Optimal (Capital)-Taxation and (Female)-Labor Force Participation –

The Importance of Labor Market Flows *

Philip Jung
Universiteit van Amsterdam

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Abstract

This paper asks how good actual US-fiscal policy was on average and over the cycle in the last 40 years compared to a Ramsey-optimal policy of a frictional labor market model. In our model labor taxes are intertemporally distortive, making the trade-off between capital and labor taxes interesting. Quantitatively, (optimal) taxation results hinge upon an estimate of the intratemporal elasticity of substitution of hours worked. We endogenize, controlling for gender, all transitions across labor market states, the "Ins and Outs of Unemployment", see Shimer (2007), to decompose this elasticity into the main labor market components. This decomposition shifts the focus away from a preference driven explanation of the elasticity to a broader concept that highlights technological and distributional aspects. We show that our model driven by a parsimonious shock structure can predict these elasticities and thereby all US-labor market flow data as documented in Shimer (2007) very well.

We find that the distortion in the labor force participation decision is important and needs to be counteracted by an optimal positive capital tax of 23%. If we allow for gender based taxes, see Alesina and Ichinol (2007), optimal capital taxes are almost zero but the female-male labor tax-ratio is 40%. The cyclical path of actual labor taxes correlates fairly close with the predicted Ramsey-optimal path.

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1 Introduction

How ”good” was actual fiscal policy over the last 40 years on average and over the cycle? Within the context of a neoclassical Ramsey-model the answer must be ”bad”, as discussed in Atkeson, Chari, and Kehoe (1999). Capital taxes were substantially too high on average and labor taxes have been far too volatile over the cycle.

This paper discusses whether the verdict changes if one considers a model with labor market frictions and a distorted labor-force participation decision. We show that the picture might differ considerably and explore the quantitative implications of this finding.

Labor market models of the Mortensen-Pissarides type can offer a new distortive element to the Ramsey problem because labor taxes can have, in general, an intertemporal distortive effect that is absent in the neoclassical world, where labor taxes are only intratemporally distortive. The trade-off between capital and labor taxes becomes more interesting. In our model this trade-off arises because, by the Ramsey assumption, the government can not tax workers who are out of the labor force and produce at home. When a worker decides to enter the main labor market she will trade the gains from working, typically the net wage, to her opportunity cost of time, which, under some assumptions, will be related to gross wages. The resulting wedge will have an intertemporal dimension by distorting the (forward looking) participation decision. Capital taxes are needed to counteract this distortion. However, positive capital tax results typically hinge upon assumptions on the available instruments.

We explore quantitatively to what extend gender based taxation, see Alesina and Ichinol (2007), will mitigate the burden of capital distortions.\footnote{In our model a simple progressive tax-code could essentially replace a gender-based tax code given that females work less than men in the market. A two bracket tax code could, therefore, achieve a similar goal!}

If labor taxes have a distortive effect on the entry decision on average the government might want to use its influence on the participation decision to smooth cyclical fluctuations, relaxing the constraint in recessions and increasing it in booms, thereby using inactive workers as a buffer stock to fluctuations. But how strong should this effect be?

To discuss the quantitative similarities of actual US-policy to the Ramsey-optimal one we first need a sensible description of the effects of fiscal policy on the margins of interest, that is on the ”Ins and Outs of Unemployment”, see Shimer (2007). These labor market transitions decompose the standard hour’s worked choice of the neoclassical model into its components and allows us to reinterpret the intratemporal elasticity of substitution that mainly governs the reaction to changes in labor tax rates in the neoclassical world. We show that the participation-elasticity is crucial and depends on three margins, the technological distance between home and market production, the substitution-elasticity...
between home-produced and market good and the heterogeneity in idiosyncratic preferences. We argue that the cyclical volatility of the transition rates convey useful information on these margins and provide a model that can structurally explain all of these flows jointly.

We proceed in 4 steps: First, we endogenize in a tractable way all transition probabilities in the Mortensen-Pissarides framework, taking explicit account of the movements from inactivity to unemployment and vice versa, as well as firing decisions and quits on the job. We utilize a discrete choice framework that allows us to deal in a straightforward manner with the particular truncations that typically complicate endogenous entry/destruction models. In particular, the model is continuously differentiable and open to Ramsey optimal policy using standard tools. Our model explicitly allows for the different labor market behavior across gender. This is essential when discussing the distortive element of labor taxes given that quantitatively females comprise a much bigger inactive group than males and they will be affected differently by changes in labor taxes than their male counterparts in general. As we will show the question how distortive labor taxes are depends on the efficiency differential between working at home and in the market. We utilize data from the recently published time-use survey of Aguiar and Hurst (2007) to identify this distance.

In a second step, we use observed measures for taxes, governmental expenditures and a TFP-measure as exogenous driving forces to compare the predictions of the model to the data for the years 1970:1 to 2004:4. Using the estimated exogenous shock processes as inputs we show that the model predicts all transition flows and other labor market variables very well. 2 We show that a structural interpretation of these flows is important to obtain an estimate of the crucial elasticities that govern the responses to tax rate changes, given that the cyclical response to shocks provides similar information.

We use the successful predictions of the model, in a third step, as an indicator that we capture in a quantitative sense important elements of the substitution elasticities and ask how a Ramsey optimal planner would have set taxes in this world. Overall we find a positive capital taxes of up to 23% in our benchmark case which suggests a quantitatively important distortion of the labor force participation margin. If we allow for gender based labor taxes, however, capital taxes were small (around 4%) and the tax burden would mainly be taken, optimally, by men. We find that the optimal female-male labor tax rate ratio is around 40%.

The quantitative impact of labor taxes suggests that the government can, potentially, use its influence on this margin to smooth out cyclical variation in employment to stabilize the business cycle. Using

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2In particular we are able to show that firing/separation rates, which we predict almost perfectly, and the job-finding probability are driven by the same common factor, the profit of the firm. This might be helpful in providing a structural interpretation on the importance of firing rates and job-finding probabilities, that has been debated between Shimer (2007) and ?.
the model we conduct, as a final step, a counterfactual experiment by asking how the labor market had looked like if the US-policy had followed a Ramsey optimal path. We show that the actual policy correlates fairly well with the actual one.

Overall, our results suggest that, quantitatively, actual US fiscal policy might not have been too far away from a constrained optimal policy along the margins considered in this paper. However, a relaxation of these constraints could lead to substantial labor market improvements.

1.1 Relation to the Literature

In the neoclassical Ramsey-literature capital taxes are typically zero, as summarized in Atkeson, Chari, and Kehoe (1999)\(^3\). Benigno and Woodford (2006b) and Benigno and Woodford (2006a) provide a time-consistent solution method to the basic stochastic neoclassical Ramsey-optimal policy model by restricting the choice set at period zero in a clever way. Their results can be viewed as a benchmark case which with to compare labor market matching models and we follow their approach. Correira (1996) shows that in the presence of a fixed factor capital taxes will be non-zero in general.\(^4\) In our model home-production can be viewed as partly taking this role. \(^7\) shows that in OLG-models capital taxes are also non-zero in general because, among other things, the intratemporal elasticity of substitution is changing over the life-cycle. Their insights are quantified in Conesa, Kitao, and Krueger (2005) who allow for a non-linear tax-code and find that capital taxes could be up to 30%. Even though our model does not feature a life-cycle structure and our mechanism is entirely different we find quantitatively a similar order of magnitude.

A very general result on optimal taxation is contained in the recent and interesting work by Albanesi and Armenter (2007) who provide a sufficient condition under which permanent (intertemporal) distortions will be zero for a broad class of models. Their idea is that the government can use a sequence of intratemporal disturbances to front load the distortions, building up enough governmental debt to pay of the expenditure-stream in the long run. These sequences of intratemporal disturbances are less distortive than a permanent intertemporal wedge. In our model their insights would translate into the result that both capital and labor taxes are zero, because they both have permanent effects. However, our model does not fall into the class of models they consider for two reasons. We assume a balanced budget rule, so the government does not have an intertemporal trade-off. This is assumed because governmental debt would induce a continuum of equilibria depending on the initial amount

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\(^3\)see Chari and Kehoe (1999) for a survey of optimal taxation in the neoclassical model, and Judd (1985) and Chamley (1986) as the classical references.

\(^4\)This result has been criticized in Armenter (2007) who shows that the result is fragile to assumptions on the incentive constraint in period zero.
of debt, which would be very hard to identify given the complexity of the model numerically. But this is not the reason why their results do not immediately translate to our model. In fact it is easy to obtain a zero capital result in our model once the intertemporal distortion is shut down. More important is that there is no intratemporal margin in the model that can be used because labor taxes are intertemporally and intratemporally distortive. Given that in the standard Ramsey problem only capital taxes are zero, while labor taxes are typically not, the same restrictions that give raise to a limiting positive labor tax will give an incentive for the government to tax capital as well.

Within the context of the standard labor market matching model (without endogenous participation or wage distortions) Domeij (2005) shows that capital taxes are zero only when the model economy is evaluated at the Hosios condition, see Hosios (1990). It is positive or negative once we move away from this efficient allocation. Quantitatively though the deviations from zero are rather small and might be driven to zero once a vacancy posting subsidy is considered. In contrast to the model in Domeij (2005), our model does feature an intensive hours worked margin.\footnote{Domeij (2005) features dis-utility of work, but it is a function of the number of workers only, not a standard intensive hours worked choice.} This implies that the basic form of the Hosios efficiency can not be applied,\footnote{The condition states in general that efficiency in matching models can be obtained if each party is precisely compensated for its contribution to the formation of the match. The matching externality arises because, when making their decisions, firms and workers do not take into account their 'congestion' effect on labor market tightness. If the Hosios condition holds, this externality can be internalized and efficiency is restored. In the standard model with a Cobb-Douglas matching technology the condition implies that the bargaining power of the worker has to be equal to the matching elasticity. But once an endogenous intensive margin, hours worked, is included, the planner’s solution would be based on direct transfers between the family and the government, leaving the hours worked margin undistorted, while the decentralized economy with a positive labor tax will distort the hours worked choice in general.} so part of our positive capital taxes might be due to the search friction, not to the labor force participation friction. However, we show that in the absence of an intertemporal distortion, the search friction induces a negative capital tax of only -0.4%, which is quantitatively negligible. Our results are not driven by the search externality.

Another positive capital tax result within the matching framework has been obtained in Arsenau and Chugh (2006). They show numerically that if labor taxes can influence the surplus of the firm (which typically is the case in a generalized Nash-Bargaining solution with a fixed outside option) capital taxes are positive in the long run. The intuition for their positive capital tax result is similar to the one in this paper. In their setup the labor tax rate influence the outside option non-proportionally which implies that the value of the match from a firms perspective is a function of labor tax rates.\footnote{This effect is ruled out in Domeij (2005).} This in turn makes labor tax rates immediately intertemporally distortive because the intertemporal

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\footnote{This effect is ruled out in Domeij (2005).}
decision to post vacancies and hire a new worker, the free entry condition, is a (linear) function of the expected profit of a match. If the government increases labor taxes and decreases capital taxes, it faces a trade-off between decreasing the stock of employed workers and increasing the stock of capital. In the absence of additional instruments capital taxes will be positive in general. A similar effect is used in Costain and Reiter (2005b) within the context of optimal cyclical policy, who show that the government can and should reduce cyclical volatility of the unemployment rate by half. They obtain this effect essentially by influencing, via tax rates, average capital over the cycle which in turn induces a change in unemployment rates by influencing the profits of a firm. In both cases the assumptions on the particularities of the bargaining process drive the results on optimal capital taxation. However, the same assumptions would also imply that observed unemployment rate fluctuations were affected by actual US tax rate changes. We do not find evidence for this effect, so our bargaining setup ensures that in our model there is no first order effect of labor tax rates on the unemployment rate. Therefore all results on optimal taxation can be traced to the labor force entry margin we wish to stress in this paper.

This margin has been analyzed in Garibaldi and Wasmer (2005) for average transition rates and in Haefke and Reiter (2006) for cyclical variations. We extend part of their insights to a considerably richer general equilibrium framework that is open to Ramsey-optimal taxation. This framework is also helpful in quantifying the effects of gender based taxation highlighted in Alesina and Ichinol (2007) given that, as we will show below, frictional labor market models add an important decomposition to the standard labor-leisure trade-off and the associated estimates on the intratemporal substitution elasticity, see ? for a survey.

Quantitatively our model relies on the ability of the Mortensen-Pissarides framework in explaining the “Ins and Outs” of unemployment, see Shimer (2007). Our model builds upon the literature that tries to explain the puzzle observed in Shimer (2005) and on the resolutions that have been proposed in, among many others, Hall and Milgrom (2007), Costain and Reiter (2005a), Shimer (2005), Jung (2005) and Hagedorn and Manovskii (2006). We expand the model to be consistent with all “Ins and Outs of Unemployment” as measured in Shimer (2007), taking explicit account of quits on the job, firings and the flows from inactivity to unemployment. To our knowledge this paper is the first that provides a structural framework in which all these flows are addressed and estimated.

The paper proceeds as follows. In section I we give a simplified version of our, rather complex, main model to show some simple results which will be confirmed in the context of a quantitative model and to build the basic intuition for the results to follow. Section II presents the main model and derives
some non-standard feature due to the discrete choice structure of the model. Section III explains our calibration strategy and shows how the model performance. Section IV discusses long run optimal capital taxation where we focus exclusively on steady states. Section V gives the corresponding cyclical properties of the model under a Ramsey-optimal rule. Section VI concludes.

2 Intuition

Before providing the full model with unemployment and labor-force participation we shortly outline the basic intuition for the results to follow and spell out the importance of different assumptions that drive our findings. We shall also discuss the particular discrete choice setting that we employ heavily throughout the main text, which is slightly non-standard in the macroeconomic literature.

