# On the sequential choice of tradable permit allocations

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**Abstract**

This paper investigates the sequential announcement of domestic emissions caps by governments in a international-based tradable pollution permit market for a transboundary pollutant. We find the sequential choice of domestic allocation caps is sub-optimal and depends on how the follower government reacts to the leader government's choice. Furthermore, the marginal damage and the degree to which allocations are substitutes or complements affects whether the leader changes from being a net permit buyer (seller) of permits to a seller (buyer).

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# 1 Introduction

The fundamental idea behind tradable pollution permit markets allows one regulator the ability to create and allocate pollution rights to firms. Due to the competitive trading of permits in the market, the pollutant can normally be controlled efficiently—where abatement efforts are efficiently distributed among firms (Coase, 1960; Montgomery, 1972). In contrast to this simple theory, many current tradable permit markets span many regulatory bodies, such as schemes that control pollution over multiple states or countries (see, for example, Ellerman et al. 2000; Ellerman et al. 2007). Due to this phenomenon, an active debate has begun to focus on the strategic issues that arise when tradable permit markets are controlled by multiple governments. As Helm (2003) has shown, allowing multiple governments to simultaneously determine a proportion of the aggregate emissions cap results in strategic behaviour that can increase aggregate emissions above the socially optimal level of emissions. Yet, the timing of governments' allocation choices has often been ignored. In particular, there has been no discussion on the consequences of allowing governments to sequentially announce emissions caps. Therefore, it is the aim of this paper to consider governments' optimal behaviour and the social optimality of a tradable permit market when multiple governments are allowed to sequentially announce their own (domestic) emissions caps.

In this paper, we investigate the effects on an international tradable pollution permit market when the level of permit allocation is determined by multiple governments. The model is split into two stages: In the first stage, two governments *sequentially* announce a level of pollution permits (domestic emissions cap) for firms under their jurisdiction (i.e. in their geographical area). In the second stage, all firms obtain a permit allocation (determined in stage one) and decide on a level of emissions to pollute in the perfectly competitive tradable permit market. We find that the sequential determination of regional emissions caps are socially sub-optimal. When the follower government's choice of emissions cap is complementary ("weakly" substitutable) the equilibrium level of aggregate allocation is closer to (further from) the socially optimal level. If the choice of emissions cap is "strongly" substitutable the equilibrium level of aggregate allocation is smaller than the socially optimal level. Our model is also reduced to consider a special case, where both governments simultaneously announce their emissions caps.

Sequential allocation choices by governments have been common place in previous tradable permit markets. The clearest example of strategic interaction exists between Member State governments in the EU-ETS. Each Member State government, through submission of their National Allocation Plan (NAP), has, among others things, the right to (non-cooperatively) determine the composition and level of their allowance allocation–albeit with the approval of the European Commission (Ellerman et al., 2007). Before the implementation of phase I (2005-2007), Member States had to notify the European Commission of their NAP by the 1st May 2004, yet as Zapfel (2007, p 23) explains:

> "[O]nly seven Member States...notified a plan close to the official date. On 7 July 2004, the date of the adoption of the Commission decisions on the first plans, nine plans were still outstanding. The last plan was received by the Commission on 3 January 2005, i.e. some nine months after the due date"

With the sequential announcement of NAPs occurring, it has been suggested by Harrison and Radov (2007, pp 41-61) that the first published draft NAP, announced by the UK, was "one of the most influential of the twenty-five Member State plans developed to implement the EU-ETS" as it was "viewed by some commentators as an attempt to influence the development of NAPs in other Member States". Furthermore, it was apparent that "[s]ome member states may in fact have delayed notification of plans...not merely for technical reasons, but also to see what standard the Commission would apply" (Zapfel, 2007 p25).

Such anecdotal evidence of sequential allocation announcements suggests that strategic behaviour may play a role in governments' choice of permit allocations. If so, it is important to consider whether the sequential announcement of domestic emission caps (and the additional information obtained) has any consequences for optimal allocation setting and social optimality. While our analysis is motivated by the clear use of sequential allocation choices in the EU-ETS, our model analyses a general international tradable permit markets with sovereign countries.

The theoretical discussions of strategic behaviour in environmental policy have been extensively investigated (see, for example, Barrett, 1994; Silva and Caplan, 1997; Ulph, 1996; 2000; Santore et al., 2001). A large part of the literature discusses the incentives for governments to act strategically in product markets with trans-boundary pollution (see Barrett (1994) for an overview). Yet very few studies have attempted to investigate strategic environmental policy in tradable permit markets. Santore et al. (2001) examine a federal-based model to investigate the incentives for US states to affect the $SO_2$ market and show that states do have an incentive to intervene in the $SO_2$ permit market and the outcome, in general, is Pareto inefficient.

Two studies relevant to our argument are Helm (2003) and D'Amato and Valentini (2006). Helm (2003) considers an international tradable permit market with $n$ non-cooperative countries and uses a two stage game where in the first stage, each country simultaneously selects a level of emissions for its representative firm. Then, in stage two, the firm from each country takes the governments allocation as given and selects a level of emissions to pollute. Helm (2003) finds, that the introduction of trading actually increases the level of aggregate emissions where "more environmentally concerned" countries choose less permits but this is offset by the selection of more permits from the "less environmentally concerned" countries. However, Helm's (2003) study is restricted to only simultaneous moves between governments. We use a similar framework to Helm (2003) in that we have a two stage game but allow for the possibility of sequential announcements. D'Amato and Valentini (2006) extend the results of Helm (2003) by including a perfectly competitive product market and provide theoretical evidence for "excessive" allocation choices and are able to obtain the social optimality of a simultaneous-moves game with two regulators. Both studies give accounts of the incentives associated with multiple governments simultaneously selecting domestic allocation caps but ignore the consequences of (often more realistic) sequential selection.

