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# On the optimal accumulation of renewable energy generating capacity

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**Abstract**

We analyze the optimal accumulation of renewable energy (backstop) generating capacity in a capital-energy economy with exhaustible fossil fuels. The analysis rests upon graphical illustrations of optimal control considerations. Due to the exhaustibility of fossil fuels the relative profitability of backstop capacity vs. capital investments increases in time. Furthermore, it turns out that the optimal economic evolution and, therefore, the steady-state levels of capital, backstop capacity, and consumption crucially depend on the capital endowment. In particular, a sufficiently large endowment gives rise to the accumulation of an excess capacity. Furthermore, a high capital endowment allows to use the full production potential of the steady-state capital stock, so that there is no mark-up on backstop costs in the steady-state. In contrast, a low capital endowment may render capacity investments non-optimal, so that the economy is in a poverty trap. Both cases are based on an intertemporal consumption trade-off. The lower the time preference rate the more beneficial the trade-off and, therefore, the lower the critical capital endowment values.

*Keywords:* Fossil Fuel, Renewable Energy, Capacity

JEL classification: Q20, Q32, Q42

## 1. Introduction

At least since the publication of the report of the Club of Rome - Meadows et al. (1972) - the sustainability of economic development is challenged. One prominent argument is

*I would like to thank Name for useful notes and comments. Anonymous reviewers have also given very valuable advice. Remaining errors are the author's sole responsibility. Please address correspondence to: Name, Address, Telephone, Fax

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the exhaustibility of fossil fuels. This pessimistic view has been widely criticized. Stiglitz (1974) and Barbier (1999) argue that a factor augmenting technology which improves sufficiently fast can ensure sustainability. Other authors, such as Chakravorty et al. (2006), Hoel (2011), Tsur and Zemel (2003), Tsur and Zemel (2005), and Kollenbach (2015), refer to renewable energy sources (backstop). In particular, solar energy is considered, as solar radiation offers a practically unlimited energy source. Consequently, a backstop is usually modeled as an unlimited resource flow. However, the utilization of solar radiation and other renewable energy sources, such as water and wind power, require sophisticated and costly technical facilities, i.e. a generating *capacity*. Obviously, the buildup of such a capacity competes with consumption and other investments possibilities, e.g. other *capital* goods, for limited funds.

This issue has been addressed Powell and Oren (1989), Wirl and Withagen (2000), Fischer et al. (2004), and Tsur and Zemel (2011). While Powell and Oren (1989) consider fossil fuels as exhaustible, they abstain from capital, so that there is no trade-off between capacity and capital investments. Wirl and Withagen (2000) and Fischer et al. (2004) focus on pollution and do not incorporate an exhaustible resource. Fossil fuels are explicitly mentioned by Tsur and Zemel (2011). However, they assume an unlimited stock. According to the results of Powell and Oren (1989) and Tsur and Zemel (2011), the complete backstop capacity is used in the steady-state, while the marginal benefits of backstop use exceed the marginal costs, i.e. there is a mark-up on marginal backstop costs.

On the one hand, capital is widely regarded as an important production factor. For example, a large part of the endogenous growth literature considers the synergy of capital and technology as the source of economic growth.[1] On the other hand, fossil fuel is exhaustible, which is, as mentioned above, the source of an ongoing discussion regarding the sustainability of economic development.[2] Consequently, fossil fuels are supposed to be limited by many authors, e.g. by Stiglitz (1974), Tsur and Zemel (2005), Barbier (1999), and Kollenbach (2015).

We analyze the optimal accumulation of backstop generating capacity in a setting with capital and a limited fossil fuel stock. For this purpose we develop a model of a capital-

[1]Cf. Barro and Sala-i Martin (2003) and Aghion et al. (1998) for comprehensive reviews of the endogenous growth theory.

[2]Cf. Meadows et al. (1972). An overview of remaining resources is given by Birol et al. (2012).

energy economy based on a modified version of the endogenous growth model of Tsur and Zemel (2005). Energy generation relies on exhaustible fossil fuels and an unlimited flow resource (backstop, e.g. solar energy). To use the latter, a limited generating capacity is needed, which can be extended by investing the composite good. Therefore, capacity investments are competing with capital investments and consumption.

Similar to Tsur and Zemel (2005, 2011), economic evolution, i.e. consumption, capital and backstop capacity accumulation, is analyzed by geometric considerations. These are based on three characteristic lines in the two-dimensional capacity-capital-space. Depending on the capital endowment seven exemplary evolution paths are described.

In contrast to Powell and Oren (1989) and Tsur and Zemel (2011), we find that a sufficiently high capital endowment gives rise to the accumulation of an excess capacity, i.e. the capacity is extended above the level used in the steady-state. This excess capacity is only used in the mid-term. Thus, the additional mid-term production must pay off the capacity investments costs in terms of lost early consumption. In other words, there exists a trade-off between early and late consumption. The lower the time preference rate the more beneficial the trade-off and, therefore, the lower the critical capital endowment that gives rise to the excess capacity. Furthermore, this case also illustrates that there may be no mark-up on marginal extraction costs in the steady-state, as found by Powell and Oren (1989) and Tsur and Zemel (2011). According to our results, the mark-up only exists, if the capital endowment is low. Moreover, a low capital endowment in combination with a high time preference rate may render capacity investments non-optimal, as the necessary consumption trade-off is not beneficial. Tsur and Zemel (2011) found that in this case the economy relies on fossil fuels. However, we consider an exhaustible stock, so that the option of a fossil-fuel based economy does not exist. Consequently, our results regarding the welfare of such an economy are more pessimistic.

The outline of the paper is as follows. The model is described in section 2. Section 3 presents the conditions for the social optimum. To illustrate the optimal evolution of the economy in section 4, we introduce the characteristic lines in 4.1 and explain the mechanism determining the evolution process in 4.2. In section 4.3 seven exemplary evolution paths are presented, while the determinants of steady-state consumption are discussed in section 4.5. Section 5 concludes.

## 2. Model

We make use of a modified version of Tsur and Zemel's (2005) endogenous growth model. In contrast to Tsur and Zemel, we abstain from technological progress but consider a limited green energy generating capacity. In the following, the assumption are briefly discussed.[3] A composite good $y$ is produced by means of the two essential production factors *capital* $k$ and *energy* $x$. The production function $F(k, x)$ is well-behaved and concave, i.e. $F_x > 0$, $F_k > 0$, $F_{xx} < 0$, $F_{kk} < 0$, $F_{kx} = F_{xk} > 0$, $J = F_{xx}F_{kk} - F_{kx}^2 > 0$, and $F(0, x) = F(k, 0) = 0$.

Energy is generated by burning exhaustible fossil fuels $b$ (black energy) or by using a renewable backstop resource $g$ (green energy).[4] The fossil fuel stock is denoted with $s$. It decreases in fossil fuel use according to[5]

$$\dot{s} = -b. \tag{1}$$

As the initial fossil fuel stock $s(0)$ is limited, $\int_0^\infty b(t)dt \leq s(0)$. The extraction costs of fossil fuels are given by the convexly increasing function $M(b)$, i.e. extraction costs depend on the current fossil fuel flow, with $M_b(b) > 0$ and $M_{bb}(b) > 0$. As we abstain from fixed costs, $M(0) = 0$.

Backstop supply is limited by the current backstop generating *capacity* $Q$. The limited capacity reflects the high capital intensity of renewable energies. In other words, $Q$ denotes a specialized capital stock necessary for backstop utilization.[6] Consequently, the stock can be increased by investing composite goods. The corresponding investments are labeled $q$, so that

$$\dot{Q} = q, \tag{2}$$

with $Q(0) > 0$ as the positive capacity endowment. Every capacity unit allows the generation of $w$ backstop units per period. Due to appropriate unit choice, we set $w = 1$,

[3] For the sake of simplicity, the time index $t$ is suppressed as long as it is not necessary for understanding.

[4] Concerning the backstop, we refer to the whole set of renewable energies, e.g. biofuel, solar, wind, and water power. One can also consider fusion power, as the required fuel, hydrogen, is abundant.

[5] We use the notation $\dot{z}$ to indicate the derivation of an arbitrary variable $z$ with respect to time $t$, i.e. $\dot{z} = \frac{dz}{dt}$.

[6] Thus, we distinguish between specialized capital $Q$ necessary for backstop generation and non-specialized capital $k$ used for production. In the following, we refer to the former as capacity and to the latter as capital.

so that

$$Q \geq g \tag{3}$$

holds at every point in time. In contrast to Tsur and Zemel (2011), the supply costs of backstop are not zero. Following Chakravorty et al. (2006) and Hoel (2011), we assume constant unit costs $m$, which cover all costs associated with renewable energy generation with the exception of capital investment, e.g. maintenance, technical wear (deprecations), the setup of a more sophisticated power grid and energy storage facilities, opportunity costs of land use, et cetera. We assume that the technical constraint associated with a higher backstop capacity, in particular with respect to the power grid, allows only a limited installation $\bar{q}$ per period, i.e

$$\bar{q} \geq q \geq 0. \tag{4}$$

By following Tsur and Zemel (2011), let $\bar{q}$ exceed net production $y^n$ at every point in time.[7] The latter is given by

$$y^n := F(k, x) - M(b) - mg, \tag{5}$$

i.e. by production net of energy costs.[8] As consumption $c$, capacity investments $q$ and capital (dis)investments $\dot{k}$ rely on the composite good $y$, the capital stock evolves according to

$$\dot{k} = F(k, x) - M(b) - mg - q - c = y^n - q - c. \tag{6}$$

Utility depends only on consumption according to the concavely increasing utility function $U(c)$, with $U_c(c) > 0$, $U_{cc}(c) < 0$, and $\lim\limits_{c \to 0} U_c(c) = \infty$.