We start with a stripped down version of the main model that abstracts from unemployment and other labor market features. Consider a standard neoclassical growth models, where a representative family consumption-insures its member. Workers can be either employed or out of the labor force. If they work in the official market, they can produce one unit of a labor good. This good serves as an input for the different final goods in the economy. If they work at home, their technology level is $A \leq 1$. With exogenous probability $\lambda$ they drop out of the main labor market, with endogenous probability $\pi_{oe}$ they enter the market. If the worker is out of the labor force she draws an idiosyncratic utility shock $\epsilon$ reflecting utility differences or random search cost between staying at home and working not captured by wages. The shock is realized after she has produced this period (but before next periods aggregate shock has materialized, when we allow for aggregate shocks). The worker makes a discrete choice between producing at home or entering the labor market and chooses the utility maximum between the two alternatives. If she decides to enter she has to pay a setup cost $s$ expressed in utility units. Overall utility from the families perspective is given by

$$V = \sum_{t=0}^{\infty} \beta^t (u(c_{1,t}, c_{2,t}, c_{3,t})) - (1 - l_t) \pi_{oe,t} s_t + (1 - l_t) E^* \epsilon_t$$

where $c$ is a consumption aggregator, $l$ is the number of employed and $E^* \epsilon$ is the expected value of the taste shock for workers out of the labor force.\(^9\)

**Discrete Choice Setup:** We rely heavily on a particular discrete choice structure throughout the paper, so we shortly describe the setup here. Whenever the worker has a (discrete) choice she obtains

\(^9\)By assumption, workers in the market do not receive taste shocks. We allow for a richer set of shocks in the main model.
an i.i.d. shock $\epsilon = \epsilon_2 - \epsilon_1$ reflecting the utility difference between the two choice states, say employed ($\epsilon_2$) and inactivity ($\epsilon_1$). We assume that $\epsilon_1$ and $\epsilon_2$ are i.i.d. extreme value distributed with zero mean, such that $\epsilon$ is logistic distributed with zero mean and variance $\frac{2\psi^2}{3}$. At the beginning of the period, before these shocks are materialized the worker decides on the cut-off level $\xi$. Whenever the shock realizes above this level, she decides to switch from inactivity to employment receiving $\epsilon_2$, while she remains inactive otherwise and obtains a utility of $\epsilon_1$. Note that for welfare considerations just the utility difference between the two alternatives matters. The star in the above expectation operator indicates that at the beginning of the period expectations are taken with respect to a truncation reflecting the fact that the worker has a choice to enter or not to enter at a later stage. This expectation and, by the law of large numbers, its average realization, is given by:

$$E^*\epsilon = -[\psi(1 - \pi_{oe}) \ln(1 - \pi_{oe}) + \psi\pi_{oe} \ln(\pi_{oe})] > 0$$

which is concave in $\pi_{oe}$ given that $\frac{\partial E^*\epsilon}{\partial \pi_{oe}} = \psi \log(\frac{1 - \pi_{oe}}{\pi_{oe}})$ and $\frac{\partial^2 E^*\epsilon}{\partial \pi_{oe}^2} < 0$.\(^{10}\) This means, that even though the shocks have a zero mean by assumption, the fact that the workers have a choice between the two alternatives will lead to a positive value due to the truncation. The option value of this choice depends on the variance $\psi$. To see this point more clearly, consider the following ad hoc value functions of a risk neutral worker, where the worker has a choice from moving to employment with

\(^{10}\)To see this, note that $\pi_{oe} \equiv F(\xi) \equiv \Pr(\epsilon_2 - \epsilon_1 \leq \xi) = \frac{1}{1 + e^{\frac{-\xi}{\psi}}}$ by the definition of a logistic random variable, where $\epsilon_1$ and $\epsilon_2$ are extreme valued i.i.d. random variables with zero mean. We denote the distribution function of the $\epsilon$ by $F(x)$ and the corresponding density by $f(x)$. So, given a cut-off level $\xi$ the expected value of the shock is given by:

$$E\epsilon_1 = \int_{-\infty}^{\infty} xf(x)F(x + \xi)dx$$

$$F(x + \xi) = e^{-e^{-x+\xi}}$$

$$f(x) = \frac{1}{\psi}ke^{\frac{-x}{\psi}}e^{-ke^{\frac{-x}{\psi}}}$$

$$k = \frac{\psi^2}{e}$$

Following cite{Anderson et. al. p.60}, a change of variable gives:

$$E\epsilon_1 = -\int_{0}^{\infty} \psi \log(t)e^{-(1+e^{-2})} - \int_{0}^{\infty} \psi \log(k)e^{-(1+e^{-2})}$$

Using:

$$\int_{0}^{\infty} e^{-st} \log(t)dt = -\frac{\log(s) - \gamma}{s}; \gamma = \text{Euler constant}$$

$$s = (1 + e^{-2})$$

7
associated value $V_e$ or remain inactive with value $V_o$:

$$
V_e = w + \beta (1 - \lambda) V_e' + \beta \lambda V_o' \\
V_o = \bar{A} + E \max \{ \beta V_e' + \epsilon_1, \beta V_o' + \epsilon_2 \}
$$

The solution to this program is equivalent to choosing the cut-off level $\epsilon$ of the alternative program$^{11}$:

$$
V_e = w + \beta (1 - \lambda) V_e' + \beta \lambda V_o' \\
V_o = \max \bar{A} + E \epsilon + \pi_{oe} \beta V_e' + (1 - \pi_{oe}) \beta V_o' \\
\pi_{oe} \equiv \frac{1}{1 + e^{-\frac{\epsilon}{\psi}}}
$$

However, the new program is continuously differentiable and can be handled much easier than the original problem, given that we can let the family choose, by monotonicity and the law of large numbers, the probabilities directly. In the sequel, we shall rely on this equivalence extensively.

**Model-Continued:** We assume that the consumption-aggregator takes the following form:

$$
\gamma c_t = (\gamma c_{1,t} + (1 - \gamma) (c_{2,t} + c_{3,t})^{\frac{\gamma_1}{\gamma}})^{\frac{1}{\gamma}}
$$

(2)

We obtain using the zero mean assumption:

$$
E\epsilon_1 = \psi \frac{\log(1 + e^{-\epsilon}) + \gamma}{(1 + e^{-\epsilon})} + \frac{\eta}{(1 + e^{-\epsilon})} \\
= \psi \frac{\log(1 + e^{-\epsilon})}{(1 + e^{-\epsilon})} \\
e^{-\epsilon} = \frac{1 - \pi_{oe}}{p_{oe}} \\
E\epsilon_1 = -\psi \pi_{oe} \log(\pi_{oe}) \\
E\epsilon_2 = -\psi (1 - \pi_{oe}) \log(1 - \pi_{oe})
$$

where the last step follows from the fact that the worker does not switch if $\epsilon_1 - \epsilon_2 \leq -\frac{\epsilon}{\psi}$ and a symmetric argument.

$^{11}$To see the equivalence note that from standard computations of an expected value of a maximum of extreme-value distribution we know that optimal choices are given by:

$$
\pi_{oe} = \frac{e^{\frac{av_{e}}{\psi}}}{(e^{\frac{av_{e}}{\psi}} + e^{\frac{av_{o}}{\psi}})}
$$

and the value can be expressed as:

$$
V_o = \bar{A} + \psi \log(e^{\frac{av_{e}}{\psi}} + e^{\frac{av_{o}}{\psi}})
$$

as shown in cite{Anderson et. al. p.60}. First order conditions of the alternative program immediately reveal the equivalence of the solution.
Here $c_1$ is the broad market good used as a numeraire (the price is normalized to one), that is storable and can be used as an investment good, while $c_2$ are goods that are produced in the market with a high elasticity of substitution relative to the home-produced good. In turn $c_3$ are home-produced goods.

We think of $c_2$ and $c_3$ as service goods, in particular child-care, as well as cooking and cleaning activities, given that we identify these goods at a later stage using the time-use survey of Aguiar and Hurst (2007) by conditioning on the main activities (time spend) of workers staying at home. A particular characteristic of these goods is the high share of labor in the production process. Consequently we assume the following production technology for perfectly competitive firms in the final good sector that produce the numeraire good:

$$y_{1,t} = k_t^\alpha l_{1,t}^{1-\alpha}$$

where $l_1$ denotes the demand of the labor good in sector 1.

Competitive firms that produce the service good just linearly transform the labor input according to:

$$y_{2,t} = l_{2,t}$$

where $l_2$ denotes the demand of the labor good in sector two.\(^{12}\)

The labor service industry is perfectly competitive and makes zero profits.

Total resources are spend, noticing that $p_2 y_2 = p c_2$ from market clearing, according to a standard constrained

$$y_{1,t} = c_{1,t} + k_{t+1} - (1-\delta)k_t + g_t$$

where $\delta$ is depreciation and $g$ is government spending, that is done using the numeraire good as input. Finally, $c_3$ is produced at home according to the linear technology:

$$c_{3,t} = A(1-l_t)$$

Prices are determined competitively and markets clear:

$$w_t = (1-\alpha)k_t^\alpha l_{1,t}^{1-\alpha}$$

$$r_t = \alpha k_t^{\alpha-1} l_{1,t}^{1-\alpha}$$

$$p_t = w_t$$

$$l_t = l_{1,t} + l_{2,t}$$

\(^{12}\)We could allow for home capital as well. Our results only depend on the assumption that home-goods are less capital intensive then good 1.
The government faces a balanced budget rule:

\[ g_t = w_t(l_t \tau_{l,t} + (r_t - \delta) k_t \tau_{c,t}) \]  

(11)

The flow governing employment is given by:

\[ l_{t+1} = l_t (1 - \lambda - \pi_{oe,t}) + \pi_{oe,t} \]  

(12)

After the realization of the shock the worker decides to switch to market activity or to work at home. She chooses the maximum of the two alternatives. The mass of workers that choose to switch by:

\[ \pi_{oe,t} = \frac{1}{1 + e^{-\beta(\Delta_{t+1} + \lambda_1)}} \]  

(13)

Here \( \Delta \) is the lagrange multiplier on having one additional worker in the labor force and reflects the utility difference between working on a job or at home evaluated from the families perspective.

\[ \Delta_t = w_t(1 - \tau_{l,t}) \lambda_{1,t} - A \lambda_{2,t} + \pi_{oe,t}s - E^* \epsilon_t + \beta(1 - \lambda - \pi_{oe,t}) \Delta_{t+1} \]  

(14)

We have the standard Euler equation:

\[ \lambda_{1,t} = \beta(1 + (r_{t+1} - \delta)(1 - \tau_{c,t+1})) \lambda_{1,t+1} \]  

(15)

where \( \lambda_1 \) and \( \lambda_2 \) are the lagrange multipliers on the aggregate budget constrained, equation (5), and the home-production constrained, equation (6), respectively. The FOC with respect to the different consumption commodities are given by:

\[ \frac{\partial u}{\partial c_{1,t}} = \lambda_{1,t} \]  

(16)

\[ \frac{\partial u}{\partial c_{2,t}} = p_t \lambda_{1,t} \]  

(17)

\[ \frac{\partial u}{\partial c_{3,t}} = \lambda_{2,t} \]  

(18)

An equilibrium of the above economy is a sequence of allocation such that given prices and probabilities, firms maximizes profits, given prices, the family optimally chooses entry rates and consumption allocation given their budget and all markets clear. Note that in the absence of distortive taxes, that is if lump sum taxes were allowed, the competitive equilibrium outcome coincides with the social planners solution. So the entry condition does not induce per se an inefficiency that needs to be corrected by means of capital taxes.
Given the model described above we can now define the Ramsey-problem from a time-less perspective as a government optimization problem that chooses allocations subject to the above economic constraints.

\[
V = \max_{z_t} \sum_{t=0}^{\infty} \{ \beta^t (u(c_t) - (1-l) \pi_{oe,t} + (1-l)E^*\epsilon_t) + \beta^{t+1} \mu_t^T f(z_t, z_{t+1}) \} 
\]

\[f(z_{t-1}, z_t) = f_t \]

Here \( z \) collects all endogenous variables of the model, \( f \) summarizes equations (1)-(18) while \( \mu_t^T \) is a column vector of lagrange multipliers attached to each constraint. We follow Benigno and Woodford (2006a) and assume a timeless perspective to avoid complications that arises due to the non-recursiveness of the standard Ramsey problem. Given that we are mainly concerned with a steady state discussion we do not want the steady state to be a function of an initial period anomaly. That is we impose a pre-commitment rule \( f_t \) that is consistent with a time-independent law of motion of the above problem, see Benigno and Woodford (2006a) for a rigorous treatment. To see that optimal capital taxes will, in general, not be zero, substitute equation (17) and (18) into equation (14) to get:

\[
\Delta_t = w_t(1 - \tau_{l,t})\lambda_{1,t} - A w_t(\frac{C_{3,t}^{c_2}}{C_{2,t}^{c_2}})\gamma_2 - 1 + \pi_{oe,t} - E^*\epsilon_t + \beta((1-l)\pi_{oe,t})E \Delta_{t+1} 
\]

Following the standard procedure, see ?, redefine the problem in terms of after tax wage \( \hat{w}_t = w_t(1-\tau_{l,t}) \) and after tax interest rate \( \hat{r} = (r - \delta)(1 - \tau_{c,t}) \). The first order condition with respect to capital \( k_{t+1}, \) (denoting with \( \mu_{t,x} \) the lagrange multiplier associated with equation \( x \)), gives:

\[
\mu_{t,5} = \mu_{t+1,5}[\beta(1 + r_{t+1} - \delta)] + \mu_{t+1,11}[\beta(r_{t+1} - \delta - \hat{r}_{t+1})] \\
+ \mu_{t+1,14}A(\frac{C_{3,t+1}}{C_{2,t+1}})\gamma_2 - 1 + \lambda_{1,t+1}(1-\alpha)r_{t+1}l_{1,t+1} 
\]

We can now see directly that capital taxes will typically not be zero except if \( A = 0 \) given that if \( \tau_{c} = 0 \) we have that \( \mu_{5} - \mu_{5}[\beta(1 + r - \delta)] + \mu_{11}[\beta(r - \delta - \hat{r}_{t+1})] = 0 \) as in the standard case, but the term \( \mu_{14}A(\frac{C_{3,t+1}}{C_{2,t+1}})\gamma_2 - 1 + \lambda_{1,t}(1-\alpha)r_{t+1}l_{1,t} \) will be non-zero as long the multiplier on the value of having an employed worker instead of an inactive worker is binding.\(^{13}\) In general the strength of the friction will be increasing in the outside opportunity \( A \) and in the degree of substitutability \( \gamma_2 \). The highlighted friction is similar to the existence of a fixed factor as in Correira (1996), given that home production

\(^{13}\)This can not be guaranteed for all parameter values, given that the marginal value of having one worker entering the labor force can be precisely zero. In this case, optimal capital taxes will be zero in the steady state as well, even if \( A \) is not zero.
plays a similar role.\textsuperscript{14} However, home-production alone is not sufficient to obtain a positive capital tax result. What matters is the additional distortion in the intertemporal choice.\textsuperscript{15}

A crucial aspect for obtaining a positive capital tax result is how the worker evaluates its home produced goods. If she evaluates the opportunity cost of her time based on the market wage, labor taxes are first order distortive and capital taxes are non-zero. That is, by driving capital taxes to zero and increasing labor taxes correspondingly, the induced change in the aggregate capital stock makes market production in terms of the output good more productive while leaving the production possibility of goods produced at home unaffected. The planner would therefore like more workers to enter the market. However, by distorting simultaneously the entry decision, the planner does not obtain the entire social surplus of an increase in the capital stock, given that it is counteracted by a decline of labor. The presence of this counteracting effect means that at some point before the capital tax is driven to zero, the planner finds it optimal to stop to avoid a crowding out of labor participation. An alternative interpretation of our setup leading to a similar conclusions would be that when deciding to enter the market and being productive the family has to buy a fixed amount $\overline{A}$ of childcare and other home-produced services in the market. The price of these goods is proportional to the gross wages paid in the market for baby-sitters, while the individual worker is interested in net wages, inducing the same wedge effect as before. Home-production could then be normalized to zero.