In our analysis, we allow one government to announce their domestic allocation cap first, that is, become the leader government. Then, after observing this action, the remaining government (the follower government) decides on an appropriate domestic allocation cap. After both governments have decided on a domestic allocation cap, the permits are then *simultaneously* distributed to participating firms in the tradable permit market. We find the sequential announcement of permits is socially sub-optimal. Aggregate emissions are chosen further from (closer to) the socially optimal level compared to the simultaneous case when the follower government's domestic allocation cap is "weakly" substitutable (complementary). In certain circumstances it is possible for the leader government's firm to change from a net supplier (buyer) of permits to a net buyer (supplier).

As international tradable permit markets become a common form of regulation, it is important to consider how governments' allocation announcements can affect the market both in the aggregate level of emissions and the social optimality of the scheme. This is especially true for the problem of climate change where the rents distributed from governments to their domestic firms have the potential to be extremely large and distortionary. Our analysis can help in explaining the consequences for aggregate emissions and social optimality when governments set allocation choices sequentially. Ideally, allocation should be centralised yet this is not always possible due to sovereign governments. When allocations are decentralised to governments social optimality and the aggregate level of emissions crucially depends on the substitutability or complementarity between allocations.

The paper is organised as follows: In Section 2 the basic model and the socially optimal case are discussed. In Section 3 the sequential announcement of allocations is discussed. We then illustrate the special case of simultaneous allocation setting. In Section 4 the simultaneous and sequential allocations are compared and finally Section 5 has some concluding remarks.

# 2 The basic model

Consider a tradable permit market for a transboundary pollutant where there are two distinct governments $k = i, j$. Each government has, under their jurisdiction, one representative polluting firm in their geographical region, which we denote as firm $k = i, j$. It is the responsibility of both governments to select a domestic emissions cap that is allocated to their representative firm.[1] The aggregation of the two domestic emissions caps determines the aggregate supply of permits in the perfectly competitive permit market. Furthermore, both firms can freely trade permits between the two regions.

Our model is similar in framework to Helm (2003) where the game is split into two stages. In stage one, governments sequentially announce a domestic emissions cap $a_k \in \mathbb{R}_+$ for $k = i, j$, to be allocated to their representative firm in order to maximise welfare in their jurisdiction. Without loss of generality, we assume that government $i$ announces an emissions cap first (the *leader* government). Government $j$ (the *follower* government), observes government $i$'s decision, and using this information, announces an emissions cap.

In stage two, the domestic emissions caps from stage one are *simultaneously* distributed to firms participating in the perfectly competitive tradable permit market.

[1] We assume throughout that the tradable permit market rules, such as rules on enforcement and monitoring, have been unanimously agreed by the governments before the market is operational. In international tradable permit markets these rules rarely distort the welfare between governments so are usually agreed by all. In contrast, the announcement of permit allocations has direct consequences for the welfare of each government and as a result, we assume that each government non-cooperatively announce their permit allocations.

Firms take the initial allocation as given and select a level of emissions to pollute $e_k \in \mathbb{R}_+$ for $k = i,\ j$. To coincide with permit allocation procedures in many existing tradable permit markets, we ignore the possibility that participating firms in the market obtain permits at different time periods. Instead, all governments distribute their chosen permit allocation to firms at one designated time period.

In order to find the subgame Nash equilibrium of this game, we use backward induction by first solving the optimal strategy of each firm (stage two) and then the governments' optimal choice of permit allocation (stage one).

## 2.1 Stage two: firms' emissions choices

In stage two, the perfectly competitive tradable permit market commences with the distribution of domestic emissions caps to participating firms (which was determined by governments in stage one). In the tradable permit market, firm $k = i, j$ takes the equilibrium permit market price $p^*$ and the allocation from its respective government $a_k$, as given. Firm $k$ selects a level of emissions $e_k$ for $k = i, j$ to maximise (minimise) profit (cost) from the tradable permit market where the cost of abatement for firm $k$ is given by $c_k(e_k)$ where $\frac{\partial c_k(e_k)}{\partial e_k} < 0$, $\frac{\partial^2 c_k(e_k)}{\partial e_k^2} > 0$ for $k = i,\ j$. Formally, firm $k$'s objective function is:

$$\max_{e_k} \quad p^*(a_k - e_k) - c_k(e_k) \qquad \text{for } k = i, j \tag{1}$$

Equation (1) shows firms' payoff from the permit market consisting of the revenue (cost) created by selling (buying) permits and the cost of abatement. Differentiating equation (1) with respect to $e_k$ gives the first order condition for firm $k$:

$$-\frac{\partial c_k(e_k)}{\partial e_k} - p^* = 0 \qquad \text{for } k = i, j \tag{2}$$

and the equilibrium market clearing condition is:

$$e_i^*(p^*) + e_j^*(p^*) = a_i + a_j \equiv a \tag{3}$$

where $e_k^*$ is the equilibrium level of emissions for firm $k = i, j$ and $a$ is the total permit supply across both regions. Equation (2) is the standard result of a perfectly competitive tradable permit market. Both firms choose a level of emissions so that their marginal abatement cost is equated to the market equilibrium permit price and as a consequence abatement effort is efficiently distributed between firms. Equation (3) is the equilibrium market clearing condition where the total amount of pollution emitted equals the aggregate supply of permits in the tradable permit market. To determine the responsiveness of the equilibrium permit price to aggregate allocation, we differentiate (2) with respect $p^*$:

$$-\frac{\partial^2 c_k(e_k)}{\partial e_k^2}\frac{\partial e_k}{\partial p^*} - 1 = 0 \qquad \text{for } k = i, j \tag{4}$$

and differentiate (3) with respect to $a_k$ :

$$\left(\frac{\partial e_i}{\partial p^*} + \frac{\partial e_j}{\partial p^*}\right)\frac{\partial p^*}{\partial a} = 1 \qquad \text{for } k = i, j \tag{5}$$