## 3. The social optimum

The social optimum is given by the maximization of welfare subject to the constraint stated above. Thus, the social planner maximizes intertemporal utility $\int\limits_0^\infty U(c(t))e^{-\rho t}dt$, with $\rho > 0$ as the time preference rate, subject to (1), (2), (3), (4), (6), $st \geq 0$, $k(t) \geq 0$, $b \geq 0$, $g \geq 0$, and $c \geq 0$. Let $\tau$, $\lambda$ and $\theta$ be the costate variables (shadow prices) associated

[7]It is also possible to let net production limit capacity investments. However, the related analysis is more complicated without providing more insight.

[8]As we abstain from technological progress and assume a concave production function, the economy cannot grow forever. Rather it converges against a steady-state. Therefore, there is always a $\bar{q}$ such that (4) holds.

with the fossil fuel stock, capital, and the backstop capacity. The Lagrange multiplier of the capacity constraint (3) is denoted with $\mu$ and the multipliers of the non-negativity conditions are $\zeta_b$, $\zeta_g$, $\zeta_q$, and $\zeta_{\bar{q}}$.[9] The current-value Lagrangian reads

$$\begin{aligned} L =& U(c) + \lambda[F(k,x) - M(b) - mg - q - c] - \tau b + \theta q + \mu\,[Q - g] \\ &+ \zeta_b b + \zeta_g g + \zeta_q q + \zeta_{\bar{q}}[\bar{q} - q]. \end{aligned} \tag{7}$$

Under the assumption of an interior optimum with respect to backstop use and fossil fuel extraction, the necessary conditions give[10]

$$U_c(c) = \lambda, \tag{8}$$

$$F_x(k,x) = M_b(b) + \frac{\tau}{\lambda} = m + \frac{\mu}{\lambda}, \tag{9}$$

$$-\lambda + \theta = \zeta_{\bar{q}} - \zeta_q. \tag{10}$$

The costate variables evolve according to

$$\dot{\lambda} = \rho\lambda - \lambda F_k(k,x), \tag{11}$$

$$\dot{\tau} = \rho\tau, \tag{12}$$

$$\dot{\theta} = \rho\theta - \mu. \tag{13}$$

The complement slackness conditions with respect to the backstop capacity constraint and the capacity investments $q$ are

$$\mu \geq 0,\;\; \mu[Q - g] = 0, \tag{14}$$

$$\zeta_q \geq 0,\;\; \zeta_q q = 0, \tag{15}$$

$$\zeta_{\bar{q}} \geq 0,\;\; \zeta_{\bar{q}}[\bar{q} - q] = 0. \tag{16}$$

Combining (8) and (11) determines the optimal consumption growth rate[11]

$$\hat{c} = \frac{F_k - \rho}{\eta}, \tag{17}$$

with $\eta$ as the positively defined elasticity of marginal utility. (17) is the well-known Ramsey rule. It states that consumption increases (decreases) as long as the marginal product

[9] The non-negativity conditions with respect to consumption and capital are omitted, because (8), (11) and the assumption $\lim_{c \to 0} U_c(c) = \infty$ ensure $c(t) > 0\ \forall t$. Since $c > 0$ requires a positive production $F(k,x) > 0$ and $F(0,x) = 0$, a capital stock of zero is ruled out, too.

[10] The assumption of simultaneous utilization of both energy sources has been used by Tsur and Zemel (2005), Kollenbach (2014), and Kollenbach (2015). See also footnote 16.

[11] The growth rate of the arbitrary variable $z$ reads $\hat{z} := \frac{1}{z}\frac{dz}{dt}$.

of capital exceeds (falls short of) the time preference rate. The absolute value of the consumption growth rate is the higher the more inelastic marginal utility, i.e. the smaller $\eta$. The Lagrangian is linear in capacity investments $q$. Therefore, the optimal *capacity investment regime* is determined by (10), (15), (16), and the maximization of $H = U(c) + \lambda[F(k,x) - M(b) - mg - q - c] - \tau b + \theta q$.[12] Lemma 1 summarizes the results.

**Lemma 1** *Capacity investments are*

- *minimal, if the relative profitability* $\theta/\lambda$ *of capacity vs. capital investments falls short of one.*
- *singular, if the relative profitability* $\theta/\lambda$ *of capacity vs. capital investments equals one.*
- *maximal, if the relative profitability* $\theta/\lambda$ *of capacity vs. capital investments exceeds one.*

*Proof:* Appendix A.1.

According to lemma 1, the relation of the shadows prices of capacity and capital determines the optimal capacity investment regime. As the shadow prices indicate the value the social planner associates with a marginal increase of the respective state variable, we refer to $\frac{\theta}{\lambda}$ as the relative profitability of capacity vs. research investments. Consequently, capacity investments are only positive, if they are at least as profitable as capital investments.

The transversality conditions, which belong to the sufficient conditions, read[13]

$$\begin{aligned} &(a): \lim_{t\to\infty} e^{-\rho t}\lambda(t)[k(t) - k^*(t)] \geq 0, \qquad (b): \lim_{t\to\infty} e^{-\rho t}\tau(t)[s(t) - s^*(t)] \geq 0, \\ &(c): \lim_{t\to\infty} e^{-\rho t}\theta(t)[Q(t) - Q^*(t)] \geq 0. \end{aligned} \tag{18}$$

Variables marked with an asterisk ($*$) denote optimal values, while unmarked variables refer to any possible path. Using transversality condition (18)(c) and lemma 1 we can prove the following proposition.

**Proposition 1** *Every evolution path* $(k(t), s(t), Q(t), c(t), b(t), g(t), q(t))$ *that exhibits positive capacity investments while the backstop capacity constraint is non-binding* ($\mu = 0$) *is not optimal.*

*Proof: Appendix A.2*

The Lagrange multiplier $\mu$ is either positive or zero. Therefore, proposition 1 directly

[12] Both Feichtinger and Hartl (1986), Satz 6.2 and Seierstad and Sydsaeter (1987), page 381, theorem 9 require the control variables to maximize the Hamiltonian.

[13] We write the transversality conditions in the form used by Feichtinger and Hartl (1986), chapter 7.2. For further literature regarding dynamic optimization see Chiang (1992) and Kamien and Schwartz (2000).

connotes that positive capacity investments can be optimal, if and only if the capacity constraint is binding.

Energy input and the energy mix are determined by (9). The sum of the marginal costs of fossil fuel $M_b(b)$ and the relative shadow price of fossil fuel vs. capital or *relative scarcity*, respectively,

$$v := \frac{\tau}{\lambda} \tag{19}$$

provides the black energy supply function.[14] Due to (11) and (12), we get

$$\hat{v} = F_k > 0. \tag{20}$$

Thus, the relative scarcity monotonically increases in time. Using Satz 6.2 of Feichtinger and Hartl (1986), or theorem 9 of Seierstad and Sydsaeter (1987), page 381, which both state that the costate variables are only functions of time $t$, we can establish a unique relationship between the relative scarcity $v$ and time $t$ on every optimal evolution path. A similar relationship exists between the fossil fuel stock $s$ and time, as the former monotonically decreases till its exhaustion at time $T$. Consequently, for every $t \in [0, T[$ we can match a unique fossil fuel stock value to every relative scarcity value, i.e. we can write $v = V(s)$, with $\frac{dV}{ds} < 0$ and $V^{max} = V(0)$ as the maximal value of the relative scarcity reached in the moment of fossil fuel exhaustion. Thus, the fossil fuel supply function reads

$$M_b(b) + V(s). \tag{21}$$

The energy demand function is given by the marginal product of energy $F_x(k, x)$, while the green energy supply function $m + \omega$ is linear in energy units. The latter consists of the marginal costs of green energy $m$ and the *mark-up*

$$\omega := \frac{\mu}{\lambda}, \tag{22}$$

which is associated with the capacity constraint $Q$.

Fig. 1 illustrates the equilibrium of the energy sector. Suppose the capacity constraint does not bind. According to (14), the multiplier $\mu$ is zero, i.e. $\omega = 0$. In this case, total energy input is determined by the intersection of the backstop supply and the energy demand function $F_x(k, x^\circ) = m$. Thus, $x^\circ$ only depends on the capital stock $k$, i.e. $x^\circ(k)$.

[14] Kollenbach (2014) calls $v$ the relative scarcity index as it sets the shadow price of fossil fuel into relation to the shadow price of capital.