The described effect will also be present once we introduce unemployment and hours worked into the model. As highlighted numerically in Arsenau and Chugh (2006) within a Nash-bargaining context of the Mortensen-Pissaridis matching model, whenever the outside option enters non-proportionally (to net wages) into the bargaining equation, capital taxes will not be zero. The effect they highlight is almost identical to the one described above. A change in capital taxes raises the social value of production, but the corresponding increase in labor taxes distort the surplus non-proportionally, leading to an increase in unemployment and creates an intertemporal distortion by affecting the

\textsuperscript{14}It is not identical because even in the absence of any home-production technology and a one good economy the effect could be obtained as long as a behavioral rule would lead to an entry condition of the above firm. Our setup is designed to provide a general equilibrium derivation of this entry condition where workers evaluate their opportunity cost of time when not working with the gross wage rate.

\textsuperscript{15}It is interesting to note that it is not enough to assume merely the existence of a home produced good, serving the role of a "fixed factor" to generate a positive capital tax result. To see this, assume that good 2 is missing in the above utility specification and the home produced good is perfectly substitutable with the market good 1. In this case the utility difference

$$
\Delta = \lambda (w(1 - \tau t) - A) + \pi_{us} s - E^* \epsilon + E \Delta'
$$

(23)

This equation implies that the worker, when entering the market, does not evaluate its time spent at home at the market wage, but with the price of the output good. One can directly see that in this case capital taxes are zero again.
free entry condition.\textsuperscript{16} Again, the key for a positive capital tax result is the distortion along the intertemporal dimension, that is in their case the free entry condition.

Finally, it is interesting to note that, independent of the described modeling assumptions, the behavior of the government or the particularities of wage setting assumptions, once unemployment will be incorporated into the model within a search and matching framework\textsuperscript{17}, the interaction of labor force participation and unemployment will lead to decentralized equilibria which can not be socially efficient. That is, the standard Hosios condition, see Hosios (1990), that shows how to trade-off the search and matching externality by choosing the bargaining weights appropriately will fail in a model with endogenous labor market participation. The reason is quite intuitive. Any model with unemployment needs to generate positive profits of a match from a firms perspective. That implies that wages cannot equal the marginal product of labor. However, when choosing to participate in the labor market, in a decentralized economy the worker will typically base her decision on actual wages, while the social planner would always base its decision on the social value, which is the marginal product of labor. Given an assumed disparity between the two, the wage can not serve its allocative role for governing unemployment and participation at the same time.\textsuperscript{18}

The question now becomes quantitative: How important are these described frictions quantitatively and what should fiscal authorities do?

The crucial margin that drives our quantitative findings is how the entry probability $\pi_{oe,t}$ reacts to changes in wage rates, either gross wages due to business cycle shocks, or net wages, due to tax rate changes. Ignoring general equilibrium effects the reaction is governed by the elasticity, that replaces the standard elasticity of hours worked:

$$\frac{\partial \pi_{oe,t}}{\partial w_{t+1}} = (1 - \pi_{oe,t}) \beta \frac{\lambda_1 t+1 (1 - \tau t+1, l - \left( c_{2,t+1} \right)^{\gamma - 1} A) w_{t+1}}{\psi}$$

Three main factors will govern this elasticity: The first factor is the underlying \textbf{heterogeneity} in \textbf{idiosyncratic preferences}. The higher the underlying variance of the idiosyncratic utility distribution or say the search cost is, the smaller will be the impact of changes (and the smaller will be the distortion on capital taxes). Intuitively $\psi$ governs the mass of workers around the cut-off level and is therefore an ultimate driving force for the reaction of the probabilities to business cycle shocks (who implicitly change net wages) as well as for the reaction to tax rate changes. We identify this margin

\textsuperscript{16}Given that we are mainly concerned with the quantitative importance of home-production, we therefore will ensure, that all positive capital tax results will be due to distortions in labor market participation, not due to distortions in the bargained wage.

\textsuperscript{17}Any other framework that generates a disparity between productivity and wages would suffice.

\textsuperscript{18}However, we show quantitatively that the resulting inefficiency is quantitatively very small.
by the volatility of the corresponding transition rates driven by aggregate shocks. The second crucial margins, driven by technology, is given by the efficiency distance between home-production and the market good \( \bar{A} \) and the associated tax distortion. We identify this margin by using the hour choice differentials across gender and employment status using the time-use survey of Aguiar and Hurst (2007), given that standard intratemporal hours worked decision across states will be monotone function of the differences in technology. The third margin, preferences for variety of the average consumer, is governed by \( \gamma_2 \). The further away home-produced and markets good are the more the consumer loves variety and will, correspondingly, demand more workers to produce this unique good at home. It is difficult to identify this margin, given that preferences and technology are hard to disentangle without information of the underlying consumption choices. We will use changes in labor tax rates to obtain some information. In the benchmark case we assume that markets do have the technological ability to provide a good of equal quality and assume perfect substitutability to rule out love for variety as a driving force for our results but shall provide an extensive sensitivity analysis with respect to this parameter choice.

3 Model

To quantify the effects we have just highlighted we now build a more realistic model of the labor market that endogenizes all transition probabilities across labor market states and study the interaction across all entry and exit margins of a frictional labor market.

3.1 Firms

We introduce 4 different sectors in the economy. The first three are perfectly competitive and are used to ease aggregation. The final good sector combines capital and a labor input good to produce the numeraire output good. This good \( Y_1 \) is used for investment and for government spending:

\[
Y_{1,t} = e^{\alpha_t} K_t^\alpha L_{1,t}^{1-\alpha}
\]

where \( K_t \) is aggregate capital, \( L_{1,t} \) is the amount of the labor good demanded in final production and \( \alpha_t \) is a technology shock assumed to follow and AR(1) process with normal increments.

The service good \( Y_{2,t} \) is also produced competitively and is labor intensive, relative to the final good sector:

\[
Y_{2,t} = L_{2,t}
\]
We view this as a close substitute to home-produced goods. \( L_{2,t} \) is the demand for the labor good produced in the service sector.

The third sector, again perfectly competitive, simply aggregates the labor good, produced in the main labor firm industry sector.

\[
L_t = m_m e_{t,m} l_{t,e,m} + m_f e_{t} l_{t,e,f} \tag{26}
\]

Let gender \( \tilde{g} \in \{m, f\} \) denote male and female respectively and denote the three labor market states \( \tilde{e} \in \{e, u, o\} \) as employment, unemployment and out of the labor force, respectively. Then \( m_{\tilde{g}} \) are the measures of males and females in the society, \( e_{t,\tilde{g}} \) are the percentage of employed males and females and \( l_{t,\tilde{g}} \) is the labor good produced in the labor service industry, where each employment relationship is viewed as a particular match between one worker and one firm.

The technology when employed or working at home is given by the homogenous production function:

\[
l_{t,\tilde{e},\tilde{g}} = A_{\tilde{e},\tilde{g}} h_{t,\tilde{e},\tilde{g}} \tag{27}
\]

where \( h_{t,\tilde{e},\tilde{g}} \) denotes the amount of hours supplied by males and females and \( A_{\tilde{e},\tilde{g}} \) denotes the technology level of a typical males or female in each of the three labor market states. We allow this level to differ across sex and employment state.

Aggregate prices from the final goods perfect competitive firms behavior are given by the standard conditions:

\[
r_t = \alpha \frac{Y_{1,t}}{K_t} \tag{28}
\]
\[
p_{t,w} = (1 - \alpha) \frac{Y_{1,t}}{L_{1,t}} \tag{29}
\]

where \( r \) denotes the interest rate and \( p_{w} \) denotes the price of the labor good. Given perfect competition \( p_{w} \) is also the price charged by the service firms.

Market clearing requires that:

\[
L_t = L_{1,t} + L_{2,t} \tag{30}
\]
3.2 Laws of motion:

The state variables of the economy are, for each gender, the measure of unemployed \( u_{\tilde{g}} \) and employed workers \( e_{\tilde{g}} \), as well as the aggregate capital stock \( K \) and the exogenous shock processes described below. Let \( \delta \) denote the depreciation rate and \( I \) denote aggregate investment, then the respective laws of motion are given by:

\[
K_{t+1} = (1 - \delta)K_t + I_t
\]

\[
e_{t+1,\tilde{g}} = (1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})e_{t,\tilde{g}} + q_{t,\tilde{g}}e_{t,\tilde{g}} + (1 - q_{t,ou,\tilde{g}})\pi_{t,ue,\tilde{g}}u_{t,\tilde{g}}
\]

\[
u_{t+1,\tilde{g}} = u_{t,\tilde{g}}(1 - (1 - q_{t,ou,\tilde{g}})\pi_{t,ue,\tilde{g}} - q_{t,uo,\tilde{g}}) + e_{t,\tilde{g}}((1 - q_{t,\tilde{g}})f_{t,\tilde{g}}) + o_{t,\tilde{g}}\pi_{t,ou,\tilde{g}}
\]

and

\[
o_{t+1,\tilde{g}} = o_{t,\tilde{g}}(1 - \pi_{t,ou,\tilde{g}}) + e_{t,\tilde{g}}(1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})q_{t,uo,f} + q_{t,uo,\tilde{g}}u_{t,\tilde{g}}
\]

where the inactivity state is redundant given that we have the accounting identity \( o_{t,\tilde{g}} = m_{\tilde{g}} - u_{t,\tilde{g}} - e_{t,\tilde{g}} \).

Recall from figure ( ) the timing convention of the model. Consider first the decision tree of an employed worker. When employed the worker first decides, (if she receives a favorable outside offer), to quit her current job and to take a new job. She does quit with probability \( q_{t,\tilde{g}} \) and will be employed next period again. If she decides to stay, the firm might decide to fire her with probability \( f_{t,\tilde{g}} \). If the firm decides to do so, the worker will be unemployed next period. If she has not received an outside offer and is not being fired she might decide to become inactive and drop out of the labor force voluntarily. She does so with probability \( q_{t,eo,\tilde{g}} \). Similarly, when unemployed, her decision process consists of deciding to become inactive or to start searching. She decides to become inactive with probability \( q_{t,uo,\tilde{g}} \) and is out of the labor force next period. If she decides to search, she is matched with an employer in the official matching market with probability \( \pi_{t,ue,\tilde{g}} \). Finally, when inactive, her only choice is to remain inactive or to start searching for a job and become unemployed next period. She does so with probability \( q_{t,ou,\tilde{g}} \). All these probabilities will be endogenized below!

Some remarks about the setup are in order: First, note that we do not allow direct flows from inactivity to employment. By definition of unemployment as a state of search any flow from inactivity to employment must go through the unemployment state, even for a short time interval. Our model treats this decision process as lasting at least a month. When mapping the model to the data, a time-aggregation bias as highlighted in ? arises. Second, our timing convention is, obviously, somewhat arbitrary. We have chosen a sequential structure to ease the already quite complex structure, and to
to avoid tie-breaking situations where a worker has to decide to quit to a new job or become inactive. The sequential structure insures that the choice problem is binary, and therefore easier to handle. 19

3.3 Family

To avoid complications with respect to the wage bargaining that arises in incomplete market setting of the model, we assume the existence of a continuum of identical families that fully insures the consumption risk of their individual members and decides on aggregate savings. Given the separability of the preferences this implies that all individuals consume the same (see Jung (2005) for a treatment of non-separable utility in a labor market context). In turn, however, the individual gives up his bargaining rights, or, alternatively, evaluates the stream of utility flows from the perspective of the aggregate family. This assumption, basically, ensures a common discount kernel, implying a common valuation of future streams of utility and allows us to neglect the distribution of capital holdings as an additional (infinite dimensional) state variable.

The representative family aggregates individual consumption and maximizes, given the law of motion of the state variables, sequences for wages and hours (derived below) and its budget constraint, consumption allocations and transition probabilities according to the inter-temporal utility function:

\[
U = \max_{\{c_1, c_2, c_3, I, a_0, f, g, n, \pi_o, \pi_u, \pi_h\}} \sum_{t=t_0}^{\infty} E_{t \rightarrow t_0} \left[ (m_m + m_f) u(c_1, c_2) \right.
\]

\[
- \sum_{\bar{g}} m_{\bar{g}} e_{t, \bar{g}}^e \phi_{t, u, \bar{g}}^e - m_{\bar{g}} o_{t, \bar{g}} \phi_{t, o, \bar{g}}^o + \text{discrete choices} \right)
\]

s.t.

Budget Constraint

Initial Conditions and Laws of Motion for states

Given sequences of \(w_{\bar{g}}, h_{e, \bar{g}}\)

Here \(u(c_1, c_2, c_3)\) is a consumption aggregator over the final good \(c_1\), the service good \(c_2\) and the home-produced good \(c_3\).

---

19 We have experimented by flipping part of the decision processes, but the particularities of the choices are not driving our results. Note that all workers are, conditional on their states, identical, and switching probabilities are quite small. So the mass of workers that stochastically first decide not to quit, and are fired afterwards, or are first not fired and then decide to quite, is almost identical. Even though the decision process is influenced by the aggregate shock, this influence is typically small quantitatively.
\[ u(c(c_1, c_2, c_3)) = \log(c(c_1, c_2, c_3)) \]

with \( c \) being a consumption aggregator

\[ c = (\gamma c_1^\gamma_1 + (1 - \gamma)(c_2^\gamma_2 + c_3^\gamma_3)^{\frac{\gamma_1}{\gamma_2}})^{\frac{1}{\gamma_1}} \] (36)

The parameters \( \gamma_1, \gamma_2 \) govern the substitution elasticity across goods and \( \gamma \) gives the share of the final good relative to the labor intensive service good.

Here \( h_{t,\tilde{e},\tilde{g}} \) is the amount of hours supplied by a worker, with gender-status \( \tilde{g} \) and employment status \( \tilde{e} \). Note that workers who are not employed, still work in the sense of using time in producing at home. \( \varphi_{\tilde{g}} \) is a normalizing constant scaling hours worked and \( \frac{1}{\gamma - 1} \) is the intratemporal elasticity of substitution.

The term \textit{discrete choices} is a short hand for the mean cost and the option value of having a (discrete) choice to change their respective labor states at the end of this period.

\[
\text{discrete choices} = \sum_{\tilde{g}} -m_{\tilde{g}}c_{\tilde{g}}^{-\varphi_{\tilde{g}}}(\varphi_{\tilde{g}}c_{\tilde{g}}^{-\varphi_{\tilde{g}}}(1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})q_{t,\tilde{g},\tilde{e},\tilde{g}}
- E^* \varepsilon_{t,\tilde{e},\tilde{g}} - (1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})E^* \varepsilon_{t,\tilde{e},\tilde{g}}
- m_{\tilde{g}}q_{t,\tilde{g}}^{-\varphi_{\tilde{g}}}(1 - \pi_{\tilde{e},\tilde{g}}) - E^* \varepsilon_{t,\tilde{e},\tilde{g}}
- m_{\tilde{g}}q_{t,\tilde{g}}^{-\varphi_{\tilde{g}}}(1 - \pi_{\tilde{e},\tilde{g}}) - E^* \varepsilon_{t,\tilde{e},\tilde{g}})
\] (37)

We express the fixed cost \( \bar{\pi} \) associated with each choice in units of average hours worked. So, for example, the utility cost of quitting an employment relation and moving to a new employer, potentially in a different town, say, are denoted by \( s_{q,\tilde{g}} \) and are suppose to capture the potentially important cost of moving, finding new friends and colleagues or being away from home. The notation for all other cost are correspondingly. For example, the cost \( s_{\bar{\pi},\tilde{g}} \) captures the setup cost of staring to search for a job.

