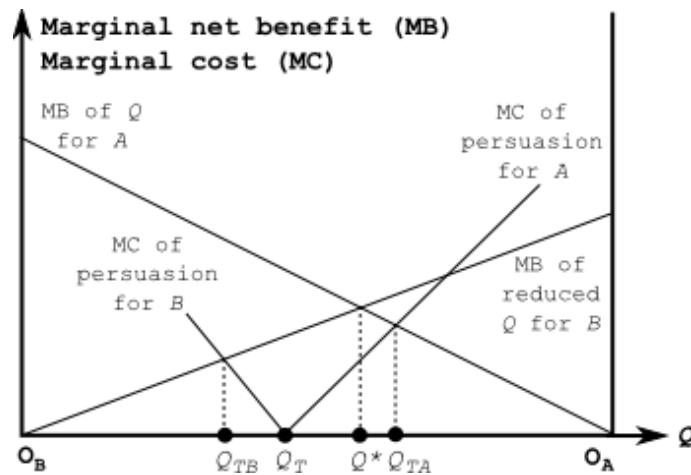
---

<sup>1</sup> These kinds of persuasion are known to the student of rhetoric as two of Aristotle’s three modes of persuasive discourse (Corbett 1990). The third is appealing to “ethos”, i.e. moral character, disposition, habit, or custom.

level derived by the equi-total sacrifice rule,  $Q_T$ . The probability for smoker  $A$  to successfully induce a variation under the equi-total sacrifice rule is  $\mu_{BA} = \alpha_A c_A$ ;  $\alpha_A > 0$ , that of non-smoker  $B$  is  $\mu_{AB} = \alpha_B c_B$ ;  $\alpha_B > 0$ . Persuasive effort is increased until the marginal benefits from a persuasion-induced variation  $\Delta Q$  equal marginal costs,  $MCP_A = c_A \Delta Q$ ;  $c_A > 0$  and  $MCP_B = c_B \Delta Q$ ;  $c_B > 0$ .

The strategic interdependence between competing persuasive efforts is not explicitly modeled, but the author notes that efforts in persuasive activities are likely to be strategic complements (p. 107). Assumed there is an equilibrium, four outcomes are possible (see Figure 1): (i)  $A$  successfully persuades  $B$  to accept the level  $Q_{TA}$ , (ii)  $B$  successfully persuades  $A$  to accept the level  $Q_{TB}$ , (iii) neither of them succeeds, or (iv) both are successful.

**Figure 1: Mutual externalities and persuasion**



Yokoyama's model covers the case of a two-person-society. Things do however become considerably more complicated if more than two individuals are involved. There are three main reasons: First, a persuasive contest in an  $n$ -person society is not simply a multiplication of the process of individual persuasion. In a two person world *public and confidential speech are identical*, but this may not be so in a world of many individuals. Secondly, the way individuals interact affects not only voting intentions but also the *intentions to persuade* others. Time and numbers are thus intertwined in a complex and sometimes counter-intuitive way. Things are, last but not least, complicated by the fact that in the real world the *time to make decisions* is limited.

This paper's purpose is to show that consent appears to be depending much on the rules of the debate. The aim is to extend Yokoyama's model to cover time-consuming persuasive activities between groups consisting of many individuals with the same preferences for the externality. The theory proposed here is thus an extension of Yokoyama's economic theory of persuasion. What it retains are its fundamental assumptions. What it modifies is the number of agents. What it adds is the dimension of time.

### 3 A dynamic theory of collective persuasion

#### 3.1 The microtechnology of collective persuasion

Imagine two groups engaging in a debate about the socially accepted level of  $Q_T$ . No one would deny that, *ceteris paribus*, a group can persuade more opponents if it has the better arguments (or rhetoric). And no one would deny that a group has, *ceteris paribus*, greater chances to win if it is larger. The chances to win are, however, not only depending on the persuasive potential of the own ideas, or the group size, but also on *how* the own group members are brought into play. This in turn depends from the rules regarding debate: Who has the right to organize interaction? What are the default or customary mechanisms? There are endless ways to pitch speaker-voters against each other. An open-ended public debate ensures that every individual hears the opinion of everybody else. The leader of a large group arguing a weak case does however have an incentive to avoid an unrestricted plenary debate when confronted with a small opposition making a convincing case for their position. On the other hand, a small group with good arguments (or rhetoric) wants to persuade enough opposition members to win a vote. It therefore has an incentive to plead its case in long plenary debates.

Several questions subsequently arise: The first is about the individual preferences. The second question is which social choice mechanism is used. The third question is how the individuals interact when they try to persuade each other. The fourth question is how the collective decision depends from the interaction between the individuals and from manipulation.

We will from now on regard  $A$  and  $B$  as homogeneous groups instead of individuals. Voice, in form of persuasion, is the only way to act, there is no exit option.

ASSUMPTION 1. A finite debate takes place in which individuals engage in persuading each other to accept their optimal level of  $Q_{Ti}$ ;  $i = A, B$ . Every individual is assumed to make Yokoyama-like persuasive attempts in regular and sufficiently short time-intervals as long the debate lasts.