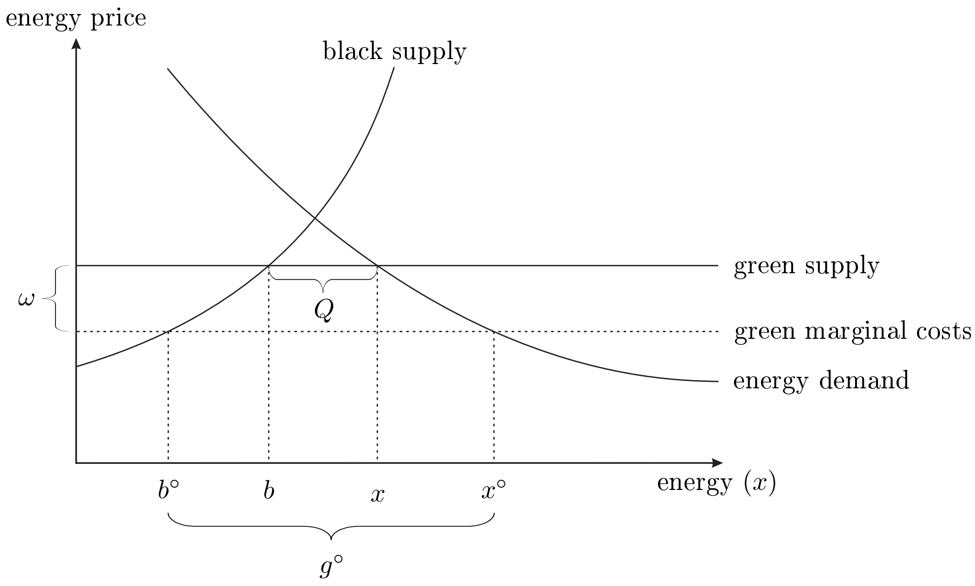



**Figure 1:** Energy sector equilibrium determined by energy demand function, black energy supply function and green energy supply function with and without a binding capacity constraint

Fossil fuel will be used if and only if its social supply costs $M_b(b) + V(s)$ are lower than that of backstop. Thus, the intersection of the two supply functions $M_b(b^\circ) + V(S) = m$ determines the fossil fuel share $b^\circ(s)$ and, therefore, the energy mix $x^\circ(k) = b^\circ(s)+g^\circ(k,s)$. The differentiation of the efficiency conditions in the energy sector with respect to capital and fossil fuel stock give[15]

$$\frac{dx^\circ}{dk} > 0, \quad \frac{db^\circ}{ds} > 0, \quad \frac{\partial g^\circ}{\partial k} > 0, \quad \frac{\partial g^\circ}{\partial s} < 0 \tag{23}$$

Because of $F_{xk} > 0$, a higher capital stock boosts energy demand ceteris paribus. Consequently, both total energy supply and backstop use increase with the capital stock. Graphically, a higher capital stock shifts the energy demand function in Fig. 1 to the right, so that both $x^{circ}$ and $g^\circ$ are higher. Due to an increasing relative scarcity, fossil fuel use decreases with the declining fossil fuel stock. As fossil fuel is substituted by backstop, the latter increases.

If the capacity constraints binds, backstop supply equals $Q$. However, total energy input and the energy mix have still to fulfill (9). Thus, total energy input is determined by $F_x(k,x) = m + \omega$ and fossil fuel extraction by $M_b(b) + V(s) = m + \omega$. Furthermore, $\omega$ has to be such that $Q = x - b$ holds. Consequently, we can write $\omega = \omega(k,s,Q)$,

[15]We get $\frac{dx}{dk} = -\frac{F_{xk}}{F_{xx}}$, $\frac{db}{ds} = -\frac{1}{M_{bb}}\frac{dV}{ds}$, $\frac{\partial g}{\partial k} = -\frac{F_{xk}}{F_{xx}}$, and $\frac{\partial g}{\partial s} = \frac{1}{M_{bb}}\frac{dV}{ds}$.

$x = x(k,s,Q)$ and $b = b(k,s,Q)$.[16] Differentiating $F_x(k, x(k,s,Q)) = m + \omega(k,s,Q)$, $M_b(b(k,s,Q) + V(s) = m + \omega(k,s,Q)$, and $x(k,s,Q) = b(k,s,Q) + Q$ with respect to capital, fossil fuel stock, and capacity gives[17]

$$\frac{\partial x}{\partial k} > 0, \quad \frac{\partial x}{\partial s} > 0, \quad \frac{\partial x}{\partial Q} > 0, \tag{24}$$

$$\frac{\partial b}{\partial k} > 0, \quad \frac{\partial b}{\partial s} > 0, \quad \frac{\partial b}{\partial Q} < 0, \tag{25}$$

$$\frac{\partial \omega}{\partial k} > 0, \quad \frac{\partial \omega}{\partial s} < 0, \quad \frac{\partial \omega}{\partial Q} < 0. \tag{26}$$

As mentioned above, due to $F_{xk} > 0$, the higher the capital stock the farther to the right the energy demand function is located in Fig. 1. Therefore, total energy input increases with the capital stock. However, the binding capacity constraint limits backstop use to $g = Q$. Consequently, the mark-up $\omega$ has to adjust such that (9) and $g = Q$ hold, i.e. such that distance between the black energy supply function and the energy demand function in the equilibrium equals $Q$. In case of an increased capital stock, this requires a higher mark-up, so that the green energy supply function is shifted upwards. Therefore, it intersects the black energy supply function at a higher energy price value, which implies more fossil fuel extraction.

The relative scarcity increases as the fossil fuel stock decreases in time. The former is represented by an upward-shift of the black energy supply function in Fig. 1. To guarantee $g = Q$, the mark-up $\omega$ has to increase, so that total energy input declines ceteris paribus. With respect to fossil fuel use, the higher relative scarcity connotes less and the higher mark-up more extraction. According to (25), the former effect dominates.

Finally, the higher the backstop capacity $Q$ the lower the mark-up $\omega$, i.e. the lower the position of the green energy supply function in Fig. 1. Consequently, total energy input increases, whereas fossil fuel extraction decreases in $Q$.

[16] Note that $\omega$ is a fraction of the multiplier $\mu$ and the costate variable $\lambda$. While the evolution of the latter is determined by (11), the former can attain every positive value, if the backstop constraint binds. If the backstop unit costs are too high, i.e. if they are located above the intersection of the fossil fuel supply and the energy demand function, backstop supply would be zero and energy generations only relies on fossil fuels. However, as the fossil fuel stock decreases in time the fossil fuel supply function shifts upwards due to the increasing relative scarcity index. Thus, zero backstop supply is only a temporary phenomenon. Following Tsur and Zemel (2005), Kollenbach (2014), and Kollenbach (2015), we assume sufficiently low backstop unit costs, so that $g > 0$.

[17] We get $\frac{\partial x}{\partial k} = \frac{F_{xk}}{M_{bb}-F_{xx}}$, $\frac{\partial x}{\partial s} = -\frac{1}{M_{bb}-F_{xx}}\frac{dV}{ds}$, $\frac{\partial x}{\partial Q} = \frac{M_{bb}}{M_{bb}-F_{xx}}$, $\frac{\partial b}{\partial k} = \frac{F_{xk}}{M_{bb}-F_{xx}}$, $\frac{\partial b}{\partial s} = -\frac{1}{M_{bb}-F_{xx}}\frac{dV}{ds}$, $\frac{\partial b}{\partial Q} = \frac{F_{xx}}{M_{bb}-F_{xx}}$, $\frac{\partial \omega}{\partial k} = \frac{M_{bb}F_{xk}}{M_{bb}-F_{xx}}$, $\frac{\partial \omega}{\partial s} = -\frac{F_{xx}}{M_{bb}-F_{xx}}\frac{dV}{ds}$, and $\frac{\partial \omega}{\partial Q} = \frac{M_{bb}F_{xx}}{M_{bb}-F_{xx}}$.

## 4. Determining optimal evolution

To analyze the optimal evolution of the economy we adapt the method of Tsur and Zemel (2005) and Tsur and Zemel (2011), which is based on *characteristic lines.* To understand the method note that the optimal capacity investment regime is given by lemma 1, while (9) determines optimal backstop and fossil fuel utilization. Thus, given the optimal capacity investments, the evolution paths of the fossil fuel stock $s$ and capacity $Q$ are determined, so that only capital $k$ remains as an independent state variable. In other words, the optimization problem of the social planner reduces to a series of single-state problems based on the optimal decision with respect to capacity investments. It turns out that both the selection of the optimal investment regime and the evolution of the capital stock depend on the relative position of three *characteristic manifolds* in the $(Q, k, s)$-space. Applying the suitable method already used by Tsur and Zemel (2005), Kollenbach (2014) and Kollenbach (2015), we illustrate these manifolds by using their projections on the $(Q, k)$-space. As these projections are lines, we refer to them as *characteristic lines.* The monotonic decrease of the fossil fuel stock in time is represented by a downward shift of the lines in the $(Q, k)$-space. As will be shown in 4.2, the characteristic lines divide the $(Q, k)$-space in subspaces with specific properties. These properties allow us to illustrate several evolution path in 4.3. As the paths illustrate the evolution of the economy in the $(Q, k)$-space, we refer to them as $(Q, k)$*-processes.*

In the following we say that the economy or the $(Q, k)$-process, respectively, is located above (on, below) the characteristic line, if $k(t) > \alpha(Q(t))$ $\Big(k(t) = \alpha(Q(t)); k(t) < \alpha(Q(t))\Big)$, with $\alpha(Q)$ denoting an arbitrary characteristic line in the $(Q, k)$-space. Due to proposition 1, capacity investments are minimal, if the constraint is non-binding. Therefore, $Q(t)$ remains constant, so that the $(Q, K)$-process reduces to changes of the capital stock. However, both capital and capacity can change, if the constraint is binding, which is assumed in the following, if not stated otherwise.