The budget constraint of the family is given by:

\[
(m_m + m_f)c_{1,t} + (m_m + m_f)p_{w,t}c_{2,t} = m_m w_{t,m} h_{t,m} e_{t,m}(1 - \tau_{t,l}) + m_m b_{t,m} u_{t,m}
+ m_m e_{t,m} q_{t,m} Ed_{t+1} + m_f w_{t,f} h_{t,f} e_{t,f}(1 - \tau_{t,f}) + m_f b_{t,f} u_{t,f}
+ m_f e_{t,f} q_{t,f} Ed_{t+1} + m_f + K_t + (r_t - \delta)(1 - \tau_{t,c}) K_t - K_{t+1} + D_t + T r_t
\] (38)

\[
(m_m + m_f)c_{3,t} = m_f A_{t,u,f} h_{t,u,f} u_{t,f} + m_m A_{t,u,m} h_{t,u,m} u_{t,m}
+ m_f A_{t,o,f} h_{t,o,f} u_{t,f} + m_m A_{t,o,m} h_{t,o,m} u_{t,m}
\] (39)
Here \( (r - \delta)(1 - \tau_c) \) is the after-depreciation, after tax capital income, \( D \) are aggregate profits that might arise in the economy and are distributed back to the owners, \( Tr \) denotes lump sum transfers or taxes from or to the government and \( \tau_c, \tau_l \) denotes the capital and labor tax while \( K \) is the aggregate capital stock. Unemployment benefits are denoted by \( b_m \) for males and \( b_f \) a for females, which we allow to be different from their male counterpart in general. The unemployment rates and out of the labor force rates are taken with respect to the mass of males and females, not with respect to total population.

Having endogenous firing rates in the model, we decided to endogenize quits on the job as well to capture the observed empirical regularity taken for example from the JOLST database, that overall destruction rates are rather acyclical, while firing rates are countercyclical and quantitatively smaller than overall destruction rates. Procyclical quits, as documented in Shimer() basically dampens fluctuations in the overall destruction rate and capture the important transitions from employment to employment, with a potentially one month intervening spell of inactivity as documented in Nagypal (). Given that we did not want to increase the already quite complex structure of the model by introducing separate matching functions for quits on the job\(^{20}\) we model the incentives to quit as receiving an initial ”golden hand check” \( Ed_{t+1} \Pi_{t+1} \) when she decides to switch her job. To mimic the behavior of on-the-job matching markets we assume that, on the job, workers can search and contact other firms much more easily than unemployed workers.\(^{21}\) This implies that an infinite number of firms stand ready to hire these workers without having to pay vacancy posting cost. We therefore have to redistribute the profits that arise. We assume that these profits are paid lump sum up front to the newly hired worker and that the standard bargaining starts from that point on, insuring that the wage each worker receives is the same and that firms hiring workers on the job make zero profits.\(^{22}\) The amount of this extra payment is the probability adjusted saved amount of vacancy posting cost the firm would incur when hiring an unemployed worker, or the value of a match from the firms perspective, so our assumptions insures that firms still make zero profits on average.

---

\(^{20}\)In fact it is straightforward to do and would also lead to pro-cyclical quitting behavior.
\(^{21}\)That is they might have access to the companies intra-net and can switch jobs within their own company or have contact to different companies through their job.
\(^{22}\)This assumption allows us to remain within a representative agent framework. We assume that wages are continuously re-bargained, and that all payments made before are sunk. This exclude the possibility of providing an optimal contract over time and shifts the extra surplus as a lump-sum payment to the beginning of the job.
The first order conditions for the different consumption and capital choices are:

\[
\frac{\partial u}{\partial c_{1,t}} = \lambda_{1,t} \tag{40}
\]

\[
\frac{\partial u}{\partial c_{2,t}} = \lambda_{1,t} \rho_{w,t} \tag{41}
\]

\[
\frac{\partial u}{\partial c_{3,t}} = \lambda_{2,t} \tag{42}
\]

\[
\lambda_{1,t} = E \beta \lambda_{1,t+1} (1 + (r_{t+1} - \delta)(1 - \tau_{c,t+1})) \tag{43}
\]

\[
\beta \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \equiv d_{t+1} \tag{44}
\]

where \(\lambda_{1,t}\) is the lagrange multiplier on the budget constraint, \(\lambda_{2,t}\) is the lagrange multiplier on the home-produced good constraint, and \(d_{t+1}\) is the definition of the discount kernel. Finally, the first order conditions on the hours choice when working at home is given by:

\[
A_{t,u,\tilde{g}}\lambda_{2,t} = \varphi \rho_{t,u,\tilde{g}}^{\rho_{e}} \tag{45}
\]

\[
A_{t,o,\tilde{g}}\lambda_{2,t} = \varphi \rho_{t,o,\tilde{g}}^{\rho_{e}} \tag{46}
\]

The hour choice when employed is derived within the bargaining setup explained next.

### 3.4 Flow values and Bargaining

We now turn to a discussion of the matching markets, where the main labor friction is located. We first derive the standard flow values of individual matches from the worker/family point of view and then from the firms point of view. These values are needed to derive the bargaining solution in the next section.

#### 3.4.1 Flow Values

Given our assumption of perfect insurance the family values the stream of labor income at the beginning of the period (that is before individual utility draws are made, but after the aggregate shock is observed) from an employed or unemployed female is given as the marginal utility of having one worker employed rather than unemployed. Define the lagrange multipliers on the law of motion of the employment state variables \(e\) and \(u\) respectively as \(\Delta_{t,e,o,\tilde{g}}\) and \(\Delta_{t,u,o,\tilde{g}}\); then\(^{23}\):

\(^{23}\text{Again, say } \Delta_{t,n,o,\tilde{g}} \text{ is redundant. One can, after some algebra, rewrite the system below, as is done in our coding, in terms of, say, } \Delta_{t,u,o,\tilde{g}} \text{ and } \Delta_{t,e,o,\tilde{g}} \text{ alone, replacing all } V_e \text{ terms. However, the decomposition below is much easier to interpret because they relate back to the flow value function equations of a standard labor market model.}\)
\[ \Delta_{t,eu,\tilde{g}} = V_{t,eu,\tilde{g}} - V_{t,au,\tilde{g}} \]
\[ \Delta_{t,co,\tilde{g}} = V_{t,co,\tilde{g}} - V_{t,ao,\tilde{g}} \]
\[ \Delta_{t,uo,\tilde{g}} = V_{t,uo,\tilde{g}} - V_{t,ao,\tilde{g}} \]  

\[ V_{t,\tilde{g}} = \lambda_{1,t} w_{t,\tilde{g}} h_{t,\tilde{g}} (1 - \tau_{t,\tilde{g}}) - \varphi_{t,\tilde{g}} \frac{h_{t,\tilde{g}} e^g}{w_{t,\tilde{g}}} q_{t,\tilde{g}} - (1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}}) q_{t,co,\tilde{g}} \varphi f_{t,co,\tilde{g}} + E^* \tilde{\varepsilon}_{t,eu,\tilde{g}} \]
\[ + \lambda_{1,t} m_{t,\tilde{g}} c_{t,\tilde{g}} E \hat{t} \|_{t+1} \Pi_{t,u,\tilde{g}} \]
\[ + E \beta (1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}}) (1 - q_{t,co,\tilde{g}}) V_{t+1,eu,\tilde{g}} \]
\[ + E \beta q_{t,\tilde{g}} V_{t+1,co,\tilde{g}} + E \beta (1 - q_{t,\tilde{g}}) f_{t,\tilde{g}} V_{t+1,au,\tilde{g}} \]
\[ + E \beta (1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}}) q_{t,co,\tilde{g}} V_{t+1,ao,\tilde{g}} + ((1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}}) E^* \tilde{\varepsilon}_{t,co,\tilde{g}} \]

\[ V_{t,u,f} = \lambda_{1,t} h_{t,\tilde{g}} + \lambda_{2,t} p_{t,u} A_{t,u,\tilde{g}} h_{t,u,\tilde{g}} - \varphi_{t,u,\tilde{g}} \pi_{t,u,\tilde{g}} (1 - \pi_{t,u,\tilde{g}}) \varphi - \varphi_{t,u,\tilde{g}} h_{t,u,\tilde{g}} + E^* \tilde{\varepsilon}_{t,u,\tilde{g}} \]
\[ + E \beta (1 - q_{t,u,\tilde{g}}) \pi_{t,u,\tilde{g}} V_{t+1,eu,\tilde{g}} + E \beta(1 - q_{t,u,\tilde{g}}) (1 - \pi_{t,u,\tilde{g}}) V_{t+1,eu,\tilde{g}} + q_{t,u,\tilde{g}} E \beta V_{t+1,ao,\tilde{g}} \]

To reiterate, the expectation (emphasized with \( E^* \)) with respect to the idiosyncratic utility components is a particular truncated value, and is derived below. Note that all wage terms are deflated by the lagrange multipliers (which one can think of as \( \frac{1}{2} \)). This essentially converts utility flows to the same units as valuations of goods and services. Turning to individual matches from the firms perspective, firms profit values are given by:

\[ \Pi_{t,\tilde{g}} = p_{t,u} A_{t,\tilde{g}} h_{t,\tilde{g}} - w_{t,\tilde{g}} h_{t,\tilde{g}} + E \hat{t} \|_{t+1} (1 - f_{t,\tilde{g}}) (1 - q_{t,\tilde{g}}) \Pi_{t+1,\tilde{g}} \]
\[ - (1 - q_{t,\tilde{g}}) f_{t,\tilde{g}} E^* \tilde{\varepsilon}_{t,u,\tilde{g}} - \] (49)

\[ \] (47)

Here \( \kappa_{F,\tilde{g}} \) denote firing costs and the idiosyncratic shock is interpreted as a random fixed cost, say energy cost, that has to be paid if the match is prolonged to the next period and is denominated in units of the final output good. We account for it in the aggregate resource constraint.\(^{24}\)

### 3.5 Bargaining and Wage Setting

It remains to specify how wages and hours are set in this economy. A standard approach would look at the Nash-bargaining over the surplus from employment to unemployment, and is given by:

\[ (V_{t,f} - V_{t,u})^{\mu} (\Pi_f)^{1-\mu} \]

\(^{24}\)At chosen parameter values, the overall fixed cost in the economy are below 0.5%.
Standard first order condition would be given by:

\[
\frac{\lambda_1 t \mu (1 - \tau_{l,t})}{(V_{t,e,f} - V_{t,u,f})} = 1 - \mu \frac{\Pi_{f,t}}{\Pi_{f,t}}
\]

As argued in the introduction, for optimal taxation the form of the wage setting is crucial. In particular, in our model taxes will have a first order effect on the profit-share of firms. This leads to a positive capital tax-rate in general, because change in labor tax rates generate a price externality that affects hiring decisions directly. Additionally, the resolution of the Shimer-puzzle within this wage setting mechanism leads to a very small match surplus and potentially counterfactual reactions to changes in unemployment benefits, as argued in Costain and Reiter (2005a).

To provide a benchmark where this effect does not occur, we follow ideas from Hall and Milgrom (2007) who argue that the threat point of the worker is essentially independent of unemployment benefits. In accordance with this view, we assume that the threat point of the worker is not unemployment, but delay or strike. That is if bargaining breaks down, the worker strikes for a month and next period will restart his bargaining. So his outside option is given by the amount of strike money she receives from her family or union supporting her strike, or the gains she has from delaying.

She might still decide to quit during the month if she receives an alternative offer and might still be fired at the end of the month (even though we need to assume here that this decision would be independent of her being at strike). Both the worker and the firm take future decisions as given, so we heavily rely on a Markov-perfect game-theoretic structure. Her utility difference when striking is given by:

\[
V_{t,e,\tilde{g}} - \tilde{V}_{t,e,\tilde{g}} = \lambda_{1,t} w_{t,\tilde{g}} h_{t,e,\tilde{g}} (1 - \tau_{l,t}) - \varphi h^{e}_{t,e,\tilde{g}} - \lambda_{1,t} strike_{t,\tilde{g}}
\]

while the threat point of the firm is given by

\[
\tilde{\Pi}_{t,\tilde{g}} = p_{t,w} A_{t,e,\tilde{g}} h_{t,e,\tilde{g}} - w_{t,\tilde{g}} h_{t,e,\tilde{g}}
\]

We therefore obtain, using again a Nash-bargaining procedure, the static wage condition:

\[
w_{t,\tilde{g}} = \mu_{\tilde{g}} p_{t,w} A_{t,e,\tilde{g}} + (1 - \mu_{\tilde{g}}) \left[ \frac{\varphi h^{e-1}_{t,e,\tilde{g}}}{\lambda_{1,t}(1 - \tau_{l})} \right] + \frac{\varphi h^{e-1}_{t,e,\tilde{g}}}{\lambda_{1,t}(1 - \tau_{l})}
\]

and the standard first order conditions for the choice of hours worked (independent of the assumed threat point) in all three sectors is given by:

\[
p_{w} A_{t,e,\tilde{g}} (1 - \tau_{l}) \lambda_{1,t} = \varphi g h^{e-1}_{t,e,\tilde{g}}
\]
due to the joint efficiency of the bargaining process for workers on the job, and the standard hours choice for inactive workers.

Note that average wages per hour will not depend on changes in taxes directly, if we assume that the strike value is a fraction of average net wage \( wh \).

\[
\text{strike}_{t, g} = \text{strike}_{g} w_t h_{t, e, g} (1 - \tau_l)
\]  

(54)

This assumption shields the wage from being affected by tax rate changes or changes in unemployment benefits. Given that \( \tilde{s} \) is arbitrary and being never played, it is unobserved. 25

This wage setting mechanism is ideally suited for our purposes in terms of long run taxation effects. It shields the wage from any non-proportional effects of labor tax-rates, as it is assumed in Arsenau and Chugh (2006) and as would be the case if we relied on the full Nash-bargaining solution with unemployment as the outside option 26. Therefore, a positive capital tax result in our model can be traced exclusively to labor-participation friction, the topic of this paper, and not to wage frictions (even though we touch upon this topic again in the result section). We view this equation as the long run steady state relation, that generates a constant profit markup unaffected by fiscal policy.