By substituting (4) into (5) we obtain:

$$\frac{\partial p^*}{\partial a} = -\left(\frac{1}{\frac{1}{\frac{\partial^2 c_i(e_i)}{\partial e_i^2}} + \frac{1}{\frac{\partial^2 c_j(e_j)}{\partial e_j^2}}}\right) < 0 \tag{6}$$

From equation (6), and the assumptions about the second derivative of the pollution abatement cost function, it is clear that as the level of aggregate emissions cap $a$ increases, the permit price decreases. We now consider the optimal behaviour of governments in stage one.

## 2.2 Stage one: governments' choice of domestic emissions cap

In stage one, governments sequentially announce a domestic emissions cap for their representative firm in order to maximise social welfare in their region. Governments have perfect knowledge of their firm's reaction in stage two. In particular, governments understand that the equilibrium permit price and the level of emissions chosen by firms are dependent on the aggregate level of permits in the market, that is $p^* = p^*(a)$ and $e_k^* = e_k^*(a)$ for $k = i, j$ where $a$ is the governments' aggregate supply of permits to the market ($a \equiv a_i + a_j$).

The welfare of government $k$ consists of net profit from its polluting firm minus the damage associated with the total level of emissions in its jurisdiction. We assume the pollutant is transboundary so that pollution from both firms cause damages to both governments. Damage is represented by $D_k(e_i+e_j)$ where $\frac{\partial D_k(e_i+e_j)}{\partial e_k}, \frac{\partial^2 D_k(e_i+e_j)}{\partial e_k^2} > 0$ for $k = i, j$. In equilibrium, as the aggregate level of emissions must equal the aggregate permit allocation, it follows from (3) that $D_k(e_i+e_j) = D_k(a_i+a_j) = D_k(a)$. Henceforth, we represent government $k$'s damage function by $D_k(a)$. We allow the damage experienced by both governments to be asymmetric in that $D_i(a) \neq D_j(a)$.

Formally, the objective function of government $k$ is:

$$\max_{a_k} W_k = p^*(a)(a_k - e_k^*(a)) - c_k(e_k^*(a)) - D_k(a) \qquad \text{for } k = i, j \tag{7}$$

where $e_i^*$, $p^*$ are the equilibrium level of emissions and permit price determined by equations (2) and (3), respectively.

In the sequential announcement game, government $i$ moves first (the leader) by announcing a level of permit allocation. Given this information, government $j$ (the follower government) selects a level of permit allocation. The sequence of play is common knowledge to both governments. It follows, then, that the difference in governments' objective functions occurs as a result of the timing of decisions.

Government $j$, the follower government, takes as given, the leader's choice of allocation. Therefore, government $j$ assumes the aggregate emissions cap $a$ is:

$$a_i + a_j \tag{8}$$

Using backward induction, government $i$, the leader, has perfect knowledge of the reaction of government $j$ and understands its choice of allocation will alter the total allocation of permits, both directly (through its own choice of allocation) and indirectly (through government $j$'s reaction to the leader's choice of allocation). As a consequence, the leader government understands that the total allocation in the market is:

$$a_i + a_j(a_i) \tag{9}$$

We show later in this paper that a special case of the model allows a game where both governments announce allocations simultaneously.

## 2.3 Socially optimal level of allocation

To aid comparisons throughout the paper, we identify the socially optimal outcome for a centralised planner.

The centralised planner aims to simultaneously choose a domestic emissions cap for both regions. The social planner's objective function is to maximise the sum of governments' welfare functions:

$$\max_{a_i, a_j} \quad W_i + W_j \tag{10}$$

which, given (7) and (8), is:

$$\max_{a_i,a_j} p^*(a)(a_i - e_i^*(a)) - c_i(e_i^*(a)) - D_i(a) + p^*(a)(a_j - e_j^*(a)) - c_j(e_j^*(a)) - D_j(a) \quad (11)$$

Differentiating equation (11) with respect to $a_i$ and $a_j$ respectively, gives:

$$\begin{aligned} &p'^*(a) \cdot (a_i - e_i^*(a)) + p^*(a)(1 - e_i^{*\prime}(a)) + p'^*(a) \cdot (a_j - e_j^*(a)) \\ &-p^*(a)e_j^{*\prime}(a) - c_i'(e_i^*(a)) - c_j'(e_j^*(a)) - \frac{\partial D_i}{\partial a}\frac{\partial a}{\partial a_i} - \frac{\partial D_j}{\partial a}\frac{\partial a}{\partial a_i} \end{aligned} \quad (12)$$