### *4.1. The characteristic lines*

The characteristic manifolds describe points in the $(Q, k, s)$-space with specific characteristics. The first manifold gives all points of the $(Q, k, s)$-space where the capacity is just sufficiently large to allow the mark-up $\omega$ to equal zero. With regard to Fig. 1, capacity $Q$ and capital stock $k$ must be such that $x = x^\circ$ and $g = g^\circ = Q$. In other words, on this manifold an economy exhibits neither an over- nor an under-capacity. Therefore, we refer to the projections of the manifold on the $(Q, k)$-space as the *sufficient capacity line* (SCL). The manifold is given by

$$F_x(k, x(k, s, Q)) = m, \tag{27}$$

which implicitly defines the function $K^C(Q, s)$. In Appendix A.3 it is shown that

$$\frac{\partial K^C}{\partial Q} = -\frac{F_{xx}}{F_{xk}} > 0, \tag{28}$$

$$\frac{\partial K^C}{\partial s} = \frac{F_{xx}}{F_{xk} M_{bb}} \frac{dV}{ds} > 0. \tag{29}$$

Thus, the sufficient capacity line continuously increases in the $(Q, k)$-space. Consider Fig. 1, for an arbitrary fossil fuel stock value, i.e. for given fossil fuel extraction $b^\circ$, a higher capacity and $Q = g^\circ$ require an upward shift of the demand function. Thus, the capital stock needs to be higher ceteris paribus.

Furthermore, (29) states the influence of the decreasing fossil fuel stock on the position of the sufficient capacity line in the $(Q, k)$-space. Since $\frac{\partial K^C}{\partial s} > 0$, the lower the fossil fuel stock the lower the position of the line. (25) shows that the lower the fossil fuel stock the lower black energy use ceteris paribus. For a given capacity, less black energy use requires a lower capital stock to ensure validity of (27). As illustrated in Fig. 2(a), the sufficient capacity line is shifted downward in time till the fossil fuel stock is exhausted at time $T$. Its initial position is given by SCL(0) and its *long-run* position valid for all $t \geq T$ by SCL(T).[18]

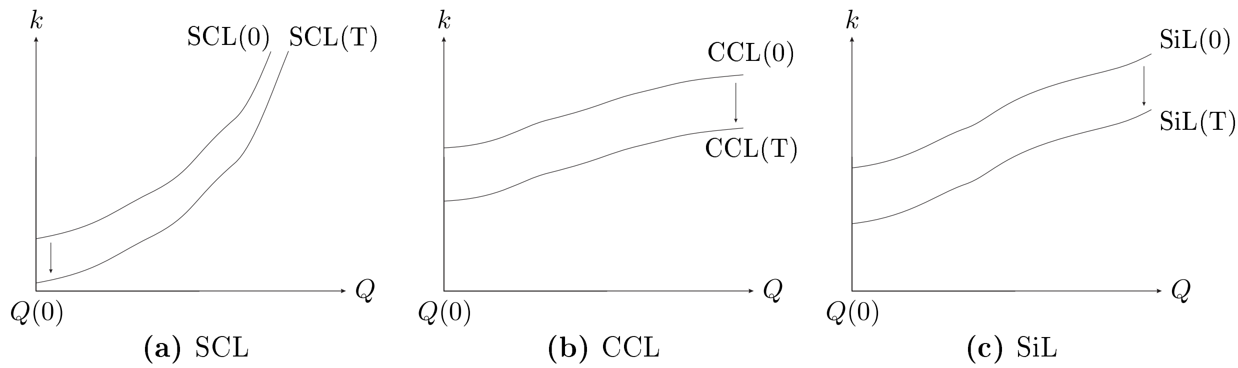



**Figure 2:** Sufficient capacity line (SCL), constant consumption line (CCL), and singular line (SiL) for $t = 0$ and $t = T$

The second characteristic manifold describes all points in the $(Q, k)$-space which allow for constant consumption and a binding capacity constraint, i.e. for $g(t) = Q(t)$ and

[18]Similar to SCL(0) and SCL(T) we refer to the projection of the manifold valid at time $0 < t < T$, i.e. for the fossil fuel stock $s(t)$, as SCL(t).

$\hat{c}(t) = 0$. Therefore, we refer to the projections of the manifold on the $(Q,k)$-space as the *constant consumption line* (CCL). According to the Ramsey-rule (17), the manifold is given by

$$F_k(k, x(k,s,Q)) = \rho, \tag{30}$$

which implicitly defines the function $K^N(Q,s)$. Appendix A.3 proves that

$$\frac{\partial K^N}{\partial Q} = -\frac{F_{kx}M_{bb}}{F_{kk}M_{bb} - J} > 0, \tag{31}$$

$$\frac{\partial K^N}{\partial s} = \frac{F_{kx}M_{bb}}{F_{kk}M_{bb} - J}\frac{dV}{ds} > 0. \tag{32}$$

According to (31), the constant consumption line continuously increases in the $(Q,k)$-space. It is noteworthy that $\frac{\partial K^N}{\partial Q} < \frac{\partial K^C}{\partial Q}$, i.e. the sufficient capacity line is located above the constant consumption line for large $Q$. (32) shows that the decreasing fossil fuel stock shifts the constant consumption line downwards in the $(Q,k)$-space till it reaches its long-run position at the fossil fuel exhaustion time $T$. The shift is caused by the ceteris paribus decline of total energy input $\frac{\partial x}{\partial s} > 0$, so that a lower capital stock is required to ensure a constant marginal product of capital. Fig. 2(b) depicts the downward shift of the constant consumption line with CCL(0) referring to the initial position of the line and CCL(T) to its long-run position.

Following Tsur and Zemel (2005), we define a *steady-state* as a situation with constant consumption, capacity and capital stock. In other words, the economy is in a steady-state, if $\dot{c} = \dot{k} = \dot{Q} = 0$. Consequently, a $(Q,k)$-process needs to be located on the constant consumption line to be in a steady-state. However, as long as the fossil fuel stock is not exhausted the constant consumption line shifts downwards in the $(Q,k)$-space. Consequently, to stay on the line a $(Q,k)$-process needs to adjust capital and/or capacity, which contradicts the definition of the steady-state. Therefore, a steady-state can only be located on the long-run constant consumption line CCL(T). In the following we refer to this line as the *steady-state line* (SSL).

**Lemma 2** *An economy can be only in a steady-state if the $(Q,k)$-process is located on the steady-state line, which is given by the long-run constant consumption line.*

The third and last characteristic manifold describes all points of the $(Q,k,s)$-space where singular capacity investments may be optimal. Hence, we refer to its projections on the $(Q,k)$-space as the *singular line* (SiL). Due to proposition 1, positive capacity investments require a binding constraint. Consequently, the singular line is defined for

$g(t) = Q(t)$. Furthermore, lemma 1 connotes that a singular investment regime requires $\theta = \lambda$ and $\dot{\theta} = \dot{\lambda}$. By substituting (11) and (13) in the latter, we get

$$F_k(k, x(k, s, Q)) = \omega(k, s, Q), \tag{33}$$

which implicitly defines the function $K^S(Q, s)$. In Appendix A.3 we prove that

$$\frac{\partial K^S}{\partial Q} = \frac{M_{bb}(F_{xx} - F_{kx})}{M_{bb}(F_{kk} - F_{kx}) - J} > 0, \tag{34}$$

$$\frac{\partial K^S}{\partial s} = \frac{F_{kx} - F_{xx}}{M_{bb}(F_{kk} - F_{kx}) - J} \frac{dV}{ds} > 0. \tag{35}$$

As the other two lines, the singular line increases in the $(Q, k)$-space. Note that $\frac{\partial K^N}{dQ} < \frac{\partial K^S}{dQ} < \frac{\partial K^C}{\partial Q}$, so that the singular line is located above the steady-state line but below the combined investment line for large $Q$. The decreasing fossil fuel stock causes a downward shift of the singular line in the $(Q, k)$-space. On the one hand, the lower the fossil fuel stock the lower total energy input ceteris paribus, so that the left-hand side of (33) is smaller. On the other hand, (26) shows that the mark-up $\omega$ increases with a falling fossil fuel stock, so that the right-hand side of (33) is higher. To ensure equality the capital stock value solving the equation decreases. Both (34) and (35) are illustrated in Fig. 2(c).

*4.2. The subspaces*

The characteristic manifolds divide the $(Q, k, s)$-space into subspaces with specific properties. According to the proofs of Appendix A.4 the properties read as follows.[19]

**Properties:**

(a) Above (below) the sufficient capacity line the economy is characterized by an under- (over-) capacity, i.e. $F_x(k, b+Q) > m$ above and $F_x(k, b+Q) < m$ below the line.

(b) Consumption increases (decreases) below (above) the constant consumption line. According to the Ramsey-rule (17), $F_k(k, x) > \rho$ below and $F_k(k, x) < \rho$ above the line.

(c) Capacity investments are minimal below the singular line and either maximal or *temporary* minimal above the line. Thus, $\frac{\theta}{\lambda} < 1$ below the singular line and either $\frac{\theta}{\lambda} > 1$ or $\frac{\theta}{\lambda} < 1$ above it.

[19] The properties with respect to the singular and the steady-state line are similar to the ones found by Tsur and Zemel (2005). Cf. also Kollenbach (2014).

(d) The singular line exerts a bonding force on the $(Q, k)$-process. In other words, if the process has reached the line, it cannot diverge from it.