However, to resolve the Shimer puzzle in a dynamic version of the model, we either need to assume that wages are sticky as we do here or profits are additionally strongly procyclical due to an additional technology effect, as is done in Costain and Reiter (2005a). As shown in Jung (2005) the above wage setting rule could not, for any parameterization of the model, generate an unemployment volatility in line with the data. A standard way to model wage stickiness, see Shimer (2005) or Hall and Milgrom (2007), would be to assume a constant outside option:

\[
\text{strike}_{t, g} = \text{strike}_{g} w_t h_{t, e, g} (1 - \tau_l)
\]  

(55)

However, this assumption would lead to a positive first order effect of taxes on unemployment, that is by choosing labor taxes the government essentially has partial control over the labor market (and capital taxes would immediately be positive). Given the arbitrariness of the wage setting assumption in these class of models we rule these effects out by assumption. We resolve this dilemma between business cycle fluctuations and long run influence of taxes by assuming the following form:

\[
\text{strike}_{t, g} = \frac{\text{strike}_{g} w_t h_{t, e, g} (1 - \tau_l)}{A_t^x}
\]

(56)

---

25The literature has not converged on a particular wage setting mechanism. Many different forms have been employed. It is fairly easy to provide, for each of this mechanisms, an underlying bargaining game that can "rationalize" or "microfound" the particular equation in use. The empirical content of such games appear to be an open question.

26We experimented with this wage rule quite a bit. However, to resolve the Shimer puzzle we had to rely on the small surplus parameterization as in Jung (2005) and Hagedorn and Manovskii (2006). The model is much harder to calibrate and has some undesirable properties.
Given that the technology level A is normalized to one in steady state, this assumption does not influence our long run optimal taxation results. By choosing $x$ appropriately, we can mimic the behavior of a sticky wage result as close as desired.\textsuperscript{27}

It is worth pointing out that our sticky wage mechanism does not depend on a small surplus of the match, as in Jung (2005) or Hagedorn and Manovskii (2006), (even though, of course, the underlying economic argument is identical given that the surplus with respect to the cost of delaying is, of course, extremely small), nor does it imply problematic responses of unemployment to governmental interventions because, by assumption, we have ruled out any first order effects on profits.

3.5.1 Truncation

As shown above the option value is given by:

$$E^*\tilde{\varepsilon}_{t,ee,\tilde{g}} = -[\psi_{\tilde{g}}(1 - q_t,\tilde{g}) \ln(1 - q_t,\tilde{g}) + \psi q_t,\tilde{g} \ln(q_t,\tilde{g})]$$

where $\psi_x$ parameterizes the variances of the different distributions. All other truncated expectation follow the identical line of reasoning.

3.6 Discrete Choices

In this section we describe the discrete choices made by the agents/family. In all cases, the choices are made after the worker’s binding wage and hour contracts for this period have been made. We assume in each case, that the bargained contract from the beginning of the period is binding and can not be renegotiated. Given that we work for tractability with i.i.d. shocks the definition of firings in this context are problematic, because in our framework it would be beneficial not to fire the worker but to lay her off and wait one period.\textsuperscript{28} Also, if renegotiation is allowed, the firm might want to compensate the worker for a bad shock to save new vacancy posting cost. Our discrete choice assumption essentially means that when making their decisions workers and firms do not look at their joint surplus, but just at their own utility differences. Even though the shocks are i.i.d. by assumption they are denoted in terms of the total value of a state. That is we view them as a summary statistic

\textsuperscript{27}A potential drawback, which is also true for the countercyclical bargaining power model of Shimer (2005) and the procyclical technology model of Costain and Reiter (2005a) is the fact, that the model does not generalize to a higher dimensional shock structure easily without imposing even stronger assumption on the joint process. However, almost all of our results can be generated by assuming a constant outside option given that the choice of $x$ turns out to deliver precisely that.

\textsuperscript{28}This fact also holds true for the standard endogenous destruction model of ?, where firings are also essentially lay-offs. This property is owned to the i.i.d. assumption, but we do not view this property as particular problematic.
over the beliefs of the worker of how much pleasure a particular work environment will deliver to him. To obtain the discrete choice probabilities we let the family/worker choose the cut-off level which is equivalent to choosing the transition probabilities directly. Note that we have assumed that, say, the firm has a right to manage and does the firing decision not taking into account the harm it inflicts upon the worker. Vice versa, the worker does not take into account, when quitting, that the firm takes a loss. So the decisions are not jointly efficient. Technically the case can be easily handled by including the decisions in the bargaining choices and let workers and firms choose them jointly. Given that the idiosyncratic shocks are hard to verify we felt that it is much more sensible to assume that workers and firms make these decisions separately.\footnote{The resulting equations, say for firings, would include the flow values of the worker as well and would essentially be a function of the total match surplus. Given that firms profit and total match surplus move in the same direction the time series properties will likely not change much.}

Using the first order conditions and the definitions of the utility flows above we get the following quitting decision:

\[
q_{t,g} = \frac{\pi_{e,e}}{1 + e^{\frac{\beta}{2}E E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} - E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}} - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} + (1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}) E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}}}{v_{q,e,g}}}
\]

where \(\pi_{e,e}\) denotes the (exogenous) probability of receiving an outside offer.\footnote{We experienced with matching markets for employed and standard vacancy postings and matching decisions that endogenize this probability. Given that the cyclical properties are very similar we decided to use the most simple framework.}

The firing decision follows a similar line of reasoning.

\[
f_{t,g} = \frac{1}{1 + e^{\frac{\beta}{2}E E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} - E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}} - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} + (1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}) E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}}}{v_{f,g}}}
\]

Here firms trade off the cost of keeping the worker while having to retrain him and pay a fixed cost or simply firing him and pay some firing cost.

The decision to quit out of labor force is given by the utility difference between the two options adjusted for the quitting cost, given that it is the last decision made in this stage.

\[
q_{t,ceo,g} = \frac{1}{1 + e^{\frac{\beta}{2}E E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} - E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}} - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} + (1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}) E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}}}{v_{q,ceo,g}}}
\]

Similarly, the decision to quit into inactivity when currently searching for a job is given by:

\[
q_{t,uo,g} = \frac{1}{1 + e^{\frac{\beta}{2}E E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} - E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}} - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}} + (1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}) E_{t+1}^{1 - \pi_{t,ue,g}^{\Delta_{t+1,ue,g}}}}{v_{q,uo,g}}}
\]
She weights the expected gains from searching against the value of being out of the labor force. Finally, the decision to enter into the unemployment pool is given by:

\[
\pi_{t,ou,\tilde{g}} = \frac{1}{1 + e^{\frac{\beta_E \Delta t + 1,ou,\tilde{g} - \beta_E \Delta t + 1,ue,\tilde{g} + \psi_{ou,\tilde{g}}}{\psi_{ou,\tilde{g}}}}}
\]

(61)

where the worker again trades-off the gain in searching and potentially receiving a job with the setup cost of doing so. Each of these shocks is parameterized by its own parameter \(\psi\) governing the idiosyncratic shock distribution.

### 3.7 Matching markets

There are separate matching markets for males and females. That is firms can observe sex and can create different markets for both types. Given that males and females generate, potentially, a different surplus once the match is formed this assumption avoids to model the decision of firms when faced with a male and a female application, which might affect the shape of the matching function. It also allows for different employment probabilities across sex, while a common matching market might not. On the other hand discrimination legislation might suggest a common matching market. We feel that the hiring process can likely be influenced in a discriminatory way, so our treatment might not conflict with this view. In any case, this assumption is not crucial for our results, given that a common matching market does work rather similar.

To close the matching market we use the standard matching functions separated by sex:

\[
M_{t,\tilde{g}} = \kappa_{\tilde{g}} ( u_{t,\tilde{g}}(1 - q_{t,uo,\tilde{g}}) )^{1-\varsigma} u_{t,m}^{\varsigma}
\]

(62)

\[
\frac{M_{t,\tilde{g}}}{v_{t,\tilde{g}}} = \theta_{t,\tilde{g}} = \kappa_{\tilde{g}} \frac{1}{x_{t,\tilde{g}}}
\]

(63)

\[
\frac{M_{t,\tilde{g}}}{u_{t,\tilde{g}}(1 - q_{t,uo,\tilde{g}})} = \pi_{t,ue,\tilde{g}} = \kappa_{\tilde{g}} x_{t,\tilde{g}}^{1-\varsigma}
\]

(64)

\[
x_{t,\tilde{g}} = \frac{v_{t,\tilde{g}}}{u_{t,\tilde{g}}(1 - q_{t,uo,\tilde{g}})}
\]

(65)

We allow \(\kappa\), the normalizing matching function coefficient, to be different for males and females. Finally the free entry condition for males and females reads as:

\[
\frac{(\kappa_{\tilde{g}})}{\theta_{t,\tilde{g}}} = Ed_{t+1} \Pi_{t+1,\tilde{g}}
\]

(66)
3.8 Government and Exogenous

Finally we assume a balanced budget rule for the government, such that
\[ G_t + T_{rt} = m_m w_{t,m} h_{t,m} e_{t,m} \tau_{t,l} + m_m b_{t,m} u_{t,m} + (r_t - \delta) \tau_{t,c} K_t, \]
\[ + m_f w_{t,f} h_{t,f} e_{t,f} \tau_{t,l} + m_f b_{t,f} u_{t,f} \]
(67)
The net replacement rates for males and females is given by
\[ b_{t,\tilde{g}} = \tilde{b}_{g,t} h_{g,t} (1 - \tau_{t,l}) \]
(68)
where \( \tilde{b}_{g} \in [0, 1) \) and is assumed to be proportional to net wages, such that it does not induce by itself a reason for a positive capital taxation.

Before turning to results on optimal taxation we need to specify the behavior of the actual US government. For the exogenous shock processes we assume that \( a_t, G_t, \tau_{t,l} \) and \( \tau_{t,c} \) follow exogenous AR(1) processes \(^{31}\), which we directly estimate from the data.

\[ G_t = \gamma e^{g_t} \]  
(69)
\[ g_{t+1} = \rho_g g_t + \epsilon_{g,t+1}, \epsilon_g \sim N(0, \sigma_g^2) \]  
(70)
\[ \tau_{t,l} = \tau_{t,l} e^{\tilde{\tau}_{t,l}} \]  
(71)
\[ \tilde{\tau}_{t,l+1} = \rho_{\tau} \tilde{\tau}_{t,l} + \epsilon_{\tau,t+1}, \epsilon_{\tau} \sim N(0, \sigma_{\tau}^2) \]  
(72)
\[ \tau_{t,c} = \tau_{t,c} e^{\tilde{\tau}_{t,c}} \]  
(73)
\[ \tilde{\tau}_{t,c+1} = \rho_{\tau} \tilde{\tau}_{t,c} + \epsilon_{\tau,c,t+1}, \epsilon_{\tau_c} \sim N(0, \sigma_{\tau_c}^2) \]  
(74)
\[ a_{t+1} = \rho a_t + \epsilon_t, \epsilon_t \sim N(0, \sigma_a^2) \]  
(75)

We let the direct transfers \( T_{rt} \) pick up the residual in the balanced budget rule. When discussing optimal policy, transfers are set to zero.

For completeness we state the aggregate resource constrained of this economy for all periods (defining aggregate consumption \( C_{1,t} \equiv (m_m + m_f) c_{1,t} \)):
\[ C_{1,t} + p_w C_{2,t} + I_t + G_t = Y_{1,t} + p_w Y_{2,t} - m_m \kappa_{t,m} x_{t,m} - m_f \kappa_{t,f} x_{t,f} - m_m e_{t,m} \kappa_{t,\tilde{g}} f_{t,m}(1 - q_{t,m}) \]
\[ - m_f e_{t,f} \kappa_{t,\tilde{g}} f_{t,f}(1 - q_{t,f}) + m_m e_{t,m}(1 - q_{t,m}) E^* \tilde{\epsilon}_{t,1,m} + m_f e_{t,f}(1 - q_{t,f}) E^* \tilde{\epsilon}_{t,1,f} \]  
(76)

3.9 Ramsey Problem

The Ramsey problem of the above economy is defined as a planners problem that maximize families utility over all endogenous taking the above described decentralized equilibrium constraints into account. We follow the setup of Benigno and Woodford (2006a) closely.

\(^{31}\)Obviously, \( G_t, \tau_{t,l} \) and \( \tau_{t,c} \) do not need to be independent, so we can allow for correlation across the shocks.
That is we define governmental problem as choosing the evolution of the state vector $z_t$ according to:

$$W = \max \begin{array}{l}
E_0 \sum_{t=t_0}^{\infty} E^{t-t_0} U(z_{t-1}, z_t, S_t)
\end{array}
$$

s.t.:

$$E_t f(z_{t-1}, z_t, z_{t+1}, S_t, S_{t+1}) = 0$$

Here $f$ collects the system of equation that characterize the above described decentralized economy, $z$ collects the vector of all endogenous variables of the model and $S$ collects the exogenous shock processes. We further assume that the initial pre-commitment rule $f_{t_0}$ is self-consistent in the sense defined in Benigno and Woodford (2006a) such that the resulting government problem becomes recursive. We derive the Lagrange constraints of the government problem using symbolical derivatives and then solve for the steady state numerically. To obtain time-series properties of the model we perturb the steady state using a first order approximation to the lagrangian problem evaluated at the optimal steady state which was shown to be equivalent to a second order approximation to welfare in Benigno and Woodford (2006a). When looking at simple tax rules and applying the Kalman filter we use a first order approximation relying, as before, on the code provided by Gomme and Klein (2006). A second order approximation has been used to see how important non-linearities might be.

4 Results

This section provides a quantitative evaluation of the properties of the model. We start out by describing how we choose parameters and obtain the processes for the exogenous driving forces of the model. Taking these processes as inputs we use the model to predict all labor market series and compare them to the actual data. Given the fit of the model we then turn to a discussion of optimal taxation results in the long run and over the cycle.

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32 The system has, in our coding around 68 variables (this is not the minimum, because we included some identities in our code) and the Ramsey problem has, due to the fact that many lagged variables become part of the states, around 147 equations. We therefore do not give the entire system here.

33 Given that the system is already quite big we rely on perturbation techniques when bringing the model to the data. Labor market models do have non-linearities that can only be seen when using global methods. However, given that we are mainly concerned with correlations and standard deviations over the cycle and our steady state discussion does not require perturbations, we chose to proceed with perturbation methods.
4.1 Calibration and Estimation

The appendix lists the parameters of the model together with the chosen targets. Most parameters can be set in a standard fashion, so our discussion will focus on the complications that arise due to the new features of the model.

**Exogenous processes:** The period of the model is one month which we aggregate to quarterly frequency when discussing cyclical properties. We report results for the period 1970:1 to 2004:4. To obtain sequences for the four exogenous series, we estimate the model with a Kalman filter using GDP, labor tax rates and government expenditure to obtain estimates of the underlying shock variances of the TFP shock, government expenditure and tax rates shocks respectively. The standard approach to obtain an exogenous TFP shock, see ? can not be used in our model, given that we have a two good structure for measured output and the labor share is not equal to the wage share. We want our model to be consistent with aggregate output and evaluate all other series relative to this. Most labor market series, in particular labor force participation show strong long run trends that must have affected output as well. We therefore choose a low frequency HP filter \( \lambda = 100000 \) to detrend all series. Given the estimates of the Kalman filter for the exogenous states, we can predict all endogenous using the model, taking the entire time-path prediction as our measure of fit.