and

$$\begin{aligned} &p'^*(a) \cdot (a_j - e_j^*(a)) + p^*(a)(1 - e_j^{*\prime}(a)) + p'^*(a) \cdot (a_i - e_i^*(a)) \\ &-p^*(a)e_i^{*\prime}(a) - c_j'(e_j^*(a)) - c_i'(e_i^*(a)) - \frac{\partial D_i}{\partial a}\frac{\partial a}{\partial a_j} - \frac{\partial D_j}{\partial a}\frac{\partial a}{\partial a_j} \end{aligned} \quad (13)$$

where $c_i'(e_i^*(a)) = \frac{\partial c_i}{\partial ei}\frac{\partial e_i}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$, $c_j'(e_j^*(a)) = \frac{\partial c_j}{\partial ej}\frac{\partial e_j}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$, $e_i^{*\prime}(a) = \frac{\partial e_i}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$, $e_j^{*\prime}(a) = \frac{\partial e_j}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$, $p'^*(a) = \frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$ and $\frac{\partial a}{\partial a_k} = 1$ for $k = i, j$. Equations (12) and (13) can be simplified by noting that, in equilibrium, the market clears so that $(a_i - e_i^*(a)) + (a_j - e_j^*(a)) = 0$. Also, from equation (2) we know that each firm will choose a level of emissions to equate their marginal abatement cost with the permit price, it follows that $-\frac{\partial c_i}{\partial ei}\frac{\partial e_i}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k} = p^*\frac{\partial e_i}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$ and $-\frac{\partial c_j}{\partial ej}\frac{\partial e_j}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k} = p^*\frac{\partial e_j}{\partial p^*}\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_k}$ for $k = i, j$. Therefore equating (12) and (13) to zero for the optimum and simplifying, we obtain:[2]

[2] Given the assumptions about the damage functions and the result from equation (6), it is clear that the second order conditions hold for optimality.

$$p^* - \frac{\partial D_i}{\partial a}\frac{\partial a}{\partial a_i} - \frac{\partial D_j}{\partial a}\frac{\partial a}{\partial a_i} = 0 \quad (14)$$

$$p^* - \frac{\partial D_i}{\partial a}\frac{\partial a}{\partial a_j} - \frac{\partial D_j}{\partial a}\frac{\partial a}{\partial a_j} = 0 \quad (15)$$

From equations (14) and (15), it is clear that for social optimality to occur, government $k$'s domestic emissions cap must be chosen so that the cost of emissions (the permit price) is equal to the sum of governments' marginal damages. In other words, each government considers the marginal damage on *both* governments when selecting a domestic emissions cap.

To investigate aggregate emissions, we sum (14) and (15) together and rearrange:

$$2p^* = \frac{\partial D_i}{\partial a}\left(\frac{\partial a}{\partial a_i} + \frac{\partial a}{\partial a_j}\right) + \frac{\partial D_j}{\partial a}\left(\frac{\partial a}{\partial a_i} + \frac{\partial a}{\partial a_j}\right) \quad (16)$$

Equation (16) shows, for the market, that at the socially optimal level of aggregate allocation, the aggregate marginal benefit of allocation (the permit price) equals the sum of governments' aggregate marginal damages of allocation (that is, each governments marginal damage caused by *both* $a_i$ and $a_j$).[3] As the central planner selects $a_i$ and $a_j$ simultaneously, the aggregate emissions cap is $a = a_i + a_j$ and it follows that $\frac{\partial a}{\partial a_k} = 1$ for $k = i, j$. For ease of comparison throughout the paper, equation (16) can be further simplified to:

$$p^* = \frac{\partial D_i}{\partial a} + \frac{\partial D_j}{\partial a} \quad (17)$$

Equation (17) shows that, for the market as a whole, the social optimum level of aggregate emissions occurs when the *aggregate* emissions cap is chosen so that the sum of governments' aggregate marginal damages (for the aggregate emissions cap) equals the permit price.

[3]The aggregate marginal benefit of allocation can also be considered as the sum of firms' marginal abatement costs.

We proceed by investigating the affects of the sequential announcement of permit allocations by governments.

# 3 Sequential announcement of permit allocations

In this section we start by examining the optimal allocation choice of the follower government and, given this, work out the optimal strategy for the leader.

The follower government, government $j$, takes the other government's domestic cap $a_i$ as given. Therefore, substituting equation (8) into (7) and differentiating with respect to $a_j$ gives government $j$'s reaction function:

$$\frac{\partial p^*}{\partial a} \cdot (a_j - e_j^*(a)) + p^* - \frac{\partial D_j(a)}{\partial a} = 0 \tag{18}$$

where $p^* = -\frac{\partial c_k(e_k)}{\partial e_k}$ for $k = i, j$, from (2). The follower government will choose an optimal level of allocation $a_j^*$ so that (18) holds. Next, we solve the leader's problem. Government $i$, the leader, understands that the follower government will react to its allocation announcement. Substituting equation (9) into (7) and differentiating with respect to $a_i$ gives:

$$\frac{\partial p^*}{\partial a} \cdot (a_i - e_i^*(a)) \left(1 + \frac{\partial a_j}{\partial a_i}\right) + p^* \left(1 - \frac{\partial e}{\partial p^*}\frac{\partial p^*}{\partial a}\left(1 + \frac{\partial a_j}{\partial a_i}\right)\right) \tag{19}$$
$$-\frac{\partial c}{\partial e^*}\frac{\partial e}{\partial p^*}\frac{\partial p^*}{\partial a}\left(1 + \frac{\partial a_j}{\partial a_i}\right) - \frac{\partial D_i(a)}{\partial a}\left(1 + \frac{\partial a_j}{\partial a_i}\right)$$