If not located on the steady-state or the singular line, the $(Q, k)$-process follows either a minimal or a maximal capacity investment regime, as shown by property (c) and the definition of the singular line. Thus, the $(Q, k)$-process approaches either the steady-state or the singular line on a most rapid approach path (MRAP). Due to proposition 1, capacity investments are only possible, if the capacity constraint binds. Therefore, property (a) directly gives lemma 3.

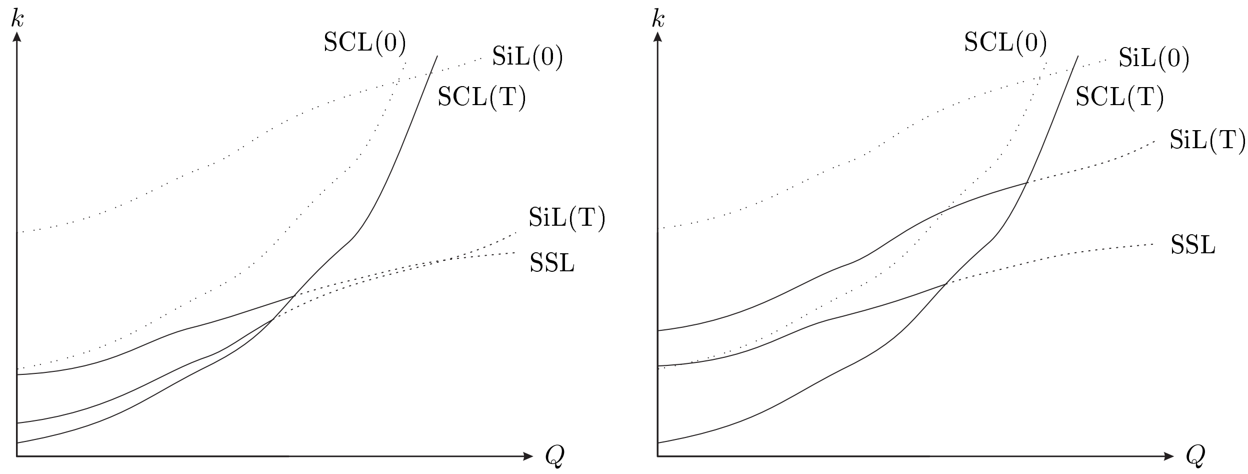
**Lemma 3** *Capacity investments below the sufficient capacity line are not optimal.*

Furthermore, the sufficient capacity line is located above the singular line for large $Q$, as $\frac{dK^S}{dQ} < \frac{dK^C}{dQ}$. According to the definition of the singular line and property (d), capacity investments are singular on the line, while the singular line binds an $(Q, k)$-process. By taking lemma 3 and the MRAP feature into account we can conclude as follows.

**Proposition 2** *The $(Q, k)$-process cannot evolve along the singular line forever. Consequently, capacity investments are minimal for late points in time.*

The downward shift of both the singular and the sufficient capacity line together with property (a) and (c) and proposition 1 illustrates how the decreasing fossil fuel stock boosts the relative profitability of capacity investments $\frac{\theta}{\lambda}$. Due to the downward shift the numbers of points above both lines increases. According to property (a), the points above the sufficient capacity line are characterized by an under-capacity, which is a requirement for capacity investments as implied by proposition 1. Furthermore, only above the singular line capacity investments may be maximal. In short, the lower the fossil fuel stock the more $(Q, k)$-combinations exist which allow for capacity investments. As Tsur and Zemel (2011) abstain from a limited fossil fuel stock, they do not optain a corresponding effect.

According to proposition 2, the $(Q, k)$-process cannot evolve along the singular line forever, so that the process has to converge against a steady-state. Therefore, whether and where the long-run singular line and the steady-state line intersect is of some importance. Suppose the two lines do not intersect in the area restricted by the long-run sufficient capacity line, as illustrated by Fig. 3(a). Furthermore, consider an $(Q, k)$-process which reaches the singular line. Due to property (d), the process has to evolve along the line, which connotes singular capacity investments. However, in the moment




**(a)** invalid intersection

**(b)** no intersection

**Figure 3:** Steady-state line (SSL), singular line (SiL) for $t = 0$ and $t = T$ of a min-max economy

the process crosses the sufficient capacity line, further capacity investments violate lemma 3. A switch into a steady-state is not possible, as the process is not located on the steady-state line and cannot leave the singular line. A similar argument holds with respect to the case illustrated in Fig. 3(b). As the long-run singular and the steady-state line do not intersect, an evolution along the singular line would imply a violation of lemma 3. Thus, in both settings singular capacity investments cannot be part of a feasible solution. Rather, capacity investments are either minimal or maximal. Therefore, we sum up both constellations under the term *min-max case*. However, if there is an intersection of the long-run singular line and the steady-state line above the long-run sufficient capacity line, a $(Q, k)$-process that evolves along the singular line can switch into a steady-state at the intersection, so that singular capacity investments can be possible. Consequently, we refer to this setting as the *singular case*.

*4.3. Economic evolution*

Following Kollenbach (2014) and Kollenbach (2015) we analyze the evolution of the economy by illustrating 7 exemplary $(Q, k)$-processes in the figures 4 and 5. These processes represent possibilities for the optimal evolution path of economy, as they are in line with the properties (a) - (d) and the MRAP feature. We distinguish between the paths by identifying them with their capital endowment $k^i$. The $(Q, k)$-processes increase the complexity of the figures 4 and 5 considerably. To keep the illustrations as simple as possible, we omit the arrows indicating the shift direction of the characteristic lines and the initial constant consumption line. Furthermore, the initial sufficient capacity and singular line are only adumbrated. Finally, it is unrewarding to illustrate every characteristic line, i.e. the projection of the corresponding manifold valid for the current fossil fuel stock value, that is reached or crossed by the $(Q, k)$-processes.

*4.3.1. The singular case*

At first we turn to the singular case, which was defined by an intersection of the long-run singular and the steady-state line above the sufficient capacity line. Fig. 4 illustrates the singular and the sufficient capacity line for $t = 0$ and $t = T$, the steady-state line (SSL) and 4 exemplary $(Q, k)$-processes $k^i$, $i = 1, ..., 5$. According to property (b), consumption increases (decreases) on all depicted paths, as long as the evolution path is located below (above) the constant consumption line.

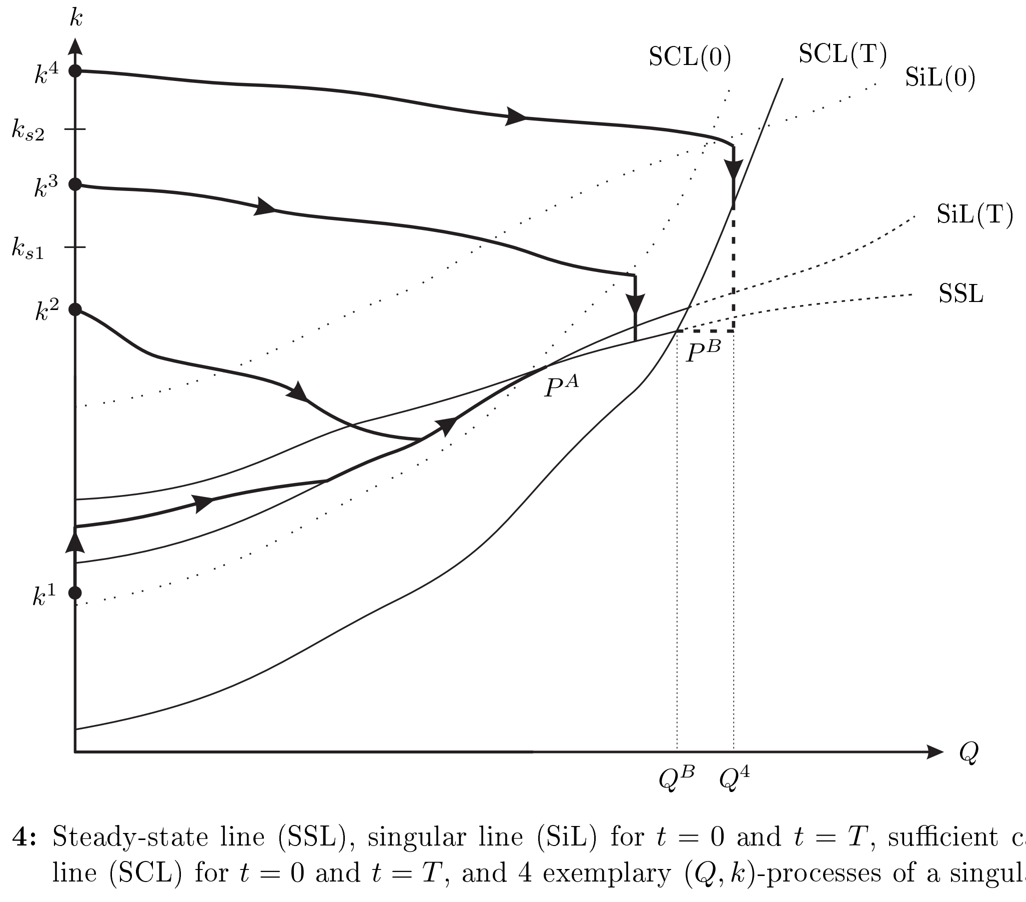



**Figure 4:** Steady-state line (SSL), singular line (SiL) for $t = 0$ and $t = T$, sufficient capacity line (SCL) for $t = 0$ and $t = T$, and 4 exemplary $(Q, k)$-processes of a singular type economy

Consider an economy with the capital endowment $k^1$, which is located below the singular line initially. According to property (c), capacity investments are minimal, so that the $(Q, k)$-process approaches the singular line from below by means of capital accumulation. In the moment the singular line is reached, the investment regime switches to singular capacity investments. In the illustrated case, the switch occurs above the long-run singular line. As the singular investment regime allows for both capacity and capital investments, the $(Q, k)$-process increases in the $(Q, k)$-space. The process evolves along the singular line till it reaches the intersection with the steady-state, i.e. the point $P^A$. Due to proposition 2, it is not optimal to evolve along the singular line forever, so that the $(Q, k)$-process switches into a steady-state at the point $P^A$. As $P^A$ is located above the long-run sufficient capacity line SCL(T), the steady-state is characterized by an under-capacity, so that the mark-up $\omega$ is positive.