As noted above, four outcomes regarding the level of the externality are possible each time individuals attempt to persuade each other.<sup>2</sup> But each time an individual has successfully been persuaded not only one vote for its group's motion is lost, but also one persuader. Let us explore debates in more detail. What characterizes persuasive appeals between members of groups? First, collective persuasion is in essence a chain of random events. To keep things simple I make the following assumption:

---

<sup>2</sup> The explanation of the individual persuasion success chance is of not interest here. There is already a large literature on the aggregation of information in committees (for a recent survey see Gerling et.al. 2005) and some interesting novel approaches on the intersection of economics and speech (e.g. Spiegler 2006). Notable are also the early works on the role of communication in Prisoners' Dilemma-experiments (e.g. Dawes/McTavish/Shaklee 1977).

ASSUMPTION 2. The random events reflecting the outcome of persuasive activities are stochastically independent.

Secondly, persuasive appeals may be either private or public.<sup>3</sup> Private appeals are typical for lobbying activities while public appeals are typical for parliamentary debate. In Yokoyama's model there is no distinction between private and public appeals because no such distinction is needed. It will be shown that it is absolutely crucial in the case of collective persuasion.

ASSUMPTION 3. Persuasive appeals are either made in public or in private. Private appeals are assumed to be made in such a way that they are exclusively noticed by the individual addressed. Public appeals are assumed to be made in such a way that they are noticed by every member of the addressed group.

In other words, it is assumed that the individuals can communicate information to other individuals in a way that eavesdropping on the private conversation is ruled out. The advantage of public appeals is that every persuader reaches every individual simultaneously. In the parliamentary tradition of democratic regimes the default mode of collective persuasion is open public discussion more often than not.

Let us first explore the effect of private appeals. Imagine two non-smokers trying to persuade two smokers about a reduction of the per capita level of smoke. The non-smokers address each of the smokers privately. There is a total of two appeals, each with a success-chance  $\mu_{AB}$ . The group of non-smokers is thus expected to persuade  $2\mu_{AB}$  smokers in the first period. Let us now assume one smoker has successfully been persuaded in the first period. In the second period both non-smokers can concentrate their efforts on the one remaining smoker. There again two appeals and the expected number of persuaded smokers is again  $2\mu_{AB}$ . Public appeals have a different effect under the assumptions made above. This makes essentially a total of four individual appeals, each with a success chance  $\mu_{AB}$ . The non-smokers, as a group, expect to persuade  $4\mu_{AB}$  smokers in the first period. Let us again assume one smoker were successfully persuaded in the first period. In the second period the non-smokers can not concentrate their efforts on the remaining smoker. The expected number of persuaded smokers is again  $4\mu_{AB}$ .

With confidential talks being the interaction mechanism the expected number of persuaded smokers relative to the number of addressed smokers is  $\mu_{AB}$  in the first period, but  $2\mu_{AB}$  in the second period. Groups relying on private appeals can concentrate their efforts but can only appeal to a number of opposition members at a time. With public appeals the expected number of persuaded smokers relative to the number of addressed smokers is  $2\mu_{AB}$  in the first *and* second period. Groups

---

<sup>3</sup> Note that interaction does not necessarily be oral. Persuasive attempts can be made via other kinds of media, like TV, blogs, e-mail, leaflets, etc.

relying on public appeals thus gain reach but sacrifice the possibility to concentrate their efforts on selected individuals. The same logic holds for an arbitrary number of individuals and leads to the following observation<sup>4</sup>:

COROLLARY 1. The number of individuals persuaded per period with private appeals is proportional to the own number of persuaders. The number of individuals persuaded with public appeals is assumed to be proportional to the product of the own number of persuaders and that of the opposition.

Free public speech is thus characterized by increasing *economies of scale* caused by a larger initial group size. The attrition caused by private interaction is characterized by constant economies of scale.<sup>5</sup>

Under the above assumptions a collective persuasion process breaks down to a series of stochastic duels between randomly selected individuals. This process has two main aspects: Firstly, collective persuasion has an element of conflict. It may, under circumstances, even be isomorphic to conflict.<sup>6</sup> While the ‘microtechnology of conflict’ affects how the committed forces of each side enter into determining the outcome of a battle (Hirshleifer 2000: 782) the *microtechnology of collective persuasion* affects how group size enters into determining the outcome of a collective persuasion process, like a debate.<sup>7</sup>

Persuasion may, secondly, not only result in manipulating the individuals’ willingness to vote, but also their willingness to persuade others. This is the reason why collective persuasion has *epidemiological* elements. Campaigning to persuade potential promoters of the own cause has elements similar to the transmission of disease. This aspect of the microtechnology of collective persuasion affects how heterogeneity enters into determining the outcome of the collective persuasion process. To keep things simple let me make the following assumption:

---

<sup>4</sup> For a formal proof see Morse/Kimball (1951): 69.