Noting equation (2), it follows that $-p^* \frac{\partial e}{\partial p^*}\frac{\partial p^*}{\partial a}\left(1 + \frac{\partial a_j}{\partial a_i}\right) - \frac{\partial c}{\partial e^*}\frac{\partial e}{\partial p^*}\frac{\partial p^*}{\partial a}\left(1 + \frac{\partial a_j}{\partial a_i}\right) = 0$. Therefore, at the optimum, equation (19) can be reduced to:

$$\frac{\partial p^*}{\partial a} \cdot (a_i - e_i^*(a)) \left(1 + \frac{\partial a_j}{\partial a_i}\right) + p^* - \frac{\partial D_i(a)}{\partial a}\left(1 + \frac{\partial a_j}{\partial a_i}\right) = 0 \tag{20}$$

Assuming that $\phi \equiv \left[1 + \frac{\partial a_j}{\partial a_i}\right]$, this can be easily expressed as:[4]

$$\frac{\partial p^*}{\partial a} \cdot (a_i - e_i^*(a)) + \frac{p^*}{\phi} - \frac{\partial D_i(a)}{\partial a} = 0 \tag{21}$$

The leader government will choose an optimal level of allocation $a_i^*$ so that (21) holds. Comparing (18) and (21), both reaction functions are similar in that three influences affect the choice of allocation (Helm, 2003). Increasing allocation increases governments' marginal damages. Second, each government benefits from the additional payoff it receives from increasing allocation, that is, the government obtains the value of the permit price for each new permit chosen ($p^*$) by either selling the additional unit or reducing the amount demanded by the additional unit. Lastly, increasing the permit allocation will increase the aggregate supply of permits. Therefore, an increase in allocation will reduce the permit price received for each additional permit bought or sold by $\frac{\partial p^*}{\partial a}$, which from (6), is negative.

The main difference between the two reaction functions arises as the leader government has additional information about the reaction of the follower government ($\phi$). From (21), it is clear that $\phi$, the *conjectural derivative*, will alter the leader's choice of allocation compared to that of the follower government (Friedman, 1983). Summing equations (18) and (21) together and rearranging, we obtain one of our main results:

**Proposition 1** *When governments sequentially determine their domestic emissions caps then the aggregate emissions cap in the market equilibrium occurs when:*

$$p^* = \frac{\phi}{1+\phi}\left[\frac{\partial D_i(a)}{\partial a} + \frac{\partial D_j(a)}{\partial a}\right] \tag{22}$$

[4] The second order conditions for the solution hold for the follower when: $\frac{\partial^2 p^*}{\partial a^2} \cdot (a_j - e_j^*(a)) - \frac{\partial^2 c_j(e_j)}{\partial e_j^2}\left(\frac{\partial e_j^*}{\partial p^*} \cdot \frac{\partial p^*}{\partial a}\right)^2 - \frac{\partial^2 D_j(a)}{\partial a^2} < 0$ and for the leader when: $(a_i - e_i^*(a))\left[\frac{\partial^2 p^*}{\partial a^2} \cdot \left(\frac{\partial a}{\partial a_i}\right)^2 + \frac{\partial p^*}{\partial a}\frac{\partial^2 a}{\partial a_i^2}\right] + 2\frac{\partial p^*}{\partial a}\frac{\partial a}{\partial a_i} - \frac{\partial e_i^*}{\partial p^*}\left[\frac{\partial p^*}{\partial a} \cdot \frac{\partial a}{\partial a_i}\right]^2 - \frac{\partial^2 D_i(a)}{\partial a^2} \cdot \left(\frac{\partial a}{\partial a_i}\right)^2 - \frac{\partial D_i(a)}{\partial a}\frac{\partial^2 a}{\partial a_i^2} < 0$. In general terms, optimality occurs when $\frac{\partial^2 p^*}{\partial a^2}$ , $\frac{\partial^2 a}{\partial a_i^2}$ are relatively small and $\frac{\partial^2 c(e_k)}{\partial e_k^2}$ is relatively large. In the following we assume that the optimality of the second order conditions holds.

*where* $\phi = \left[1 + \frac{\partial a_j}{\partial a_i}\right]$*, government i (the leader) chooses a domestic cap from (21) and government j (the follower government) chooses a domestic cap from (18).*

Proposition 1 presents an expression which relates governments' aggregate marginal damages with the permit price (given an aggregate emissions cap). From equation (22) it is immediate that $\frac{\phi}{1+\phi} \neq 1, \forall \phi$. It follows by comparing (17) and (22) that the aggregate emissions in the market equilibrium will never reach the socially optimal level of aggregate emissions. Both governments do not take into consideration the effect of their permit allocation on the other government's damage function and, as the result, aggregate emissions are not socially optimal. We return to this in the following section.

As with any Stackelberg (leader-follower) model, the leader's knowledge of whether the follower government selects allocation as a substitute or complement is crucial to the level of allocation chosen. When $\phi < 1$, the follower government's allocation choice is negatively related to the choice made by the leader—the follower government's choice of domestic allocation is a substitute. We denote two types of substitute: "weak" and "strong". For "weak" substitutes the follower government's response is relatively insensitive ($\phi \in (0, 1)$) and for "strong" substitutes the reaction is relatively sensitive ($\phi \in (-\infty, -1)$). Further, when $\phi > 1$, the follower government's choice of allocation is a complement (i.e. the follower government increases allocation when the leader government increases allocation).