**Calibration of Parameters - Basics:** Our parameters are detailed in n table (2). With respect

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34 The CPS labor market series separated by sex obtained from \cite{Shimer:07} start in 1976 only. We use his merged series which he obtained from \cite{Beakly} and \cite{} for the years before to obtain a longer time-series. The mean rates are calibrated to the years 1976:1 to 2004:12.

35 We initialize our Kalman filter in 1948:1 to obtain some estimates of the initial state.

36 It is well known that this filtering procedure causes problems in obtaining "true" innovations given that the HP-filter will uses forward looking variables. However, if we use a linear trend in the Kalman estimation the results on the frequency we consider are almost unchanged. In predicting all variables, on business cycle frequency, if \( \lambda = 1600 \) or using a BP-filter of Christiano-Fitzgerald for 2 to 8 years, the model would predict even better, so the low order HP-filter makes it harder given the strong trends in the data for certain labor market series. Our model is not designed to explain this trends within a business cycle estimation procedure.

37 The fact that we basically employ a RBC interpretation using (essentially) one technology shock does not imply that we believe other shocks to be unimportant. However the estimation results in Jung and Kuester (2007) based on a simplified labor market model with monetary and wage frictions suggest, that the basic mechanism of strongly pro-cyclical profits driven by a wage friction as the driving force for labor market volatility does survive the extension to a more elaborate model featuring a more realistic set of shocks. In addition a parsimonious shock structure simplifies the analysis on where and how the basic mechanism works and where it can be improved.

38 Box 1 standard aggregate macroeconomic targets we wish our model to match, box 2 lists preference related parameters, box 3 list parameters related to the government, box 4 describes the technology parameters detailed in table(1),
to standard labor market parameters, we set the elasticity of the matching function to .5 as in Hall (2005), set the vacancy posting cost to 11% of the average monthly productivity of the match and the hiring probability to 70%, as argued for in Hagedorn and Manovskii (2006). The bargaining power and the strike value are not jointly identifiable in our framework, so we set the bargaining power arbitrary and innocently to 50% and choose the strike value such that the unemployment rate is 5.4% for males and 6% for females as in the data. We target the right amount of unemployment volatility by setting $x$, the sticky-wage parameter in our wage setting equation to match the volatility of the unemployment to population rate, side-stepping the debate surrounding Shimer’s work. Note that our choice of $x$ is almost equivalent to a fixed outside option in a standard model, given that procyclical wage income is counteracted by procyclical technology, making the strike-value acyclical. Our results still hinge on a small surplus calibration. In our calibration vacancy posting cost and thereby profits from a match from the firms perspective are small, implying that the resulting wedge is small and wages are close to the competitive equilibrium outcome. Note, however, that the surplus for the family (and thereby the total match surplus) is substantial in our calibration. The family would pay 3.1 times the average monthly net wage rate (or, equivalently one quarterly wage), to shift a male worker directly from inactivity to work, and 1 month net wage for a female.

**Calibration of Parameters - New Features:** We now turn to a discussion of the new parameters in our model: We harmonize as many preference parameters across males and females as possible. The first set of new parameters are related to the utility cost parameter $\tilde{s}_{g\varrho}$ and the variances $\tilde{\psi}_{g}$ of the discrete choice distributions. We target these parameters by the mean transition rates separated by sex and the variances of the choice probabilities obtained from our estimation procedure. We also try to harmonize as much as possible the utility parameters governing the average utility cost across inactivity and unemployment.

---

39 Small surplus calibrations have been criticized forcefully, on a priori grounds. This might be correct but it implies also that labor markets were, on average, far away from the competitive outcome. Scholars arguing in favor of substantial profits must acknowledge that the world is substantially away from the neoclassical competitive benchmark.

40 Note that our model can sustain a substantial surplus relative to the calibration of say Hagedorn and Manovskii (2006) because our bargaining threat point is an abstract strike value that reflects labor market inefficiencies without relating them to the real surplus. This trick, used in Hall and Milgrom (2007), of course implies that the surplus relative to the threat point is still minuscule.

41 Given that our parameter setting is subject to the constraint that the value of employment must be bigger than the value of unemployment and the value of inactivity, the range of possible parameters is limit given all other targets. We were unable to generate enough volatility in the transition rate from inactivity to unemployment and from unemployment to inactivity. We chose the parameter governing search-cost $s_{ou}$ to be zero, the boundary value. We varied the distribution parameter of $\psi_{ou,m}$ within the range of permissible values such that the value of employment is bigger than the value of inactivity and picked the local maximum. However, the parameter is hard to identify and does not influence the cyclical...
sex and let the differences between sex be captured by the underlying distributions of the shock. As discussed in the intuition section the variances are informative about the key elasticities that will drive the reaction of the model to changes in the tax rate.

**Time-Aggregation:** An immediate problem of targeting the mean rates is the fact that our model does not have a direct transition from inactivity to employment, which is quantitatively non-negligible. By definition these flows do not exist\(^{42}\) and are an immediate consequence of the time-aggregation bias highlighted in Shimer (2007). However, in our view the main problem is not so much the aggregation from a continuous time framework to a discrete time framework, but the treatment of the flows from employment to inactivity and vice versa. As shown in Nagypal (2005) around 40% of the transitions from employment to inactivity are followed by a flow to employment with one intervening month. Naturally what might happen is that workers have searched on the job, obtained a new job but due to some time frictions the starting date is postponed by a month. Potentially the worker has to move, or the worker simply takes a vacation. In fact we view part of these flows as quits on the job. We deal with this problem by looking at the net flows from employment to inactivity. According to the data of Fallick and Fleischman (2004) the flows from employment to inactivity are roughly 2 times as large as the reversed flows. So we treat half of the flows from employment to inactivity as real quits, and the other flows as part of the job-to job transitions. Table (3) lists our chosen targets. The stars indicates that these variables are not freely chosen but need to adjust to fulfill the requirements of a probability transition matrix. Note that our job-finding probability, while at odds with the unadjusted monthly rates, is close to the average job-finding probability reported in Shimer (2005), who argues that this rate was on average 45% over the last 60 years. This is a consequence of our treatment of the net-flows from inactivity to employment.

**Technology across state and sex:** The second set of non-standard parameters are related to the productive capability across sex on the job relative to the home-market. This margin is of crucial importance for our long run optimality results, so we will provide an extensive sensitivity analysis to our choice. In our benchmark scenario we chose the productivity differences between males and females working in the market. We use the time-use survey data of Aguiar and Hurst (2007) and compare the total amount of time spend working of an employed worker relative to a non-employed worker using the definitions of work/leisure as highlighted in Aguiar and Hurst (2007). We find, that on average, a female supplies .77 as much time as their male counterpart to market work. However, properties much.

\(^{42}\)By definition a worker who obtains a job must have been matched to an employer, and must have exerted a time-search cost, at least by answering the phone or appearing to an interview. So, in any case, there must have been a time where he was actively searching, which implies that he must have been unemployed in between.
employed females do not enjoy more leisure, but use the time-difference mainly working at home. More crucially for our purpose is the relative amount of time supplied when out of the labor force compared to the employment state. While the classification, what accounts as leisure, and what accounts as work, is somewhat arbitrary, see the critic by ?; this effect is reduced by looking at the quotient, given that a miss-specification shows up both in the numerator and the denominator. In our benchmark calibration inactive males spent roughly 50% less time in undertaking productive activities (or enjoy 50% more leisure $h_{m} = 1/2$), while females enjoy roughly 1/3 more leisure ($h_{f} = 2/3$). Given that employed females work, on the job and at home, jointly as much as males when employed, this implies that inactive females do devote a substantial amount of time to activities that should be economically valued by society. In terms of efficiency units ($A_{g,e}$) this implies that females are roughly 40% as productive at home than their employed counterparts, while males are only 26% as efficient at home as on the job.43

Preferences: The third set of parameters is related to the utility specification. As discussed in the intuition section, our basic mechanism supposes that, say a female, when making her entry decision into the labor market, evaluates her time spend in the home market at the opportunity cost of time, which is the market wage. We therefore work with a log utility specification of overall consumption to be consistent with balanced growth, and choose, as a benchmark, the case of perfect substitutability between the home produced good and the market service good. We set the elasticity between the final good and the service good ($\gamma_1$) to .4 in line with the findings of ? and set the share $\gamma$ to .5, which implies that 10% of the working population works in sectors with a high substitution elasticity to home-produced goods. Our results are not sensitive to the choice of these last two parameters. Perfect substitutability might appear as a strong assumption given the findings. Note however, that ? look at a two good economy, while our argument relies strongly on the ability of say a female to buy child care in the market. We discuss the sensitivity of our results with respect to this important elasticity extensively below.

Government: Finally, with respect to parameters influencing governmental policy, we set the labor tax rate to 24% as observed in the data, choose a government expenditure to output rate of 20% and set the capital tax residually (28.7%).44 In our model unemployment benefits payed to the unemployed would work as a subsidy to search, given that our model does not differentiate properly between

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43The fact that females work less hours in the market than males implies in our model that females are less efficient. We normalize $A_m = 1$ and the hour differential (and also the wage differential) between males and females implies that $A_f = .68$. What matters for our purposes is not so much the absolute number, but the relative distance to home-production. Here $A_{o,m} = A_{u,m} = .26$ and $A_{o,f} = A_{u,f} = .28$.

44The data would suggest a rate of around 30% for the before depreciation capital tax, and around 50% for the after depreciation capital tax, see \cite{RupertGomme:07} and \cite{Tchilinguarian}.
unemployed coming from employment state and unemployed coming from inactivity.\textsuperscript{45} We therefore treat unemployment benefits as transfers to the family that do not influence the decision of starting to search when inactive.\textsuperscript{46}

We estimate the autocorrelation and variances of our AR(1)-processes directly from the data and let the cyclical residual be picked up by lump-sum transfers to fulfill the balanced budget.\textsuperscript{47}

\textbf{4.1.1 Evaluation of the Model}

This section summarizes the properties of the model with respect to key labor market variables. Table (4) gives the basic moments of the model and Figure(1) to Figure(4) visualize the performance of the model for selected series:\textsuperscript{48}

\textbf{Employment States:}

The first fact to highlight is that the model predicts the unemployment rate very well, as figure (2) makes clear. The actual series correlates with .92 to the predicted series. Given that unemployment and output are highly correlated and we targeted the relative standard deviation of unemployment to output in the calibration, this result is not entirely surprising, though still a reconfirming feature of the model. It shows that the Mortensen-Pissarides model with a wage-frictions is an excellent model for discussing cyclical properties of labor market variables, in contrast to recent claims in the literature.\textsuperscript{49} The model overpredicts the volatility in the employment-population rate for both males and females and overpredicts the correlation for females while it matches almost perfectly the reaction for males. This is mainly due to an overshooting in the oil-price crises in the eighties and at the end of the sample, which might be driven by the strong labor tax decline, that might be miss-measured.

\textbf{Nipa and BLS-aggregates:}

The model predicts, see figure (3), investment almost perfectly, while it under-estimates the volatility

\textsuperscript{45}To capture this effect the model would have to introduce an additional state and additional flows making the model even more complex. Given that the role of unemployment benefits is not the main focus of the paper we chose to assume it away.

\textsuperscript{46}Obviously, the model can easily be recalibrated and unemployment benefits can be included. None of our basic results would change.

\textsuperscript{47}Our estimated autocorrelation-coefficient, given the low order HP-filter, for the TFP shock is only .86, significantly lower than is typically used in the literature. However, labor market models offer considerably more persistence than the neoclassical model, so our model does not need as much "exogenous" autocorrelation as most competitive models.

\textsuperscript{48}All data are in log, given that our mean rates do not always correspond to the data-means and might otherwise have a feedback effect into the standard deviation.

\textsuperscript{49}This does not mean that the, ad hoc, wage friction is a satisfying modeling device, but it does mean, that alternative explanations have to be at least as good along this margin and better in some other dimension. This has yet to be shown.
in consumption, but captures the correlation correctly. The model fails to match the wage per hour series obtained from the BLS for non-farm business almost completely, at least during the late eighties up to now, even though the correlation-coefficient comes close. The model correlates very well with the total hour measure from the BLS, but underpredicts the volatility. This is mainly due to an underprediction of the average hours worked per person, given that total employment is matched rather accurately.

Two remarks about this failure are in order: first, note that Hagedorn and Manovskii (2006) use the role of wages over the cycle as a target to explain the success of the Mortensen-Pissarides framework in explaining unemployment volatility. We caution to judge the ability of the Mortensen-Pissarides model to generate unemployment volatility by a matched wage-elasticities alone, either by claiming a success or by claiming a failure, given the picture provided above. As the prediction makes clear the correlation-coefficient (and likely the elasticity) is not a sufficient statistic for capturing the wage dynamics.

However, second, it is unlikely that any standard RBC model will be able to explain the behavior of the above wage series given that between 1994 and 2000, say, common believe, our TFP-shock and our hours worked measure suggest a substantial boom of the US economy, while wages suggest a severe depression. If these data are actually correct, our model is unable to explain them and the wage friction we highlight is problematic. However, the wage and hour concepts used by the BLS is clearly not consistent with other wage and hour series used in the literature, so some caution is appropriate. This point is emphasized in who argues that Nipa compensation is seriously miss-measured.

**Labor Market Probabilities:**

As mentioned in the calibration section, we were unable to generate enough volatility for the unemployment to inactivity flows and vice versa. However, given that we understate both rates, part of the failure might actually cancel out. As the picture, see figure (4), and the tables, see table (4) and (5), make clear the model captures the correlation to output as well as the entire cyclical movements very well. In fact, it is interesting to note that transition rates from unemployment to inactivity are

\footnote{Note that in principle, given that we use government expenditure and output in our estimation procedure and match investment by implication, consumption should be matched perfectly as well, being the residual. Of course, given that we filter all series separately the HP-filter distorts the identity, so does foreign trade.}

\footnote{Note, however, that the model makes strong assumption how wages are split, but this must not correspond to wages payed. The timing of wage payments is undefined given that both the family and the firm would be willing to shift the discounted flows back and force in time. Some small cost involved in changing payed wages over the quarter could give raise to very different observed wage-payments. However, obviously, this argument is a cheap way out of the problem, making the theory not falsifiable.}
positively correlated with the business cycle, while the transition rates from inactivity to unemployment is negatively correlated with the business cycle, in the data and the model. Intuitively one might have expected that in a boom more, say, females would start searching, not less. Apparently the wealth effect overturns this result, given that the family as a whole is richer in a boom and does not need to force that many females with a potentially negative shock realization to enter the search pool. Flows from inactivity to employment are procyclical in the data, which we capture in our model by strongly procyclical quits. Quits to inactivity are countercyclical in the model and the data though our model overpredicts the correlation. The model explains firings very convincingly, if we abstract from high-frequency noise that surrounds the estimate. In our model, firing rates (or separations from employment to unemployment) and the job-finding probability are driven by the same variable, II, the surplus of the match from the firms perspective, which corresponds to the prediction of the standard model. Given the picture the standard insights offered from the Mortensen-Pissarides framework appear to be correct. This also implies that the debate on the importance of the “Ins” versus the “Outs” appears less severe given that both rates are driven by the identical force.