<sup>5</sup> This explains certain election campaign strategies. Campaigners e.g. recently came up with the following idea to get voters to the polls for the democratic primary election candidate Joe Lieberman in Connecticut. They hang “Vote Joe” signs on as many doorknobs as possible, and returned after a few hours. If the signs were gone by the time they returned they would go and hassle the voters, thus concentrating their forces (The Economist, August 12th 2006, 33). Unfortunately for Lieberman’s campaigners it was a windy day, and when the campaigners returned all the signs were gone.

<sup>6</sup> This explains why many terms used in the context of political competition are derived from military terminology. The word congress is for instance derived from the Roman word for combat, “congressus”. The term commission is derived from Roman military jargon, too, and refers to an official document issued by a government conferring on the recipient the rank of an officer in the armed forces. In the German tradition of consensual decision-making a vote on a highly controversial issue with a close margin is literally called a combat-vote (“Kampfabstimmung”). The words campaign and debate are derived from 13th century French: “campagne” marks an open country suited to military maneuvers and “debâtre” meant ‘to beat down completely’.

<sup>7</sup> One model concentrating on the fine-structure of such kinds of interaction is the Chase-Osipov-Lanchester attrition model. The model is better known as the Lanchester model, after British aeronautics pioneer F.W. Lanchester (1868-1945).

ASSUMPTION 4. A constant proportion  $0 \leq \beta_i \leq 1$ ; ( $i = A, B$ ) of group  $i$  individuals is of a type that will be engaging in persuasive activities directed at members of the own group after having successfully been persuaded. Individuals of this type are equally distributed within the population.

This means, in other words, that persuasion may induce campaigners to switch sides although their preferences over  $Q$  have not changed. Although it is a highly interesting question whether certain kinds of argumentation have this effect while others have not this issue is not taken up here.

The debate ends with a collective decision on the level of  $Q_T$ . Regarding the social choice rule I make the following assumption:

ASSUMPTION 5. A simple majority vote is taken on the conflicting motions regarding the intended per capita variation of  $Q$  at some point in time. Each individual is assumed to have one and only one vote.

As said earlier, collective persuasion is almost always subject to rules. Such institutional arrangements provide for a binding set of rules regarding control over the organization of meetings, debate etc. These rules are seen as “the procedural safeguard that protects the individual and the group in their exercise of the rights of free speech, free assembly, and the freedom to unite in organization for the achievement of common aims. These rights, too, are meaningless, and the timeless freedoms they define can be lost if parliamentary procedure is not observed.” (Sturgis 2000: 2) I want to concentrate on the most important of these rules, namely the duration of the debate (i.e. the right to fix the exact timing of a vote on a motion), and the right to organize the debate. Some terminology is needed<sup>8</sup>:

DEFINITION 1. Let  $L$  denote the set of the opinion leaders and the control set  $\tilde{L}$  denote a subset of  $L$ . Let  $V$  denote the right to decide on the timing of the vote and  $M$  the right to decide which of the available interaction mechanism will be used by both groups. Let the procedures (or rules of order)  $R$  denote a set  $\{V, M\}$ . Let  $\tilde{R}$  denote a subset of  $R$ . The mapping  $\varphi: \tilde{L} \rightarrow \tilde{R}$  shall be called the *control structure* over the procedures.

A control structure reflects, in other words, which opinion leader is in the position to exclude the other(s) from control over the procedures. If the procedures do not allocate control rights, there must be *default rules*, otherwise debate will degenerate into a chaotic shouting-contest. If there is an equal distribution of control rights both opinion leaders have a veto over each other’s decisions. We shall such a control structure symmetric:

---

<sup>8</sup> The control structure over the rights to take certain decisions with an influence over the outcome of the debate (and potentially the vote) is modeled here similarly to the control structure over a firm’s assets in the governance-theory of the firm literature. Indeed, in my model parties forming factions in a committee are treated as firms (i.e. networks of agents) ‘producing’ persuasion. Seen through this lens, control rights over certain procedural decisions indeed appear as important assets.

DEFINITION 2. A *symmetric control structure* is given if the control structure reflects an equal distribution of power, i.e.  $\varphi(\{\tilde{L} \mid \tilde{L} = L \vee \tilde{L} = \emptyset\}) = \tilde{R}$ .

Let us now assume the opinion leader of  $A$  were acting as chairwoman of the committee. If leader  $A$  alone is in charge of certain procedural decisions we shall call such a control structure exclusive:

DEFINITION 3. *Exclusive control* is given if the control set  $\tilde{L}$  contains exactly one opinion leader while the control structure includes both rights, i.e.  $\varphi(\{\tilde{L} \mid \tilde{L} = \{A\} \vee \{B\}\}) = \tilde{R}$ .

Exclusive control exists, in other words, when only one group leader has the right to exclude the opposition leader from procedural decisions. This reflects an unequal distribution of procedural power.