The $k^2$-path illustrates a similar long-run evolution pattern. However, the capital endowment $k^2$ is exceeds the singular and the sufficient capacity line initially, so that, in line with the properties (a) and (c) capacity investments are maximal. According to the assumption $\bar{q} > y^n$, the maximal investment regime connotes a decreasing capital stock. Consequently, the $(Q, k)$-process approaches the singular line from above to evolve along it to the steady-state $P^A$.

Consider the evolution of the $k^2$-path at early points in time in more detail. Due to the maximal investment regime, net production is completely spent for capacity investments. Consequently, it can not be used for consumption. However, at later points in time a higher capacity gives rise to more energy input and, therefore, to more net production. Thus, there is a consumption trade-off. The social planner is willing to trade-off more current for future consumption the lower the time preference rate. In terms of the evolution path, the lower the time preference rate the flatter the decreasing part of the $(Q, k)$-process.

For a given time preference rate we an mark a threshold $k_{s1}$. The $(Q, k)$-process approaches the steady-state $P^A$, only if the capital endowment does not exceed this threshold. Otherwise, as illustrated by the $k^3$-path, the economy accumulates more capacity than is used in $P^A$. However, the $(Q, k)$-process cannot approach the singular line, as this would imply an evolution along the line above the steady-state and below the sufficient capacity line. According to property (b) and proposition 2, both are not optimal. Thus, at some point in time characterized by a capacity $Q(t)$ exceeding the level of $P^A$, the maximal investment regime is abandoned above the singular line in favor of minimal capacity investments. Henceforth, the $(Q, k)$-process approaches the steady-state line by means of capital stock adjustments. As long as the steady-state is located between $P^A$ and $P^B$ - the intersection of the steady-state and the long-run sufficient capacity line - it is characterized by an under-capacity, i.e. a positive mark-up $\omega$.

Using a similar argument as with respect to $k_{s1}$, we can mark a second threshold $k_{s2}$. If the capital endowment equals $k_{s2}$, the switch from maximal to minimal capacity investments occurs at the moment the capacity $Q(t)$ equals $Q^B$, so that the process converges against $P^B$. Consequently, the mark-up in the steady-state is zero. However, if the capital endowment exceeds the second threshold, as does $k^4$, the switch occurs at a higher capacity level. In other words, more capacity is accumulated than utilized in $P^B$. Given a completely used capacity $Q^4$ the steady-state would be located below the long-run sufficient capacity line, which is not feasible. Consequently, the steady-state of a $k^4$-type economy is characterized by an over-capacity. To determine the steady-state, recall that, given an over-capacity, (9) requires $F_x(k, x) = m$ to hold while the long-run sufficient capacity line is defined by $F_x(k, Q) = m$ and $x = Q$. Thus, the steady-state needs to be located on the long-run sufficient capacity line. In other words, the intersection of the steady-state and the long-run sufficient capacity line $P^B$ determines steady-state capital stock and energy input $x^B = Q^B < Q^4$. The economy approaches this steady-state by evolving along the sufficient capacity line with $g(t) < Q^4$.

**Proposition 3** *If the capital endowment $k(0)$ of a singular type economy exceeds the threshold $k_{s2}$, steady-state backstop use falls short of the accumulated backstop capacity, i.e. the economy is characterized by an over-capacity in the steady-state.*

To rationalize the $k^4$-path, notice that the capacity constraint is binding on the section of the vertically falling part of the evolution path that is located above the sufficient capacity line. On this section both energy input and the capital stock exceed their steady-state values, while capacity investments are zero. Therefore, consumption is higher than in the steady-state. If this mid-term excess consumption is sufficiently high and the time preference rate low, it outweighs the negative effect of capacity investments on consumption at earlier points of time.

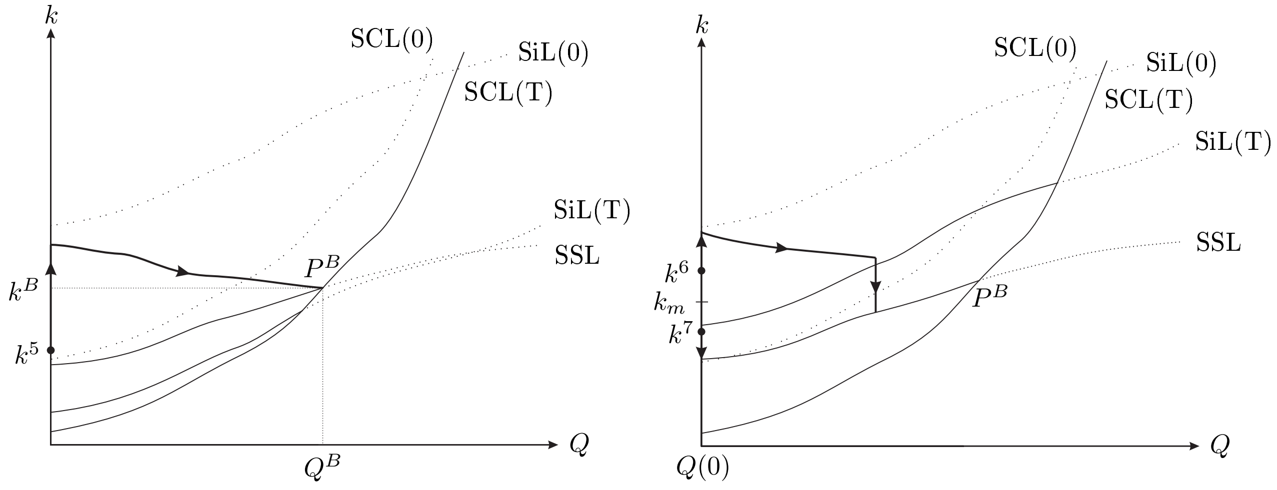
The location of the thresholds $k_{s1}$ and $k_{s2}$ crucially depends on the time preference rate. As stated above, the higher the preference rate the less beneficial the trade-off of current for future consumption. During a maximal capacity investment regime, $q = \bar{q} > y^n$, so that net production is completely used for investments. As consumption must be positive, (6) connotes a declining capital stock. Thus, the higher the time preference rate the higher the consumption in early periods. Consequently, the decreasing part of the evolution path are the steeper and the thresholds higher.

With the $k^4$-path and the threshold $k_{s2}$ we extend the results of Powell and Oren (1989) and Tsur and Zemel (2011). According to the former, the steady-state benefit of backstop always exceeds marginal backstop costs in absence of capital, i.e. there is always a positive mark-up. Consequently, less backstop is used than without a backstop capacity constraint. The threshold $k_{s2}$ and the $k^4$-path show that also the opposite can be true, if the capital endowment is sufficiently large. If the capital endowment equals $k_{s2}$ the $(Q, k)$-process approaches the steady-state $P^B$, so that the mark-up is zero. In case of the $k^4$-path, an over-capacity exists in the steady-state, which also implies a non-existing mark-up. In other words, a high capital endowment guarantees the utilization of the full production potential of the steady-state capital stock. Furthermore, the possibility of a capital-driven excess capacity is neither obtained by Powell and Oren (1989) nor by Tsur and Zemel (2011). According to Fischer et al. (2004), a high initial pollution may also cause an excess capacity. However, Fischer et al. (2004) do not consider capital. Finally, Tsur and Zemel (2011) also do not discuss the possibility of maximal capacity investments, which we show to be optimal in case of a sufficiently large capital endowment.

### *4.4. The min-max case*

After having discussed the singular case, we turn to the min-max case, which is characterized by the non-existence of an intersection between the long-run singular SiL(T) and the steady-state line SSL above the long-run sufficient capacity line SCL(T). As illustrated in figures 3 and 5, either the singular line intersects the steady-state line below the sufficient capacity line or it is located above the steady-state line for all $Q$. Recall that in both cases the $(Q, k)$-process cannot evolve along the singular line. Due to property (d), this rules out every $(Q, k)$-process which approaches the singular line.