To what extent the choice of allocations are substitutes or complements depends on the functional forms placed on firms' abatement costs and governments' damage functions. Bárcena-Ruiz (2006) and Kennedy (1994) have shown that, for the case of environmental taxes, the selection of substitutes or complements depends on the extent to which pollution "spillovers" to the other government, that is, to what extent the pollutant is transboundary. A similar logic applies here: parameters in the functional form of the abatement cost and damage functions will determine the characteristics of allocation choice. However, we abstract from the causes of what determines allocation choices to be substitutes or complements, and instead focus on the optimal behaviour and social optimality of the permit market when the characteristics of allocation choices have been ex-ante determined. From equation (22), it is immediate that a special case exists when $\phi = 1$.

### 3.1 Special case: Cournot-Nash game $\phi = 1$

Assume that $\phi \equiv \left[1 + \frac{\partial a_j}{\partial a_i}\right] = 1$, where the conjectural derivative is zero, $\frac{\partial a_j}{\partial a_i} = 0$. In this game, the leader government takes as given, the follower government's level of allocation. This means that both governments simultaneously announce allocations—a Cournot-Nash game. This can be seen more clearly by substituting $\phi = 1$ into equations (21) and (22) and summing so that:

**Corollary 2** *When $\phi = 1$, governments $k = i, j$ simultaneously announce permit allocations so that their reaction functions are*

$$\frac{\partial p^*}{\partial a} \cdot (a_k - e_k^*(a)) + p^* - \frac{\partial D_k(a)}{\partial a} = 0 \tag{23}$$

*and the aggregate emissions cap, at the market equilibrium, occurs when*

$$p^* = \frac{1}{2}\left[\frac{\partial D_i(a)}{\partial a} + \frac{\partial D_j(a)}{\partial a}\right] \tag{24}$$

*for $k = i, j$.*

This is in line with Helm (2003) and D'Amato and Valentini (2006). Comparing equation (17) with (24) shows that when domestic caps are chosen simultaneously, the socially optimal level of allocation (emissions) is not achieved. It follows that decentralising the allocation process to separate governments actually increases the aggregate level of emissions relative to the socially optimal level of emissions. Similar to the sequential game, this occurs as government $k$ does not take into consideration the affect of it's emissions on the other government's damage function.

# 4 Sequential vs. simultaneous announcement of permit allocations

In this section, we directly compare the social optimality of simultaneously and sequentially announcing domestic emissions caps. Furthermore, we show the sequential announcements of domestic emissions caps can significantly alter whether the leader government's prefers its firm to be either a net permit buyer or seller.

Comparing the socially optimal level of allocation (17) with the levels for the simultaneous (22) and sequential (24) games, shows that as $\frac{\phi}{1+\phi} \neq 1 \ \forall \ \phi$, allowing governments the option to determine their own domestic permit cap, either sequentially or simultaneously, is socially sub-optimal.

First, let us consider $\frac{\partial a_j}{\partial a_i} > 0$ where the follower government increases its allocation when it observes an increase in leader government's allocation. Under this scenario, the sequentially determined aggregate cap will be larger than the socially optimal level (i.e. $\frac{\phi}{1+\phi} < 1$) but smaller than the cap in the simultaneous game (i.e. $\frac{\phi}{1+\phi} > 1/2$). Further, as $\frac{\partial a_j}{\partial a_i} \to \infty$, one observes an aggregate emissions cap converging to the socially optimal level. Given any increase in the leader's allocation this will result in an increase in allocation from the follower government which will further depreciate the permit price. Therefore, the leader government may consider reducing allocation in order to prevent a dramatic fall in the equilibrium permit price. The degree to which this happens depends on the sensitivity of both the price change (given by (6)) and the follower government's reaction (18).

Second, the follower government may also announce allocation as a "weak" substitute $\frac{\partial a_j}{\partial a_i} \in (-1, 0)$. In this case, the sequentially determined cap will still be larger than the socially optimal level (i.e. $\frac{\phi}{1+\phi} < 1$) but now the cap is larger than the cap in the simultaneous game (i.e. $\frac{\phi}{1+\phi} < 1/2$). Intuitively, the follower government's reaction does not outweigh an increase in the leader's allocation and, as a consequence, aggregate emissions increase. Again, the degree to which aggregate allocation changes depends on the sensitivity of the price change and the follower government's reaction.

Finally, when $\frac{\partial a_j}{\partial a_i} < -2$ the follower government announces its allocation as a "strong" substitute and we find the sequentially determined aggregate emissions cap is now lower than the socially optimal allocation and the simultaneous game (i.e. $(\frac{\phi}{1+\phi} > 1)$) and as $\frac{\partial a_j}{\partial a_i} \to -\infty$, the aggregate emissions cap converges towards the socially optimal level.[5] Intuitively, as the leader government increases its allocation, the follower government reduces allocation, to such an extent, that the aggregate level of emissions is now lower than socially optimal. In summary we have:

**Corollary 3** *If domestic emissions caps are chosen sequentially, then:*

- When $\frac{\partial a_j}{\partial a_i} > 0$ the aggregate cap is larger than the socially optimal level and smaller than the simultaneous allocation.
- When $\frac{\partial a_j}{\partial a_i} = 0$ the aggregate cap is larger than the socially optimal level and is identical to the simultaneous allocation.
- When $\frac{\partial a_j}{\partial a_i} \in (-1, 0)$ the aggregate cap is larger than the socially optimal level and the simultaneous allocation.
- When $\frac{\partial a_j}{\partial a_i} < -2$ the aggregate cap is smaller than the socially optimal level and the simultaneous allocation.