DEFINITION 4. *Mixed control* is given if there are two control sets  $\tilde{L}_V$  and  $\tilde{L}_M$  containing exactly one opinion leader and two control structures,  $\varphi_V$  and  $\varphi_M$ , assigning each leader different exclusive control rights, i.e.  $\varphi_V(\{\tilde{L} \mid \tilde{L} = \{A\}\}) = \{V\}$  in connection with  $\varphi_M(\{\tilde{L} \mid \tilde{L} = \{B\}\}) = \{M\}$  or  $\varphi_V(\{\tilde{L} \mid \tilde{L} = \{B\}\}) = \{V\}$  in connection with  $\varphi_M(\{\tilde{L} \mid \tilde{L} = \{A\}\}) = \{M\}$ .

This simple structure allows us to analyze democratic procedures with regard to the regulation of debates. Parliamentary rules of order usually contain strict regulations of this kind.<sup>9</sup> There are, of course, many more rights incorporated in such rule-systems, but one does not need to deal with all. Often rights are proxies for others: the right to impose an interruption of the session, a break or an adjournment amounts to a right to change the interaction mechanism from open public discussion to confidential talks. Why this may be of crucial importance shall be shown in the next sections, where we will explore ‘persuasive democracy’.

---

<sup>9</sup> The procedures of e.g. the Canadian House of Commons (see Stanford 1995 and Marlot/Montpetit 2000) contain detailed provisions concerning the length of members’ speeches during debate. In most cases, the maximum length of a speech is either 20 minutes or 10 minutes. However, there are exceptions. In some cases members can speak for an unlimited amount of time. In many cases, the Prime Minister and the Leader of the Opposition are not subject to any time limit. Certain rules exist, too, that allow the government to “curtail debate” in cases when it is felt a decision would otherwise not be taken in reasonable time, or not taken at all. The “time allocation rule”, for instance, allows for specific lengths of time to be set aside for the consideration of a bill. It can be used by the government to impose strict limits on the time for debate. In most cases, time is allocated in terms of sitting days or hours. When there is no agreement between the parties, the amount of time allocated may not be less than one day.



### 3.2 Open public discussion: plenary debate

Let us now explore the effect of persuasion on the equi-total sacrifice rule between two homogenous groups when the inter-group persuasive process is organized as open public discussion. It follows from Corollary 1 that each group faces an expected loss rate of unpersuaded campaigners proportional to the product of both groups' size in a plenary debate. The quadratic inhomogeneous system of differential equations

$$\begin{aligned}\dot{A} &= -\mu_{AB}A(t)B(t) - \mu_{AB}A(t)\beta_A[A_0 - A(t)], \\ \dot{B} &= -\mu_{BA}B(t)A(t) - \mu_{BA}B(t)\beta_B[B_0 - B(t)]\end{aligned}\quad (1)$$

gives an impression of the attrition caused by the public speeches of the opposition. Although not to be seen as a perfectly realistic model it offers some quite interesting insights into the interdependence between collective persuasion and majority votes. It reflects attrition between groups of swing-voters brought about by a 'playing tag'-like mechanism, combining the elements of conflict and the diffusion of individual characteristics in the populations.<sup>10</sup> For  $\beta_i = 0$ ;  $i = A, B$  the system (1) reduces to simple Chase-Osipov-Lanchester formulation.

The variables  $A(t)$  and  $B(t)$  reflect how many individuals are expected to *communicate* with the aim of persuading others at each point in time. But who is expected to *vote* for which motion? Of course, unpersuaded members of  $A$  will vote for their group's motion,  $Q_{TA}$ . So will those members of  $B$  which have successfully been persuaded. Vice-versa, both unpersuaded members of  $B$  and those members of  $A$  which have successfully been persuaded will vote for the motion  $Q_{TB}$ . The voting intentions  $V_A(t), V_B(t)$  are then given by

$$\begin{aligned}V_A(t) &= A(t) + [B_0 - B(t)], \\ V_B(t) &= B(t) + [A_0 - A(t)].\end{aligned}\quad (2)$$

Some graphs are shown in Figure 2. Here  $A_0 = 30$ ,  $B_0 = 20$ ;  $\mu_{AB} = 0.04$ ,  $\mu_{BA} = 0.01$  and  $\beta_A, \beta_B = 0.5$ . The expected values  $A(t)$  and  $B(t)$  are asymptotically approaching zero for  $t \rightarrow \infty$  while  $V_A(t)$  and  $V_B(t)$  are asymptotically approaching their equilibrium levels 30 and 20.

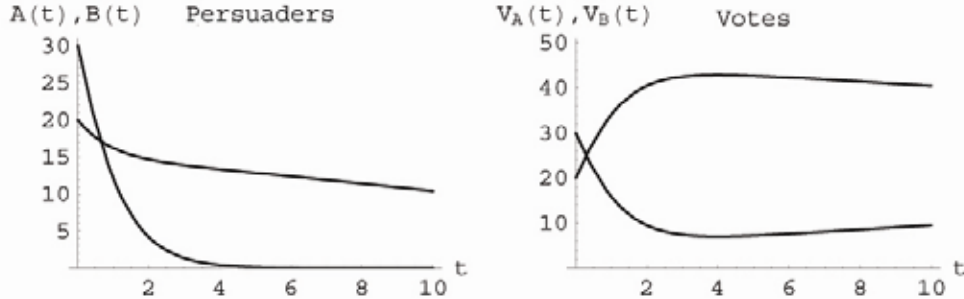
The debate can be decomposed into two phases: in the first phase of the debate  $A$  has a single majority for the own motion  $Q_{TA}$ . Voting in  $t^* = 0.2939$  would result in a draw. In the second phase of the debate  $B$  has a majority for  $Q_{TB}$ . Assuming  $t$  to be corresponding to hours, this would mean that after roughly 20 minutes of

---

<sup>10</sup> The general solutions and more details on the model can be found in Pitsoulis/Werthebach (2006).

plenary debate enough members of the incumbent majority  $B$  could be expected to be persuaded for  $A$  to win a majority vote.