Consider at first the case of Fig. 5(a). The singular line is located above the steady-state line in the area of the $(Q, k)$-space restricted by the long-run sufficient capacity line. According to property (c), minimal capacity investments above the singular line are only a temporary option. However, recall that the singular line is defined for a binding capacity constraint. Consequently, it does not exist at the point $P^B$, which is the only possible steady-state the $(Q, k)$-process can converge to. If the capital endowment is sufficiently large, the $(Q, k)$-process is similar to the $k^3$- or $k^4$-path. In the illustrated case of the $k^5$-path, the capital endowment falls short of the singular line at early points in




**(a)** invalid intersection

**(b)** no intersection

**Figure 5:** Steady-state line (SSL), singular line (SiL) for $t = 0$ and $t = T$, sufficient capacity line for $t = 0$ and $t = T$, and 3 exemplary $(Q, k)$-processes of a min-max type economy

time, so that capacity investments are minimal. Consequently, capital is accumulated. As singular capacity investments are not optimal, there is no switch from minimal to singular investments at the moment the evolution path reaches the singular line. Rather, capacity investments remain minimal while the capital stock is increased. Due to this investment regime and the scarcity driven downward shift of the singular line, the $(Q, k)$-process surpasses the singular line. Above the singular line maximal capacity investments are optimal. Consequently, the $(Q, k)$-process converges against the steady-state $P^B$. Since this steady-state is the only feasible one in the illustrated setting, capital investments need to be sufficiently high at early points in time. Otherwise, the $(Q, k)$-process would surpass the singular line below $k^B$, so that the steady-state $P^B$ cannot be reached.

In the case illustrated by Fig. 5(b), the singular line is always located above the steady-state line. Thus, every point on the steady-state line is a feasible steady-state. If the capital endowment is sufficiently large, the $(Q, k)$-process is similar to the $k^3$- or the $k^4$-path. In case of a small capital endowment, such as $k^6$, the $(Q, k)$-process evolves as the $k^5$-path at early points in time, i.e. it surpasses the singular line due to capital accumulation to switch to maximal capacity investments above the singular line. As the steady-state line is located below the singular line for all $Q$, there needs to be a switch from maximal capacity investments to minimal one as long as the $(Q, k)$-process is located above the singular line. In the illustrated case the accumulated capital stock is not high enough to reach the steady-state $P^B$. Instead, maximal capacity investments are abandoned at a smaller level. Afterwards, the $(Q, k)$-process converges against a steady-state by means of

capital adjustments. Obviously, the higher the capital stock at the moment of the switch to maximal capacity investments the higher the reachable steady-state values of capital and capacity and, therefore, of consumption. However, high early capital investments require a small time preference rate, so that the trade-off of early for later consumption is beneficial. Thus, the lower the rate the higher steady-state consumption.
If the capital endowment is too low given the time preference rate, the trade-off is generally not beneficial. In other words, we can mark a threshold $k_m$. Only if the capital at least equals this threshold, capital and capacity investments are possible. Otherwise, as illustrated by the $k^7$-path, the $(Q, k)$-process approaches the steady-state line at $Q(0)$ by means of capital stock reduction. Thus, the $k^7$-path illustrates a poverty trap in the sense of Tsur and Zemel (2005), Kollenbach (2014), and Kollenbach (2015). However, our result is not driven by a lack of research expenditures, but by a lack of backstop capacity investments. The location of the threshold $k_m$ crucially depends on the time preference rate. The lower (higher) the rate the higher (lower) the value of long-run consumption and, therefore, the lower (higher) the threshold.

**Proposition 4** *If the capital endowment $k(0)$ of a min-max type economy falls short of the threshold $k_m$, backstop investments are minimal for all points in time.*

Similar to the $k^1$- and $k^2$-paths, the whole min-max case augments the results of Powell and Oren (1989) and Tsur and Zemel (2011). In particular, the non-optimality of singular capacity investments and, therefore, the dependence on maximal ones for the realization of an improved steady-state contrasts with Tsur and Zemel (2011), who focus on singular investments. Nonetheless, the $k^7$-path bears a resemblance to the fossil fuel based economy of Tsur and Zemel (2011). With an unlimited fossil fuel stock, energy demand can be satisfied by fossil fuels. Consequently, Tsur and Zemel (2011) do not identify the corresponding evolution path as a poverty trap. However, with a limited stock, this possibility of a fossil fuel based economy does not exist. Consequently, the economy relies on the more expensive backstop, so that the welfare prospects are less bright than in the setting of Tsur and Zemel (2011).

### *4.5. Determinants of steady-state consumption*

As shown in the previous two sections the steady-state depends not only on capital endowment and time preference rate, but also on the long-run position of the characteristic lines. Steady-state consumption is the higher the larger the steady-state capital stock and capacity, i.e. the further in the north-east of the $(Q, k)$-space the steady is located. In terms of the characteristic lines this requires a high position of the steady-state line and a low position of both the sufficient capacity line and the singular line. Furthermore, the latter two should be rather flat, so that the number of $(Q, k)$-combinations allowing for and requiring capacity investments, respectively, is high.

Inspecting (27) shows that, ceteris paribus, the marginal product of energy is the smaller the smaller the capital stock. Thus, low backstop unit costs give rise to a low position of the sufficient capacity line. Fig. 1 illustrates how lower backstop unit costs increase the advantageousness of capacity investments. On the one hand, lower backstop unit costs increase the amount of backstop that would be used given a sufficiently large capacity ($g^{\circ}$). On the other hand, the possible maximal energy input ($x^{\circ}$) is boosted, which would increase production.

Using a similar formal argument with respect to (30) shows that the position of the steady-state line is the higher the smaller the time preference rate. The economic intuition is the same as mentioned with regard to the thresholds. The lower the time preference rate the more beneficial the trade-off of early for later consumption.

Finally, (28) and (34) show that the sufficient capacity and the singular line are the flatter the lower $|F_{xx}|$, which connotes a flat energy demand function in Fig. 1 and, therefore, a high elasticity of the marginal product of energy $\epsilon_{F_x,x} = F_{xx}\frac{x}{F_x}$. The effects of a flat energy demand function are similar to the ones of low backstop unit costs. That is, both the potential backstop input $g^{\circ}$ and the maximal possible energy input $x^{\circ}$ are the higher the lower $|F_{xx}|$ ceteris paribus.

A high elasticity of marginal extraction costs $\epsilon_{M_b,b} = M_{bb}\frac{b}{M_b}$ has no effect on the slope of the sufficient capacity line in the $(Q, k)$-space. However, the higher $M_{bb}$ the closer the right-hand side of both (31) and (34) to unity. In other words, $\frac{\partial K^N}{\partial Q}$ and $\frac{\partial K^S}{\partial Q}$ resemble parallels for elastic marginal extraction costs, which implies an intersection of the steady-state line and the long-run singular line in the far north-east of the $(Q, k)$-space. Furthermore, (29) shows that the impact of the decreasing fossil fuel stock on the position of the sufficient capacity line is weak if $M_{bb}$ is high. Thus, the initial position of the line is close to the long-run position. To explain these effects we refer again to Fig. 1. A high $M_{bb}$ implies a steep fossil fuel supply function. Consequently, the economy relies heavily on backstop from the very beginning. Furthermore, the scarcity induced reduction of fossil fuel use is small in absolute terms. On the one hand, the high reliance on backstop gives rise to a high relative profitability of capacity investments $\frac{\theta}{\lambda}$. On the other hand, the relative profitability does not increase much in time, due to the small capacity induced reduction of fossil fuel use. The results are summarized in proposition 5.

**Proposition 5** *Ceteris paribus, steady-state consumption is the higher the lower the time preference rate, the lower the unit costs of backstop, and the higher the elasticities of marginal extraction costs $\epsilon_{M_b,b}$ and of the marginal product of energy with respect to energy $\epsilon_{F_x,x}$.*

Tsur and Zemel (2011) consider constant unit costs of fossil fuel extraction and no marginal backstop costs, while Powell and Oren (1989) abstains from capital and fossil fuel extraction costs. Therefore, the corresponding results of proposition 5 cannot be obtained in their models.

## 5. Conclusion

We analyze how a capital-energy economy should invest in backstop capacity and how the limited but extendable capacity affects the evolution of the economy. For this purpose we determine the social optimum of the economy given a limited fossil fuel stock and a limited capacity necessary for the utilization of a renewable backstop energy source. The capacity can be extended by investing the composite good. Therefore, capacity investments compete with capital investments and consumption for limited funds. Similar to Tsur and Zemel (2005, 2011), our analysis of the optimal evolution path is based on the relative position of three characteristic lines in the capacity-capital space.
According to our results, the steady-state crucially depends on the capital endowment and the type of the economy. If singular capacity investments are possible (singular type economy) and the capital endowment is located between two critical values, the steady-state levels of backstop capacity, capital and, therefore, consumption are the higher the higher the capital endowment. However, if the capital endowment falls short of the lower critical value or exceeds the upper one, the capital endowment dependence of the steady-state vanishes. In the case that the capital endowment falls short of the upper critical value, the steady-state is characterized by an under-capacity, i.e. there is a positive mark-up on backstop costs. In contrast, if the capital endowment exceeds the upper critical value, the economy accumulates more backstop capacity than is used in the steady-state. In other words, a sufficiently high capital endowment gives rise to the accumulation of an

excess backstop capacity. Consequently, the full production potential of the steady-state capital stock is used. Thus, the marginal product of backstop equals marginal backstop costs, so that there is no mark-up on backstop costs. The excess capacity is only used in the mid-term but not in the steady-state. Therefore, the negative consumption effect of capacity investments in early periods is outweighed by increased mid-term consumption. In other words, there is a trade-off between early and late consumption. Obviously, the lower the time preference rate the more beneficial the trade-off and, consequently, the lower the critical values.