Finally, it is worthwhile to mention that the model generates the right beveridge-curve despite the fact that firing rates are endogenous. To summarize, the model, driven by a parsimonious shock structure does capture the basic movements and correlation signs of all probabilities correctly, and predicts the time-path of all labor market transitions and states in our view to a reasonable extent. A richer shock structure as well as an interaction with some standard macroeconomic frictions not considered in this model might enhance the fit. The price to pay though is the loss of a parsimonious shock structure. Also, a much better understanding of the behavior of hours worked and wages per hour is needed to claim a real success.

4.1.2 Optimal Long Run Capital Taxes

Having established that the model at our benchmark calibration provides a reasonable description of the labor market, we now turn to discuss the quantitative implications of the model for long run optimal taxation. We proceed as follows: we first solve the model for an observed labor tax rate and solve for the resulting parameters using the stated calibration targets (or the particular deviation from the above described targets, we are interested in). Given these parametric values we then ask how the Beveridge-curve is not valid in our model. Even if firings are double or triple as volatile as quits on the job, and, correspondingly, destruction rates were counter-cyclical, we would still obtain a negative correlation between unemployment rates and vacancies. What is crucial is the endogeneity of both rates, while Shimer makes them exogenously fluctuating.

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52It is interesting to note that the argument of Shimer (2005), that fluctuating destruction rates would destroy the Beveridge-curve is not valid in our model. Even if firings are double or triple as volatile as quits on the job, and, correspondingly, destruction rates were counter-cyclical, we would still obtain a negative correlation between unemployment rates and vacancies. What is crucial is the endogeneity of both rates, while Shimer makes them exogenously fluctuating.
Ramsey planner would set tax rates optimally. Table 6 summarizes our results where data in bold emphasize the benchmark calibration.

Before turning to the main results we first explore the performance of the model in special circumstances. Given that the model has many non-standard features that might drive our results it is helpful to show that the model behaves well in certain limiting cases:

**Zero Government Expenditure and Matching Friction:**
We consider two extreme cases: In the first extreme case we shut down all governmental expenditures (so consumption takes up its part) and assume that $\gamma = 0$ and, correspondingly, that labor and capital taxes were also zero initially. Box 4 gives the optimal labor and capital taxes in this case. Optimal tax rates are non-zero but quantitatively small. The reason for a positive capital tax is the distortion that arises due to the labor force participation friction. Recall that in the planner’s problem the socially optimal entry behavior would be based on the comparison between the price of the labor good and the price of the home-produced good. In the decentralized solution this decision is based on gross wages. Given that in our models firms make positive profits once matched to a worker, wages are not equal to the marginal product of labor. This wedge distorts the entry decision of inactive workers. The planner uses its influence to distort the intertemporal participation decision by subsidizing labor to corrected for the friction. The optimal Ramsey plan is to use labor taxes to subsidize wages to counter this wedge. Quantitatively this wedge is a direct function of the assumed amount of vacancy posting cost, which in turn determine directly the amount of profits that goes to the firms.

In the second extreme case, discussed in the first column of Box 1, we assume that there is no home-production technology available. That is workers at home do not supply any labor. Correspondingly there is no labor-force participation friction. Again, optimal capital taxes are close to zero, yet slightly negative. Without the unemployment part of the model as described in the introduction, capital taxes were precisely zero and we were back in the neoclassical world. However, the matching market introduces a small friction given that we do not evaluate the model at the Hosios efficiency condition, which does not hold in our model. Therefore the search friction leads to a deviation from zero capital taxation, but for small equilibrium profits this friction is quantitatively negligible.

Having shown that at the extreme cases our model is, almost, back at the neoclassical zero capital tax result we now turn to a quantitative assessment of the importance of labor force participation.

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53 It is hard to keep all targets identical if we look at some extreme cases. So we fix the average utility fixed cost at its benchmark values and allow the distributions to adjust to ensure that, for in each case, the initial labor market states and mean probabilities are identical.

54 This is not a numerical error, but is a function of the vacancy posting cost.
friction, when the government needs to finance a stream of expenditures but has only an insufficient set of direct instruments at its disposal to accomplish its spending pattern.

**Technology:**

The bold case in box one gives our preferred calibration and is the benchmark case. We see that the Ramsey planner wants to reduce capital taxes from 28.6% relative to our initial parameterization to around 23% and increase labor taxes instead from 24% to 25.8%. This implies that in particular for females the employment to population rate drops by 2%. In our benchmark case we see that the resulting friction is substantial. Taking capital taxes of zero as a (neoclassical) benchmark the entry decision we wish to stress in this paper can be quite substantial and amounts to an Ramsey-optimal capital tax of 23%. The other columns in box 1 make clear that our quantitative result hinges strongly on the technological distance. Recall from the discussion of the simplified model that the crucial distance driving the friction was given by \( w(1 - \tau_l - A)\lambda_1 \). If goods are close substitutes the planner would want to tax them at the same rate. By assumption we do not allow him to do so. Reducing capital taxes towards zero forces the government to increase labor taxes. The more efficient workers can produce at home the stronger is the influence of tax rates on their decisions. We used hours worked as our identifying mechanism to estimate this distance. However, this estimate is crude at best. If, in particular females, are less productive at home than in our benchmark case, the resulting friction will be softened considerably. For example, by doubling the technological distance capital taxes can be reduced by 2/3 in the long run. Labor taxes are much less distortive and the world moves towards the neoclassical benchmark again. Note also that, by the choice of our wage friction, the unemployment to population rate is almost unaffected by tax rate changes.

**Preferences:**

The second important margin, as discussed in the introduction, is given by preferences. In our preferred setup we assume that the market has the technological ability to reproduce home-produced goods perfectly. However, if society values home-produced goods, in particular child care, as strongly different goods, labor taxes again lose part of their intertemporal distortion. Box 2 shows this case: The lower the substitution technology the more unique home-produced goods become. This implies that the family, valuing variety, will happily assign workers to produce this unique good that cannot be

55 So, at our benchmark parameterization, female labor force participation rate is roughly linear in labor tax rates. For example, if we fix the capital tax at 28.6% and reduce government expenditure such that, balancing the budget, labor taxes are around 14%, then the 10 percentage point decline in labor taxes leads to an 11 percentage point increase in female employment and a 3 percentage point increase for males.

56 Of course, if the distance is actually smaller, which might be the case at least for subgroups of the population, labor taxes would be even much more distortive.

37
produced otherwise. The described wedge between gross and net wages weakens, and the love for variety overturns the trade-off of having to tax a similar good at different rates.

Let us defend why the extreme case of perfect substitution might be a good benchmark case. First, the reason why we introduced the three good structure was, essentially, to derive within an equilibrium model the trade-off an inactive worker, say a female, faces when entering the main labor market. She has to buy a fixed amount of child care or food preparation in the market, that is the time watching the child while she is away, or the time saved in not having to prepare a meal. This costs the market price of child care, proportional to gross wages (of low skilled workers), or the market price of prepared food, which are all in gross terms. In contrast, the time spend at home is in net terms, because the government cannot tax home-produced goods by assumption. So an alternative interpretation of our three good setup would be a two good model where workers are forced to buy a fixed amount of services to be able to work. Second, in our identification scheme using time spend in home-production we look at the relative differences in time spend. That is, while spending time playing with your own child instead of buying child-care time in the market is clearly a different good and likely not perfectly substitutable, but is not captured in our measure. Employed females spend almost as much "active" time with their children than inactive females. What matters is the difference in time used. We argue that a professional child nurse performance a very similar task when watching the sleeping child as the mother.

**Distribution:**

Our discussion of the results in table (6) is complicated by the fact that a change in one target has implications for other steady state relation not mentioned so far. In example the consumption ratio between the goods changes as well as the amount of workers allocated to the production of each good. Also, to fulfill all other targets, the variances of the distribution change across case. In particular we have not re-estimated the model in each case to match the volatilities but have fixed the average preferences at the benchmark case and allowed the underlying distribution to vary to ensure that the model is evaluated at the same average transition rates and labor market states. In the benchmark case, if we ignore the second moments of the model and vary the underlying distribution labor taxes can be more or less distortive. For example, if we reduce the cost for quitting from employment to inactivity by half, and, correspondingly increase the variance roughly by 2, capital taxes would increase from 23% to 29%, which is sizable. Similar results can be obtained for all other rates. A good estimate of the variance of the idiosyncratic distributions clearly matters quantitatively, though its impact is slightly less important than the two other margins mentioned.

57 Changes in Firings and Quits on the job are, though, much less distortive than changing distributions that impact inactivity rates.
The role of the intratemporal substitution elasticity: Does the intratemporal substitution elasticity matter at all? So far, our discussion has circled around the new dimensions of this model relative to the standard labor-leisure choice. However, in our calibration strategy the intratemporal elasticity does matter strongly, because it pins down, implicitly, the technological distance described above. The relative hours worked choice across labor market states is a direct function of this elasticity. Making it, say, less elastic, implies that for given targets above the technological distance is increased, reducing the labor force participation friction and weakening the counteractive force of capital taxes. In this sense it is still a crucial parameter. However, note, that, given separability of preferences and the particular functional form chosen, our elasticity implies, for example in the case of a 5% increase of labor taxes in column 2 a decline in hours worked on the job by .66% for males.\(^{58}\) Understanding the impact of taxes on labor force participation decisions appears quantitative more important than understanding the standard leisure choice for employed workers.

**Gender Based Taxes:**

Positive capital tax results hinge, typically, upon the number of available instruments. In our model we have forced the government to use a uniform tax rate for males and females evaluated at our benchmark parameters. The final column in table (6) relaxes this constraints and allows the government to set different labor tax rates for males and females. Gender based tax rates have been popularized by Alesina and Ichinol (2007). In our case gender based taxes would be very powerful. Optimally, female labor tax rates are basically cut to half relative to the optimal uniform tax rate, while male tax rates would increase by 7% points. As a result, first, labor force participation rates across males and females would almost equalize and roughly half of the inactive female population would be driven into the main labor market. Second, not reported, given the strong wealth effect and tax effect (output increases by 20% due to the increase in capital and labor), males hours worked would drop dramatically from .3 to .26, while females hours worked choice would increase from .231 to .24. Capital taxes would be almost back at zero (roughly 4%).

Of course, proposing discriminatory tax rates based on gender, would also imply, by the same logic, to use tax rates on, say, age, given that life-cycle data would suggest a significant correlation of age on wealth which in turn will drive hours worked choices. However, given the obvious problems of political economics involved in these proposals one can interpret the above results as indicative for proposing a progressive tax code. Given that female work income, due to a significant lower hour choice, is very different from males, one could use, in our model, a simple two bracket tax rate code based on income or some tax rebates, to achieve a similar goal without provoking by using a discriminatory tax.

\(^{58}\) Across steady states the wealth effect appears to be much more important then the pure substitution effect.
Summary of the Results:
Before voting for a labor tax increase (or equivalently before voting on a zero capital tax) as suggested in Atkeson, Chari, and Kehoe (1999) it appears important to ensure that the resulting friction is indeed small. To state it explicitly, it is clear that, in principle, the government has other mechanisms at its disposal than a capital tax to counteract the above problem. Our results should not be interpreted as a universal call for capital taxation per se. They could, though, be interpreted as evidence that labor taxes have, potentially, substantial intertemporal distortions associated with them that have been neglected in the neoclassical world.

Our analysis has focused on average tax rates and has highlighted different margins that are quantitative very important. What drives the above results, though, is not necessarily the average tax rate we used in this paper as a proxy for the average distortion but the difference between tax rates and work subsidies. For example, northern european countries, having substantially higher labor tax rates, are able to sustain a high labor force participation rate because they have, at the same time, substantial subsidies for child care. This in turn weakens the above described distortion of labor taxes and possibly even subsidizes the entry decision. What matters in our analysis is the relative distance between working at home and working on the job and the relative distortion of labor tax rates net of subsidies.

4.2 Optimal Fluctuations
Having shown that in the long run in our model labor tax rates have, potentially, a strong impact on the behavior in particular of females to enter the main market, we now ask, whether the government should use its influence on this margin to smooth the cyclical volatility of output by using, say females, as a buffer stock? By raising labor taxes in booms and lowering it in recessions the government clearly can influence employment volatility.

Should the government therefore deviate from the standard neoclassical description that optimal labor taxes should not vary much over the cycle? The answer is that the neoclassical prescription still holds. The reason that the government does not want to use its influence too strongly is that in our model the government takes into account the idiosyncratic shock distributions and the substantial cost for some workers to move. Moving, say females, in and out of the labor force over the cycle is simply costly activity.

Ramsey-Optimal labor tax path:
To show this we use the estimated exogenous TFP shock and the governmental expenditure shock

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59 Or, more simply, ensuring that different policy instruments like subsidizing child care or lowering tax rates on service products related to home-produced goods become smaller at the same time.
as given, and ask how the economy would have evolved if US taxes had followed a Ramsey optimal rule. We find that the logarithm of the optimal labor tax rate should have been half as volatile than the measured actual tax rate. However, the cyclical properties are fairly similar. Figure (5) plots the predicted tax series as well as the predicted employment sequences for males and females under an optimal rule. We see that the access volatility in employment rates is reduced under the optimal one compared to the volatility we showed above under an exogenous policy rule, but the reduction is only modest. In fact part of our excess volatility on employment rates might have been driven by "labor tax rate shocks" who were fully anticipated by the agents as part of a governmental rule. In any case, the cyclical path of labor market variables would not be very different under an optimal rule. Can we therefore conclude that the explicit modeling of the inactivity margin is unimportant for cyclical policy?

**What can go wrong:**

To show how important endogenous transition rates might be figure (6) plots the behavior of the model when we fix capital taxes at the steady state value and let labor tax rates only balance the budget. This happens in some models who specify, say, a total income tax rule (typically jointly with a debt stabilization rule). In this case, labor taxes are counter-cyclical! Even though overall volatility of tax rates increases only modestly compared to the actual and unemployment volatility is not strongly affected, we see a strong impact on employment rates. This is due to a strong cyclical behavior of the probability to enter the market. The search flow is quickly absorbed into employment rates and employment volatility strongly increases, so does output volatility. Counter-cyclical labor tax rate were a disaster.

This example makes clear that even though observed transition rates might not have contributed much to the fluctuation in the employment to population ratio as claimed in Shimer (2007), this should, in our view, not lead to the conclusion of not modeling these transitions carefully when conducting policy experiments. A wrong policy can have very bad consequences along these margins.