**Figure 2: Open public debate**



A first observation is that the outcome of the vote is not independent from the control structure:

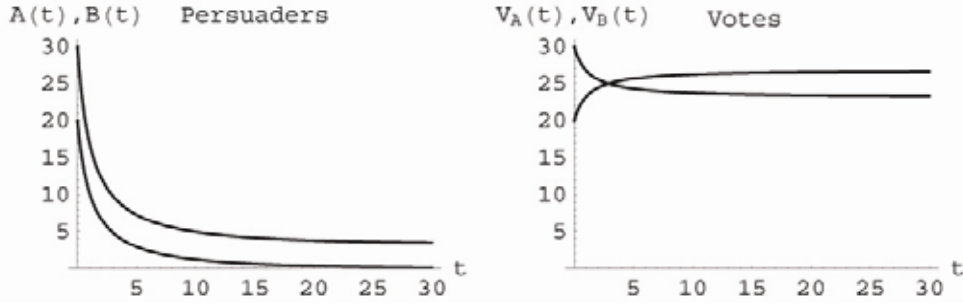
1. If the control structure is symmetric, the equilibrium levels are  $V_A^*(t) = 20$  and  $V_B^*(t) = 30$ .
2. Asymmetric control structures may result in non-equilibrium outcomes:
  - a. If the leadership of  $A$  can be excluded from decisions over the timing of the vote, faction  $B$  can win a majority vote by deciding to vote at some point in time in the interval  $]t^*, \infty[$ .
  - b. If the leadership of  $B$  can be excluded from decisions over the timing of the vote, faction  $A$  can win a majority vote in the interval  $[0, t^*[$ .

An interesting phenomenon is worth to be mentioned in this context:

**COROLLARY 2.** Interaction may result in *voting paradoxes* of the kind that an incumbent majority ‘wins’ the debate (i.e. completely persuades all opposition persuaders), but nevertheless loses the vote.

Such a paradox emerges for  $A_0 = 30, B_0 = 20$ ;  $\mu_{AB} = 0.04, \mu_{BA} = 0.03$  and  $\beta_A, \beta_B = 0$  (see Figure 3). Under these circumstances the outcome of the debate is a Pyrrhic victory in the sense that  $A$  sacrifices too many voters in the debate but can not compensate by persuading enough opposition members to vote for its proposal.

Figure 3: Voting paradox I



### 3.3 Confidential talks: private lobbying

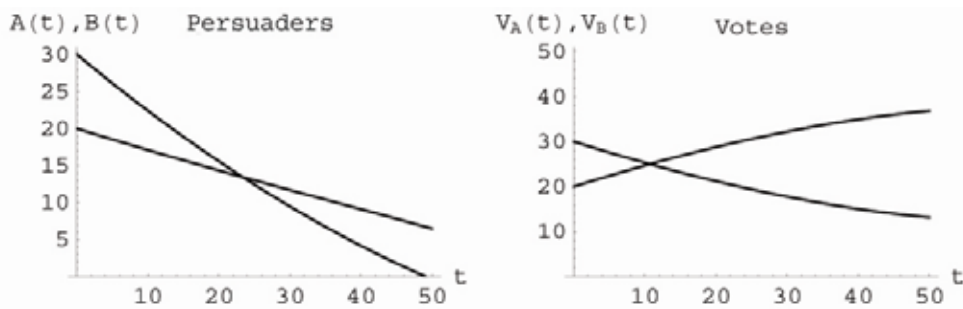
The observation that exclusive control rights over the timing of the vote may be crucial for the outcome is of course more or less trivial, but seen in connection with other interaction mechanisms some interesting cases do emerge. These cases shall be explored now.

According to Corollary 1 each group faces an expected loss rate of unpersuaded individuals proportional to the number of opposed agents in private talks. Assumed the persuasion probabilities are the same as in plenary debate (which in reality might not be so) the linear pseudo-inhomogeneous system of differential equations

$$\begin{aligned} \dot{A} &= \begin{cases} -\mu_{AB}B(t) - \mu_{AB}\beta_A[A_0 - A(t)] & : B(t) > 0 \\ 0 & : B(t) = 0, \end{cases} \\ \dot{B} &= \begin{cases} -\mu_{BA}A(t) - \mu_{BA}\beta_B[B_0 - B(t)] & : A(t) > 0 \\ 0 & : A(t) = 0 \end{cases} \end{aligned} \quad (3)$$

reflects the attrition caused by the private lobbying of the opposition (see Figure 4). For  $A_0 = 30, B_0 = 20$ ;  $\mu_{AB} = 0.04, \mu_{BA} = 0.01$  and  $\beta_A, \beta_B = 0.5$ , a vote in  $t^* = 10.6874$  would result in a draw. An earlier vote would result in a single majority for the motion  $Q_{TA}$ . A later vote would result in a majority for  $Q_{TB}$ .