To provide tractability, we focus on the most realistic scenario where $\phi > 0$. That is, to make comparisons between the simultaneous and sequential games, we focus on scenarios where the follower government's reaction is either a complement or a "weak" substitute and simply refer to them as complements and substitutes. From Corollary 3, then, we consider the scenario where aggregate allocation is larger than the socially optimal level.

[5] We exclude $\frac{\partial a_j}{\partial a_i} \in [-2, -1]$ due to the asymptotic behaviour of $\frac{\phi}{1+\phi}$.

To further compare the results of simultaneous and sequential allocation, we follow Helm (2003) by denoting "low damage" governments when the government experiences $p^* > \frac{\partial D_k(a)}{\partial a}$ and "high damage" governments when $p^* < \frac{\partial D_k(a)}{\partial a}$. Given we know $\frac{\partial p^*}{\partial a} < 0$ from equation (6), it follows from equation (23) that in equilibrium,"low damage" governments will choose allocation so that their firm will be a net seller of permits and the firm in "high damage" governments must be net a buyer of permits. Intuitively, if "high damage" governments increased their allocation, their damages would increase more than the payoff they would receive from doing so. Therefore, they prefer their firm to be a net buyer of permits. Conversely, "low damage" governments receive a higher price than their damage for each unit of allocation chosen, so would prefer to increase allocation and allow their firm to be a net seller of permits.

An interesting result occurs when we investigate the consequences of switching between a simultaneous and sequential game. In a sequential allocation game the leader government can adapt it's allocation choice in full knowledge of the reaction of the follower government. In fact, in certain circumstances the leader government may completely alter it's behaviour between the simultaneous and sequential games. For $\phi > 0$, comparing (21) with $\phi = 1$ and $\phi \neq 1$ reveals that a government in a simultaneous game that switches to become a leader government in a sequential game may have an incentive to alter its allocation of permits so that its firm changes from a net seller (buyer) to net buyer (seller). We find that:

**Corollary 4** *If a government changes from a simultaneous player to a leader government in a sequential allocation then:*

*(i) For* $\phi \in (1, \infty)$, $\exists\ \phi^*$ *where* $\forall\ \phi \geq \phi^*$ *such that* $p^* > \dfrac{\partial D_i(a)}{\partial a}$ *and* $\dfrac{p^*}{\phi} < \dfrac{\partial D_i(a)}{\partial a}$ *and*

*(ii) For* $\phi \in (0, 1)$, $\exists\ \phi^*$ *where* $\forall\ \phi \in (0, \phi^*]$ *such that* $p^* < \dfrac{\partial D_i(a)}{\partial a}$ *and* $\dfrac{p^*}{\phi} > \dfrac{\partial D_i(a)}{\partial a}$

Corollary (4) shows that, due to the additional information about the reaction of the follower government, it is possible for a firm who is a net seller (buyer) in the simultaneous allocation game to become a net buyer (seller) in the sequential game. The intuition is clear. In case (i) the firm is initially a net seller of permits, then their government moves first to become the leader in the sequential game. If the follower government in the sequential game chooses allocation so that $\phi \in (1, \infty)$, then any increase in the leader's permit allocation will be met with an increase in the the follower's allocation which depreciates the permit price. Indeed as shown in Corollary (4) case (i), there will be a threshold value of the follower government's reaction ($\phi^*$) which will depreciate the price to such an extent that the leader government selects an allocation so that its firm becomes a net buyer of permits. In case (ii), the firm (in the leader government) is initially a net buyer and the follower government announces allocation so that $\phi \in (0, 1)$. For the leader government, a threshold value of $\phi^*$ exists in which the substitution of permits is so low that, the price of permits becomes "too high" and the leader government selects allocation so that their firm becomes a net supplier of permits.

Two other cases exist, (namely, $p^* > \frac{\partial D_i(a)}{\partial a}$ with $\phi \in (0, 1)$ and $p^* < \frac{\partial D_i(a)}{\partial a}$ and $\phi > 1$) for which this combination actually strengthens the behaviour of the leader, so that the firm in a leader government continues to be a net seller (buyer).

The choice of whether the leader government decides allocation so that its firm becomes a net buyer or seller of permits and consequently whether the follower government's firm will be a net supplier or buyer of permits, can also be viewed through the leader's marginal damage relative to the follower government. This can be seen by subtracting (18) from (21) which gives:

$$\frac{\partial p^*}{\partial a} \cdot (a_i - e_i^*(a) - a_j + e_j^*(a)) + p^* \left( \frac{1 - \phi}{\phi} \right) = \frac{\partial D_i(a)}{\partial a} - \frac{\partial D_j(a)}{\partial a} \qquad (25)$$

Noting that, in equilibrium, $a_i - e_i^*(a) = -a_j + e_j^*(a)$ and denoting $\frac{\partial p^*}{\partial a} = p^{*\prime}$ equation (25) becomes:

$$2p^{*'} \cdot (a_i - e_i^*(a)) + p^* \left(\frac{1-\phi}{\phi}\right) = \frac{\partial D_i(a)}{\partial a} - \frac{\partial D_j(a)}{\partial a} \tag{26}$$

From equation (26) we have the following Proposition:

**Proposition 5** *If* $\frac{\partial D_i(a)}{\partial a} > \frac{\partial D_j(a)}{\partial a}$ *then the leader government announces an allocation so that:*