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60 The actual tax rate shows high volatility in the end of the sample, in line with the behavior of our wage per hour measure (and also overall compensation measure from which labor taxes have been inferred) measure. Again, even though we followed a standard procedure from macroeconomics, see ?, in constructing our labor tax measure it is clear that actual tax rates might have behaved rather differently given that NIPA compensation might be seriously miss-measured. The time-series are, therefore, at best a crude approximation, but the best that was available to me for the time period considered. In particular we suspect that the volatility is strongly overstated.

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41
5 Conclusion

This paper has disentangled the intratemporal elasticity of substitution into its main labor market components within a general equilibrium framework. We showed that the technological distance, distributional heterogeneity and the valuation of home-produced goods by society are crucial margins in understanding labor market flows and in conducting optimal fiscal policy. We gave conditions under which labor taxes are intertemporally distortive and showed that, quantitatively, the distortion can be substantial.

Our results suggest that a vote in favor of zero capital taxes should be accompanied by an argument that insures that labor taxes are not intertemporally distortive or that other instruments are available to counter the distortion. Progressive taxes, or, if one prefers, gender based taxes, could take this role. Cutting taxes by half for females and increasing tax rates for males could equalize labor force participation rates quite dramatically.

The model relies on a discrete choice setting introducing idiosyncratic utility cost typically absent in the standard model. We view these distributions as an insurance mechanism against miss-specification of the model. Given that our model can not control for females with and without a child, we hope to capture with these distributions precisely this underlying heterogeneity. However, a richer model that explicitly controls for this dimension will surely provide a more accurate picture of the distorting effect of labor tax rates. Also, a more careful treatment of the tax-code is an obvious next step. In this model we proxy for the average distortion by using the average tax rate. However, what matter for labor force participation is not the average tax rate per se, but the tax rate net of subsidies. Families receive many direct or indirect transfers that also vary greatly across country. A comparison based on average tax rates alone might be seriously misleading. Finally, embedding the above structure into an incomplete market model might be an important next step to control more carefully for the distorting wealth effect. Whether these types of models can be made consistent with the aggregate labor market dynamics documented in this paper will be an interesting challenge.
References


43


6 Appendix

The appendix summarizes our calibration procedure and shows the predictions of our model.

6.1 Calibration

We first describe the data used in the estimation procedure. Table (??) shows total hours worked obtained from the American Time Use Survey, table (2) gives the parameter values chosen together with the description of the targets and table (3) gives the average monthly probabilities we target.

6.1.1 Data

Aggregate data are nominal output, consumption (non-durable and services), investment (plus durable consumption) and government expenditure taken from the NIPA and deflated by the GDP-deflator and expressed in per capita terms. Average labor and capital taxes are obtained using the procedure outlined in ?. Total Hours worked and wages per hour in non-farm business are taken from the BLS as well as aggregate labor market states. Labor market probabilities are taken from Shimer (2007). In the plots we use his aggregate measure and his time-aggregation correction downloaded from his homepage. For the HP-filter employed this measure turns out to be very similar with respect to its cyclical properties as the monthly unadjusted rates. For the years before 1976 his data are taken from ? and ?. The monthly rates separated by sex are taken from the CPS micro-data using the (adjusted) code again kindly provided by Shimer (2007).

6.1.2 American Time Use Survey

<table>
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<tr>
<th>Year</th>
<th>Work-Male (Emp)</th>
<th>Work-Female (Emp)</th>
<th>Work-Male (Inactive)</th>
<th>Work-Female (Inactive)</th>
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</thead>
<tbody>
<tr>
<td>1975</td>
<td>51.55</td>
<td>53.47</td>
<td>22.7</td>
<td>39.04</td>
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<tr>
<td>1985</td>
<td>53.39</td>
<td>52.82</td>
<td>27.95</td>
<td>36.78</td>
</tr>
<tr>
<td>1993</td>
<td>53.63</td>
<td>52.97</td>
<td>28.64</td>
<td>35.15</td>
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<tr>
<td>2003</td>
<td>54.15</td>
<td>52.2</td>
<td>24.13</td>
<td>33.37</td>
</tr>
</tbody>
</table>

Notes: The data have been made publicly available by ? and we follow their definitions. We drop all people under the age of 21 and over 65 as well as retired workers from the sample. We then condition on age (not reported), sex and employment status. We do not use fixed sample weights controlling for changes in demographics across surveys as in ? but report averages across our demographic cells, given that changes in these cells are our primary focus. Our definition of work uses their definition, for details see ?: work = work core +home production
work core = regular-work+working-at-home+overtime+moonlighting
home production = meals + housework + home car maintenance + home other + garden - pet + obtaining - goods
### 6.1.3 Calibration - Parameters

#### Table 2: Basic Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
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</thead>
<tbody>
<tr>
<td>Box 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β</td>
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<td>Capital-output-ratio=4</td>
</tr>
<tr>
<td>α</td>
<td>0.400</td>
<td>Capital share=.4</td>
</tr>
<tr>
<td>δ</td>
<td>0.004</td>
<td>Investment to output ratio:.2</td>
</tr>
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<td>ρ</td>
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<td>95 (quarterly)</td>
</tr>
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<td>m_m</td>
<td>1.000</td>
<td>Population normalization</td>
</tr>
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<td>m_f</td>
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<td>Population normalization</td>
</tr>
<tr>
<td>Box 2</td>
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<tr>
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<td>Equal across sex</td>
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<td>10% in labor sector in the market</td>
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</tr>
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<tr>
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<td>σ_f</td>
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</tr>
<tr>
<td>σ_1,f</td>
<td>0.947</td>
<td>Average  f_f</td>
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<tr>
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<td>Average  q_f</td>
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<tr>
<td>σ_1,q,m</td>
<td>12.45</td>
<td>* Std(q_m)</td>
</tr>
<tr>
<td>σ_1,q,f</td>
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<td>* Std(q_f)</td>
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<td>σ_1,q, m</td>
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<td>Std(q_f)</td>
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<td>*Std(q_m)</td>
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<td>σ_1,q</td>
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<td>σ_1,q</td>
<td>0.75</td>
<td>Wage Stickiness</td>
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</table>

**Notes:** This table gives the numerical parameters used and the attempted targets the particular parameter influences the most. The stars indicate that we were unable to achieve this target. We set  \( \sigma_{1,m} \) to the boundary value 0. Given all other targets the influence of  \( \sigma_{1,m} \) on the variance is small and we were unable to generate enough volatility there, given that again we hit the boundary that the value of employment must be bigger than the value of inactivity on average. We varied the parameter in the allowable range and set it to the value that gave the maximal variance, even though the impact of this parameter on the results is rather small.
### 6.1.4 Average Probabilities

**Table 3: Mean-Probabilities**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model</th>
<th>CPS Monthly</th>
<th>Shimer (2007)</th>
<th>Fallick et al. (94)</th>
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<tbody>
<tr>
<td>Job to Job (Male)</td>
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<td>0.025</td>
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<td></td>
</tr>
<tr>
<td>Job to Job (Female)</td>
<td>0.025</td>
<td>0.025</td>
<td></td>
<td></td>
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<tr>
<td>Job to Job (Average)</td>
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<td>0.025</td>
<td>-</td>
<td>0.025</td>
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<tr>
<td>Firing (Male)</td>
<td>0.017</td>
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<td></td>
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<tr>
<td>Firing (Female)</td>
<td>0.013</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firing (Average)</td>
<td>0.015</td>
<td>0.015</td>
<td>0.021</td>
<td>0.013</td>
</tr>
<tr>
<td>Quit E to Inactiv (Male)</td>
<td>0.011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quit E to Inactiv (Female)</td>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Quit E to Inactiv (Average)</td>
<td>0.014</td>
<td>0.029</td>
<td>0.029</td>
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<tr>
<td>Job-Finding (Male)*</td>
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<tr>
<td>Job-Finding (Female)*</td>
<td>0.500</td>
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<tr>
<td>Quit U-Inactivity (Male)</td>
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<td>Quit U-Inactivity (Female)</td>
<td>0.270</td>
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<td></td>
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<tr>
<td>Quit U-Inactivity (Average)</td>
<td>0.215</td>
<td>0.219</td>
<td>0.311</td>
<td>0.230</td>
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<tr>
<td>Inactivity-U (Male)*</td>
<td>0.058</td>
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<tr>
<td>Inactivity-U (Female)*</td>
<td>0.043</td>
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</tr>
<tr>
<td>Inactivity-U (Average)*</td>
<td>0.048</td>
<td>0.026</td>
<td>0.044</td>
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</tr>
<tr>
<td>Inactivity-E (Male)</td>
<td>-</td>
<td>0.055</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactivity-E (Female)</td>
<td>-</td>
<td>0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactivity-E (Average)</td>
<td>-</td>
<td>0.046</td>
<td>0.044</td>
<td>0.048</td>
</tr>
<tr>
<td>E (Male)</td>
<td>0.710</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>U (Male)*</td>
<td>0.041</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O (Male)</td>
<td>0.250</td>
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<tr>
<td>E (Female)</td>
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<tr>
<td>U (Female)*</td>
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<tr>
<td>O (Female)</td>
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**Notes:**
6.2 Basic Results

Table (4) and table (5) give the basic correlations and standard deviations.

<table>
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<tr>
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<th>Relative Std to Y</th>
<th>Corr with Output</th>
<th>Corr(Model,Data)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td></td>
<td>Std</td>
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<tr>
<td>Y</td>
<td>0.022</td>
<td>0.022</td>
<td>1.000</td>
</tr>
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<td>C</td>
<td>0.344</td>
<td>0.690</td>
<td>0.771</td>
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<tr>
<td>I</td>
<td>3.857</td>
<td>3.551</td>
<td>0.965</td>
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<tr>
<td>G</td>
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<td>1.427</td>
<td>0.227</td>
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<td>1.550</td>
<td>0.332</td>
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<td>$\tau_c$</td>
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<td>2.208</td>
<td>0.424</td>
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<tr>
<td>Hours</td>
<td>0.791</td>
<td>1.218</td>
<td>0.915</td>
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<td>0.259</td>
<td>0.724</td>
<td>0.375</td>
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<td>-0.709</td>
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<td><strong>Transition Probabilities:</strong></td>
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<td></td>
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<tr>
<td>U-Rate</td>
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<td>6.986</td>
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<td>Vacancies</td>
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<td>EU</td>
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<td>3.671</td>
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<tr>
<td>EN</td>
<td>1.977</td>
<td>1.931</td>
<td>0.658</td>
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<tr>
<td>UE</td>
<td>7.829</td>
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*Notes:*
Table 5: Basic Properties (HP=1600)

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<td>Model</td>
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<td>Std</td>
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<td>Employed (Male)</td>
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<td>Employed (Female)</td>
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<td>Out of labor (Female)</td>
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Transition Probabilities:

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<th>EN</th>
<th>UE</th>
<th>UN</th>
<th>NU</th>
<th>NE</th>
<th>Q</th>
</tr>
</thead>
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<td></td>
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<tr>
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<td></td>
<td></td>
<td>0.729</td>
<td>0.269</td>
<td>0.588</td>
<td>0.614</td>
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<td>0.729</td>
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<td>0.614</td>
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<td>-</td>
<td>-</td>
<td>0.588</td>
<td>0.614</td>
<td>0.435</td>
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</tr>
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<td></td>
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<td>-</td>
<td>-</td>
<td>0.588</td>
<td>0.614</td>
<td>0.435</td>
<td></td>
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</tr>
</tbody>
</table>

Notes:
6.3 Graphs

Figure 1: Exogenous Shock Processes

The figure shows:
The figure shows:
Figure 3: Nipa and BLS-Aggregates

The figure shows:
The figure shows: Predicted vs actual probabilities.
## 6.4 Long Run Taxation Results

Table 6: Basic Optimal Taxation Result

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Technology: Box 1</th>
<th>Preferences: Box 2</th>
<th>No Gov</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{o,m}$</td>
<td>0 0.068 0.125 0.192 0.269</td>
<td>$\gamma_{2} = .9$</td>
<td>$\gamma_{2} = .8$</td>
<td>$\gamma_{2} = .7$</td>
</tr>
<tr>
<td>$A_{o,f}$</td>
<td>0 0.111 0.160 0.215 0.275</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(b_{o,m}/h_{m})$</td>
<td>0 0.200 0.300 0.400 0.500</td>
<td>0.500 0.500 0.500</td>
<td>0.500 0.500 0.500</td>
<td>0.500 0.500 0.500</td>
</tr>
<tr>
<td>$(b_{o,f}/h_{f})$</td>
<td>0 0.360 0.460 0.560 0.660</td>
<td>0.660 0.660 0.660</td>
<td>0.660 0.660 0.660</td>
<td>0.660 0.660 0.660</td>
</tr>
</tbody>
</table>

| Change in Endogenous: | | | | | |
|-----------------------| | | | | |
| $U - Male$ | 0.041 0.041 0.041 0.041 0.041 | 0.041 0.041 0.041 0.041 0.041 | 0.040 0.035 |
| $U - Female$ | 0.034 0.035 0.035 0.035 0.035 | 0.035 0.035 0.035 0.035 0.035 | 0.033 0.024 |
| $O - Male$ | 0.26 0.263 0.263 0.263 0.259 | 0.266 0.268 0.268 0.268 0.249 | 0.249 0.164 |
| $O - Female$ | 0.46 0.480 0.485 0.485 0.470 | 0.481 0.474 0.468 0.468 0.436 | 0.436 0.213 |
| $E - Male$ | 0.697 0.696 0.696 0.696 0.700 | 0.692 0.691 0.691 0.691 0.711 | 0.711 0.801 |
| $E - Female$ | 0.49 0.486 0.480 0.480 0.496 | 0.485 0.491 0.498 0.498 0.531 | 0.531 0.763 |

| Optimal Taxation: | | | | | |
|--------------------| | | | | |
| $\tau_{l,m}$ | 0.29 0.291 0.286 0.276 0.258 | 0.282 0.290 0.293 0.293 0.293 | 0.293 |
| $\tau_{c}$ | -0.004 0.039 0.085 0.152 0.231 | 0.139 0.079 0.048 0.048 0.048 | 0.048 |

Notes: The table describes changes in optimal taxation for different assumption on technology and preferences. The first box gives changes in technology, the second gives changes in preferences and the third box assumes the absence of the government. The final box shows what happens if we allow for gender based taxation. In each case productivity on the job is left unchanged, that is $A_{m} = 1$ for males and $A_{f} = 0.675$ for females. Other targets have been hold constant, except the variances of the idiosyncratic distributions which have been kept at the benchmark case for simplicity. $E$, $U$ and $O$ give the respective employment, unemployment and inactive population rates. Data in bold are the benchmark calibration.
6.5 Optimal Cyclical Variations

The figure shows:

Figure 5: Optimal Labor Tax Rates

The figure shows:

Figure 6: Counter-cyclical Tax Rate