Figure 4: Private talks (lobbying)



#### 4 Consent engineering: How to manufacture a majority

We have so far discussed the effect of Yokoyama-like collective persuasion between two groups under two typical interaction mechanisms. Open public debate between all individuals, on the one hand, satisfies the democratic axiom of free public speech. It is however characterized by increasing economies of scale in persuasion caused by a larger initial group size and is hence preferred by majorities. Lobbying in the form of confidential talks, on the other hand, violates this axiom, but is preferred by minorities. It is obvious that plenary debate will therefore be favored by the former when the aim is to persuade the latter.

In the previous examples faction  $B$  would win the vote independently of how the interaction was organized. Depending on the parameter constellation there may yet be different majority vote winners under different interaction mechanisms. Who wins the vote is then first and foremost a function of the allocation control rights, and only secondly of numbers and persuasive strength. The crucial question is How important is control over the organization of interaction?

CLAIM 1. The way the groups interact may be decisive for the outcome of the majority vote.

In order to verify the claim it is necessary to show that there are sufficient conditions under which interaction bears out to be the decisive factor. Procedural rules are then dispositive.

PROPOSITION 1. *Interaction determines the outcome of the collective persuasion process under the following circumstances: If*

$$\frac{\mu_{BA}}{\mu_{AB}} \frac{B(t)}{A(t)} < \frac{B_0^2 + 2\beta_A[A_0 - A(t)]B_0}{A_0^2 + 2\beta_B[B_0 - B(t)]A_0} < \frac{\mu_{BA}}{\mu_{AB}} \quad (4)$$

*party B wins in a plenary debate but A if interaction is private. If*

$$\frac{\mu_{BA}}{\mu_{AB}} \frac{B(t)}{A(t)} > \frac{B_0^2 + 2\beta_A B_0[A_0 - A(t)]}{A_0^2 + 2\beta_B A_0[B_0 - B(t)]} > \frac{\mu_{BA}}{\mu_{AB}} \quad (5)$$

*party A wins in a plenary debate but B if interaction is private.*

PROOF. In simplified notation, system (1) can be written as

$$\mu_{BA} B[A + \beta_B(B_0 - B)]dA = \mu_{AB} A[B + \beta_A(A_0 - A)]dB. \quad (6)$$

Integrate the LHS from  $A_0$  to  $A(t)$  and the RHS from  $B_0$  to  $B(t)$  while treating all other variables as constant. After rearranging we obtain

$$\begin{aligned}
& \mu_{AB}AB^2 + 2\mu_{AB}AB\beta_A(A_0 - A) - \mu_{BA}A^2B - 2\mu_{BA}AB\beta_B(B_0 - B) \\
= & \mu_{AB}AB_0^2 + 2\mu_{AB}AB_0\beta_A(A_0 - A) - \mu_{BA}A_0^2B - 2\mu_{BA}A_0B\beta_B(B_0 - B)
\end{aligned} \tag{7}$$

We are interested in the stalemate condition. A stalemate is reflected by setting the LHS of (7) to zero. The RHS of (7) can then be written as

$$\frac{\mu_{BA}}{\mu_{AB}} \frac{B}{A} = \frac{B_0^2 + 2\beta_A(A_0 - A)B_0}{A_0^2 + 2\beta_B(B_0 - B)A_0}. \tag{8}$$

Although it includes quadratic terms this is actually an expression of Lanchester's 'linear law' (for more information see e.g. Coleman 1982). Note that for  $A(t) = A_0$  and  $B(t) = B_0$  (8) reduces to the familiar formulation of the linear law  $\mu_{BA} / \mu_{AB} = B_0 / A_0$ . In the same manner we can calculate the stalemate condition for system (3). Not surprisingly, we obtain

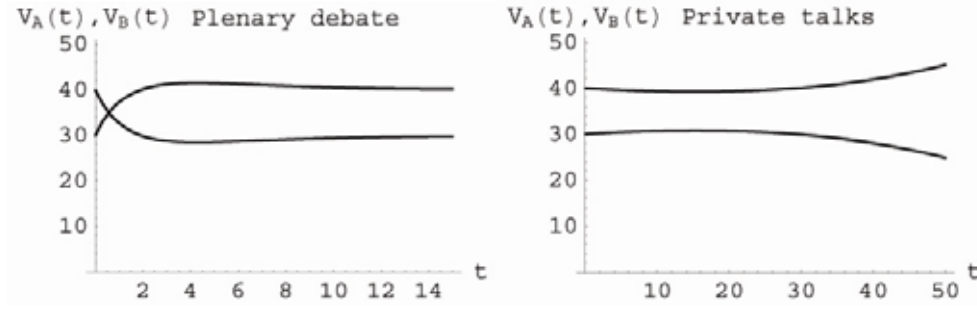
$$\frac{\mu_{BA}}{\mu_{AB}} = \frac{B_0^2 + 2\beta_A(A_0 - A)B_0}{A_0^2 + 2\beta_B(B_0 - B)A_0}, \tag{9}$$