- *when* $\phi \in (0,1)$ *either (i)* $a_i < e_i^*$ *or (ii)* $a_i > e_i^*$ *such that* $\left|2p^{*'}(a_i - e_i^*(a))\right| < p^* \left(\frac{1-\phi}{\phi}\right)$
- *when* $\phi > 1$ *then* $a_i < e_i^*$ *such that* $2p^{*'}(a_i - e_i^*(a)) > \left|p^* \left(\frac{1-\phi}{\phi}\right)\right|$

*If* $\frac{\partial D_i(a)}{\partial a} < \frac{\partial D_j(a)}{\partial a}$ *then the leader government announces an allocation so that*

- *when* $\phi \in (0,1)$ *then* $a_i > e_i^*$ *such that* $\left|2p^{*'}(a_i - e_i^*(a))\right| > p^* \left(\frac{1-\phi}{\phi}\right)$
- *when* $\phi > 1$ *then either (i)* $a_i > e_i^*$ *or (ii)* $a_i < e_i^*$ *such that* $2p^{*'}(a_i - e_i^*(a)) < \left|p^* \left(\frac{1-\phi}{\phi}\right)\right|$

Proposition 5 shows that not only does the leader government allow its firm to be a net supplier/ demander based on its relative marginal damage it also depends on the reaction of the follower government. When $\frac{\partial D_i(a)}{\partial a} > \frac{\partial D_j(a)}{\partial a}$, it is intuitive that the leader, which has larger marginal damage, would allow its firm to be a net buyer of permits. Yet in certain circumstances, although the leader government has relative higher marginal damages, it will choose to increase allocation and allow its firm to become a net seller of permits. When the follower government chooses allocation in a substitutable fashion, it may be optimal for the leader government to increase its permit allocation even when it has relatively high marginal damages. The leader government will increase allocation as long as $\left|2p^{*'}(a_i - e_i^*(a))\right| < p^* \left(\frac{1-\phi}{\phi}\right)$.

Alternatively assuming that the leader government has lower marginal damage than the follower government $\frac{\partial D_i(a)}{\partial a} < \frac{\partial D_j(a)}{\partial a}$, it is feasible that the leader government's firm is a net seller of permits. Yet, as the follower government increases allocation (when the leader government increases allocation), the permit price may be depreciated to such an extent that the leader government firm becomes a net buyer of permits. The leader government may choose to be a net buyer when $2p^{*\prime}(a_i - e_i^*(a)) < \left| p^* \left( \frac{1-\phi}{\phi} \right) \right|$.

This counter-intuitive result is due to the additional information the leader government obtains about the follower government. However, similar to D'Amato and Valentini (2006), for the special case when governments simultaneously announce allocation caps ($\phi = 1$), the above result is simplified:

**Corollary 6** *For* $\phi = 1$,

*(i) If* $\dfrac{\partial D_i(a)}{\partial a} > \dfrac{\partial D_j(a)}{\partial a}$ *then government* $i$ *announces an allocation so that* $a_i < e_i^*$

*(ii) If* $\dfrac{\partial D_i(a)}{\partial a} < \dfrac{\partial D_j(a)}{\partial a}$ *then government* $i$ *announces an allocation so that* $a_i > e_i^*$

When the leader government takes the other government's choice as given, the government with the largest (smallest) marginal damage will always allow its firm to be a net buyer (supplier) of permits. From corollary (6), it can easily be shown that government $j$ will be be a net buyer (seller) when government $i$ is a net seller (buyer) based on their relative marginal damages.

# 5 Conclusion

The purpose of this paper is to investigate the consequences of sequentially announcing domestic allocation caps in an international tradable permit market. In the first stage of our game, governments sequentially announce their domestic allocation caps to their representative firm. In stage two, their representative firm, given this information, selects a level of emissions to pollute in the perfectly competitive tradable permit market. To the best of our knowledge, no study has investigated the consequences of allowing governments to sequentially announce their allocation choices. However, it is apparent from existing tradable permit markets, such as the European Emissions Trading Scheme, that sequential allocation setting is prevalent. The sequential setting of permit allocations may be a result, not of officially sanctioned rules or regulations in the tradable permit market, but due to heterogeneous factors that affect the timing of states' permit allocation selections, such as the different efficiency levels of government bureaucracy. For this reason alone, it is important to understand the consequences of numerous governments announcing domestic allocation caps at separate times.

We find that allowing governments to decide and announce their allocation cap is socially sub-optimal for sequential setting of permits (we also show this for the simultaneous case). We show that under the sequential setting of domestic emissions caps, the aggregate emissions is chosen closer to (further from) the socially optimal level when the follower government's domestic allocation cap is complementary ("weakly" substitutable). In fact, the degree to which the follower government changes allocation due to the leader's choice may, in certain circumstances, change the leader government from being a net buyer (seller) of permits to a net seller (buyer).

Designers of tradable markets, need to be fully aware of the potential consequences of allowing governments to simultaneously or sequentially allocate permits. From this analysis it appears that simultaneous and sequential allocation setting will be socially sub-optimal. However, under a sequential allocation setting game, the leader government may choose an allocation cap nearer the socially optimal level (compared to the simultaneous game) when the follower government reacts to the leaders choice as if the allocation caps were complements.

This paper suggests that previous attempts to model the strategic behaviour of governments in international tradable permit markets have neglected the important issue of timing. When designing tradable permit markets, one must consider the potential consequences for social optimality when domestic allocation caps are sequentially determined.

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