which is another formulation of Lanchester's 'square law'. Note that for  $A(t) = A_0$  and  $B(t) = B_0$  (9) reduces to the familiar formulation of the square law  $\sqrt{\mu_{BA} / \mu_{AB}} = B_0 / A_0$ . If the LHS of (8) or (9) is greater than the RHS group  $B$  can be expected to win the collective persuasion process, if it is smaller, group  $A$  can be expected to win the collective persuasion process. ■

Note that Proposition 1 establishes only a sufficient condition for winning the majority vote, because interaction may still result in voting paradoxes like the one referred to in Corollary 2. The interpretation of (8) and (9) is that a faction's larger initial numeric strength and the presence of opposition defectors can offset a lower per-member persuasion probability and vice versa. Equation (8) shows that in plenary debate a faction initially has to be two times as effective to stalemate an opposition twice as large. Equation (9) shows that it has to be four times as effective if interaction is confidential.

Proposition 1 can best be explained by help of a short example. Assume  $A_0 = 40$ ;  $B_0 = 30$ ;  $\mu_{AB} = 0.03$ ;  $\mu_{BA} = 0.02$ ;  $\beta_A = 0.3$  and  $\beta_B = 0.5$ . In this case  $B$ 's initial numeric strength is 75% compared to that of  $A$ , but  $A$ 's persuasion probability is 67% compared to that of  $B$ . Who should win the vote? The initially larger group,  $A$ , or the one with the better arguments,  $B$ ? Interaction mechanisms link group size and persuasiveness of arguments. According to Proposition 1 we may expect  $A$  to win the collective persuasion process if interaction is private and  $B$  if interaction is plenary debate ( $0.67 < 0.76 < 0.82$ ). It is easily confirmed that the winner of the collective persuasion process wins the vote, too (see Figure 5).

Figure 5: Voting paradox II



Different combinations of group composition, interaction characteristics and control structures (it is assumed that plenary debate is the default interaction mechanism) will result in different majority vote results. These are summarized in Table 1. Interestingly, the incumbent majority faction  $A$  loses the vote under any symmetric procedural control structure, but wins under any mixed control structure.

Table 1: Control rights, interaction, and majority decisions

Procedural Control Structure	Interaction mechanism Default: Plenary Debate	Majority vote winner
Symmetric $\varphi(\{\tilde{L}   \tilde{L} = \{A, B\}\}) = \{V\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{A, B\}\}) = \{M\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{A, B\} \vee \tilde{L} = \emptyset\}) = \{V, M\}$	Plenary debate Plenary debate Plenary debate	$B$ $B$ $B$
Exclusive $\varphi(\{\tilde{L}   \tilde{L} = \{A\}\}) = \{V\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{B\}\}) = \{V\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{A\}\}) = \{M\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{B\}\}) = \{M\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{A\}\}) = \{V, M\}$ $\varphi(\{\tilde{L}   \tilde{L} = \{B\}\}) = \{V, M\}$	Plenary debate Plenary debate Private talks Plenary debate Dispensable Plenary debate	$A$ $B$ $A$ $B$ $A$ $B$
Mixed $\varphi_V(\{\tilde{L}   \tilde{L} = \{A\}\}) = \{V\}$ and $\varphi_M(\{\tilde{L}   \tilde{L} = \{B\}\}) = \{M\}$ $\varphi_V(\{\tilde{L}   \tilde{L} = \{B\}\}) = \{V\}$ and $\varphi_M(\{\tilde{L}   \tilde{L} = \{A\}\}) = \{M\}$	Dispensable Private talks	$A$ $A$

In this case consent is invariably depending on the distribution of procedural power between the groups. It may thus be engineered by abusing procedural power. Several strategies are usually employed to avoid plenary debate and enable a group to switch to private talks. The *strategic use of adjournments* is such a strategy. Adjournments or session breaks can be exploited with the aim of concen-

trating one's campaigners on opposition members.<sup>11</sup> The *organization of meetings* may, too, be exploited to control the interaction. This would explain why party leaders organize conferences for days on end to decide questions of principle. Such meetings open many possibilities to switch between the interaction mechanisms, alternating plenary debates and confidential talks, and thus many possibilities to influence the outcome of an interaction process. *Agenda-setter power*, too, may strategically be employed to arrange agenda items in such a way that debate is artificially curtailed, extended, or even avoided. Procedural power, truly, has a subtle 'dark side'.

## 5 A concluding remark

While the extension of Yokoyama's original model proposed here is simple and suggestive, rather than refined and definitive, it serves as a new framework by which the importance of collective interaction – and control over it – is exposed and is easier made the vehicle for further analysis. Factions within a committee, like a parliament or a jury, were modeled in analogy to firms producing voting outcomes by pitching their persuaders against those of the opposition. The link between group size and persuasiveness of arguments is formed by the interaction mechanisms, which were interpreted as microtechnologies of political competition. Interaction mechanisms differ significantly in the ability to concentrate one's efforts on the opposition campaigners. As a result, interaction mechanisms vary in their (dynamic) economies of scale in the persuasion of opponents.

It was shown that is not only essential to be in a position in which you can make people stop talking, but also to decide *how interaction is organized*. The fundamental differences between open public discussion and private talks may indeed prove essential for the outcome of the vote. They must be seen in connection with procedural arrangements like parliamentary procedures and rules of order. Usually, such rules confer rights to take certain decisions to a chairwoman or chairman or the majority speaker. It is the rule rather than the exemption that these procedures are inherited as part of tradition and not rationally designed.

Reference was earlier made to the procedures of the Canadian House of Commons. Another interesting example is Robert's Rules of Order and Parliamentary Procedure, which is the most commonly adopted parliamentary authority among political, literary, scientific, benevolent, and religious societies in the United States.<sup>12</sup> The preface summarizes the rules' purpose (Robert et.al. 2000: xix, italics added):

---

<sup>11</sup> The author himself has witnessed several cases where one faction's victory was snatched from the jaws of certain defeat by the skillful strategic use of adjournments and concentrated lobbying in between meetings. Further examples can be found in Pitsoulis/Werthebach (2006).

<sup>12</sup> US Courts have ruled that it is binding upon assemblies that have formally adopted it. The first edition was published in 1876 by General Henry Martyn Roberts (1837-1923), who was frustrated by the constant partisan confrontations over proper procedure in church meetings he presided over. Robert's procedures are loosely modeled after those of the US House of Representatives.

The object of Rules of Order is to assist an assembly to accomplish in the best possible manner the work for which it was designed. To do this it is necessary to restrain the individual somewhat, as the right of an individual, in any community, to do what he pleases, is incompatible with the interests of the whole. Where there is no law, but every man does what is right in his own eyes, there is the least of real liberty. [...] *Whether these forms be in all cases the most rational or not is really not of so great importance. It is much more material that there should be a rule to go by than what that rule is; that there may be a uniformity of proceeding in business, not subject to the caprice of the chairman or captiousness of the members.* It is very material that order, decency, and regularity be preserved in a dignified public body.

No one denies that rules of order are needed for effective government by discussion, but it seems that it is indeed of immense importance *what* the rules are, and whether they are the most rational. The reason is that they determine who is in charge of organizing interaction, and thus who may hold the power to manufacture a majority. Perhaps democracy is ineffective if no-one can stop people talking. But it may be ineffective, too, if someone can stop people talking.

## References

- Buchanan, J.M., 1954, Social Choice, Democracy, and Free Markets, *Journal of Political Economy* 62: 334-43.
- Coase, R.H., 1960, The Problem of Social Cost, *Journal of Law and Economics* 3: 1-44.
- Coleman, C.S., 1982, Combat Models, in W.J. Lucas (ed.), *Differential Equation Models*, New York: 109-31.
- Corbett, E.P.J., 1990, *Classical Rhetoric for the Modern Student*, 3rd ed., Oxford.
- Dawes, R.M., J. McTavish, H. Shaklee, 1977, Behavior, Communication, and Assumptions about Other People's Behavior in a Commons Dilemma Situation, *Journal of Personality and Social Psychology* 35: 1-11.
- Gerling, K., H.P. Gruener, A. Kiel, E. Schulte, 2005, Information Acquisition and Decision Making in Committees, *European Journal of Political Economy* 21: 563-597.
- Hirshleifer, J., 2000, The Macrotechnology of Conflict, *Journal of Conflict Resolution* 44(6): 773-92.



Marleau, R., C. Montpetit (eds.), 2000, House of Commons Procedure and Practice, Ottawa.

Morse, P.E., G.E. Kimball, 1951, Methods of Operations Research, New York.

Nurmi, H., 1998, Voting Paradoxes and Referenda, *Social Choice and Welfare* 15: 333-50.

Pitsoulis, A., G. Werthebach, 2006, A Dynamic Theory of Collective Persuasion, Manuscript.

Robert, H.M., W.J. Evans, D.H. Honeman, T.J. Balch, 2000, Robert's Rules of Order, 10th ed., Cambridge.

Spiegler, R., 2006, Argumentation in Multi-issue Debates. *Social Choice and Welfare* 26: 385-402.

Stanford, G., 1995, Bourinot's Rules of Order: A Manual on the Practices and Usages of the House of Commons of Canada and on the Procedure at Public Assemblies, Ottawa.

Sturgis, A., 2000, The Standard Code of Parliamentary Procedure, 4th ed., New York.

Yokoyama, A., 1991, An Economic Theory of Persuasion, *Public Choice* 71: 101-15.