If the economy type does not allow for singular capacity investments (min-max type economy) and the capital endowment falls short of a critical value, there are no capacity investments. In this case, the consumption trade-off is not beneficial and the economy is in a poverty trap.[20]

With capital, exhaustible fossil fuels and positive backstop unit costs, we integrate several aspects, which are not all considered by Powell and Oren (1989), Fischer et al. (2004), Wirl and Withagen (2000), and Tsur and Zemel (2011).[21] In particular the capital endowment can affect the results considerably. The possibility of a capital-driven excess capacity is not obtained by Powell and Oren (1989) and Tsur and Zemel (2011). Furthermore, Powell and Oren (1989) and Tsur and Zemel (2011) postulate a mark-up on marginal backstop costs in the steady-state, so that not the full potential of the capital stock is used. According to our results, such a mark-up only exists if the capital endowment is low. Thus, our analysis provides a more optimistic view concerning the evolution prospects of economies with a high capital endowment.

The exhaustible fossil fuel stock gives rise to two important results. Firstly, the decrease of the stock in time boosts the relative profitability of capacity investments. Moreover, in combination with a low capital endowment it gives rise to the poverty trap. While an economy without capacity investments is also obtained by Tsur and Zemel (2011), they do not find the economy to be in a poverty trap. Due to their assumption of an unlimited fossil fuel stock, energy generation can heavily rely on fossil fuel for all time. In our setting of a limited fossil fuel stock, this possibility of a fossil fuel based economy does not exist. Thus, our analysis suggests a more pessimistic view concerning the welfare of such an

[20] A poverty trap is also found by Tsur and Zemel (2005), Kollenbach (2014), and Kollenbach (2015). However, the cause of our poverty trap is not a lack of research expenditures but of capacity investments.

[21] Recall that Fischer et al. (2004) and Wirl and Withagen (2000) focus on the effects of pollution.

economy.

Our present model makes use of several simplifying assumptions. In particular, we abstain from a stock dependence of fossil fuel extraction costs. As shown by Kollenbach (2015), a stock dependence may cause the economically but not physically exhaustion of fossil fuels. In this case, the stock left in situ depends on the availability of backstop, i.e. the speed of capacity accumulation. Our results may also be affected by endogenous technological progress and pollution, which both may boost potential backstop utilization and, therefore, the relative profitability of capacity investments. In particular, technology progress may give rise to everlasting growth.

## A. Appendix

### *A.1. Proof of Lemma 1*

The maximization of the Hamiltonian gives

$$\begin{aligned} q = 0, \text{ if } & -\lambda + \theta < 0, \\ 0 \leq q \leq \bar{q}, \text{ if } & -\lambda + \theta = 0, \\ q = \bar{q}, \text{ if } & -\lambda + \theta > 0. \end{aligned} \tag{A.1}$$

If $\theta < \lambda$, capacity investments are zero. As $\zeta_{\bar{q}} = 0$, (10) connotes $\zeta_q = \lambda - \theta > 0$, which is in line with (15). In case of singular investments, $\theta - \lambda = \zeta_q = \zeta_{\bar{q}} = 0$, and $\zeta_{\bar{q}} = \theta - \lambda > 0$, $\zeta_q = 0$, if capacity investments are maximal.

### *A.2. Proof of Proposition 1*

The capacity constraint can be non-binding either for a limited time interval $[t_1, t_2]$, with $0 \leq t_1 < t_2 < \infty$, or an unlimited time interval $[t_3, \infty[$, with $0 \leq t_3 < \infty$. At first, consider the case of the limited time interval. Suppose two investment plans. The first one stipulates positive capacity investments at some point in time $t_1 \leq t < t_2$. The second one resembles the first one but reallocates one marginal investment unit from capacity to capital investments at time $t$. At time $t_2$ this marginal capital unit is disinvested and used for a capacity investment, so that capacity at time $t_2$ is identical under both investment plans. The reallocation at time $t$ has no effect on energy input, as the constraint is non-binding during $t_1 \leq t < t_2$. However, the reallocation increases the capital stock, so that production is higher during the time interval $[t, t_2[$. This additional production can be used either for consumption or capital accumulation. In both cases, the second investment plan is superior to the first one. This argumentation can be repeated until there are no capacity investments at time $t$.

Consider now the unlimited time interval $[t_3, \infty[$. As the capacity constraint is non-binding, $\mu(t) = 0$ for all $t \in [t_3, \infty[$. Suppose that $q(t) > 0$ at the same point in time $t_3 \leq t < \infty$. According to lemma 1, $\theta(t) \geq \lambda(t)$ is required. Since the shadow price of the capital stock $\lambda$ is positive for all points in time, $q(t) > 0$ implies $\theta(t) > 0$. However, due to (13) and $\mu = 0$ we can write $\theta(\tilde{t}) = \theta_t e^{\rho \tilde{t}}$, with $\theta_t > 0$ and $\tilde{t} \geq t$. Substituting into (18)(c) gives $\lim_{\tilde{t}\to\infty} \theta_t[Q(\tilde{t}) - Q^*(\tilde{t})] \geq 0$. Due to the capacity investments at time $t$ there exists at least one feasible evolution path with $\lim_{\tilde{t}\to\infty} [Q(\tilde{t}) - Q^*(\tilde{t})] < 0$. Thus, the capacity investments $q(t) > 0$ violate the transversality condition. This violation together with the upper paragraph prove proposition 1.

*A.3. Slope of the characteristic manifolds*

To determine the slope of the sufficient capacity line we substitute $K^C(Q, s)$ in (27) and differentiate with respect to $Q$ and $s$, which yields

$$\begin{aligned} &\left[F_{xk} + F_{xx}\frac{\partial x}{\partial k}\right]\frac{\partial K^C}{\partial Q} + F_{xx}\frac{\partial x}{\partial Q} = 0 \\ \Leftrightarrow &\frac{\partial K^C}{\partial Q} = -\frac{F_{xx}}{F_{xk}} > 0. \end{aligned} \tag{A.2}$$

$$\begin{aligned} &\left[F_{xk} + F_{xx}\frac{\partial x}{\partial k}\right]\frac{\partial K^C}{\partial s} + F_{xx}\frac{\partial x}{\partial s} = 0 \\ \Leftrightarrow &\frac{\partial K^C}{\partial s} = \frac{F_{xx}}{F_{xk}M_{bb}}\frac{dV}{ds} > 0. \end{aligned} \tag{A.3}$$

Analogously, the slope of the constant consumption line is determined by substituting $K^N(Q, s)$ in (30) and differentiate with respect to $Q$ and $s$. We get

$$\begin{aligned} &\left[F_{kk} + F_{kx}\frac{\partial x}{\partial k}\right]\frac{\partial K^N}{\partial Q} + F_{kx}\frac{\partial x}{\partial Q} = 0 \\ \Leftrightarrow &\frac{\partial K^N}{\partial Q} = -\frac{F_{kx}M_{bb}}{F_{kk}M_{bb} - J} > 0. \end{aligned} \tag{A.4}$$

$$\begin{aligned} &\left[F_{kk} + F_{kx}\frac{\partial x}{\partial k}\right]\frac{\partial K^N}{\partial s} + F_{kx}\frac{\partial x}{\partial s} = 0 \\ \Leftrightarrow &\frac{\partial K^N}{\partial s} = \frac{F_{kx}}{F_{kk}M_{bb} - J}\frac{dV}{ds} > 0. \end{aligned} \tag{A.5}$$

The slope of the singular line is determined by substituting $K^S(Q,s)$ in (33) and differentiating with respect to $Q$ and $s$.

$$\begin{aligned}
&\left[F_{kk}+F_{kx}\frac{\partial x}{\partial k}\right]\frac{\partial K^S}{\partial Q}+F_{kx}\frac{\partial x}{\partial Q}=\frac{\partial \omega}{\partial Q}+\frac{\partial \omega}{\partial k}\frac{\partial K^S}{\partial Q}\\
\Leftrightarrow&\frac{\partial K^S}{\partial Q}=\frac{M_{bb}(F_{xx}-F_{kx})}{M_{bb}(F_{kk}-F_{kx})-J}>0, \qquad \text{(A.6)}\\
&\left[F_{kk}+F_{kx}\frac{\partial x}{\partial k}\right]\frac{\partial K^S}{\partial s}+F_{kx}\frac{\partial x}{\partial s}=\frac{\partial \omega}{\partial k}\frac{\partial K^S}{\partial s}+\frac{\partial \omega}{\partial s}\\
\Leftrightarrow&\frac{\partial K^S}{\partial s}=\frac{F_{kx}-F_{xx}}{M_{bb}(F_{kk}-F_{kx})-J}\frac{dV}{ds}>0. \qquad \text{(A.7)}
\end{aligned}$$

*A.4. Properties of subspaces*

In the following we prove the properties of the subspaces, determined by the characteristic manifolds. As we adapt Tsur & Zemel's (2005) model, the proofs follow or modify the proofs of Tsur & Zemel's appendix.

**SCL**

Define $\Lambda_C$ as

$$\Lambda_C(k,s,Q):=F_x(k,x(k,s,Q))-m. \qquad \text{(A.8)}$$

According to the definition of the sufficient capacity line, $\Lambda_C(K^C(Q,s),s,Q)=0$. Differentiating $\Lambda_C$ with respect to capital gives $\frac{\partial \Lambda_C}{\partial k}=\frac{M_{bb}F_{xk}}{M_{bb}-F_{xx}}>0$. Thus, $F_x(k,x(k,s,Q))>m$ above and $F_x(k,x(k,s,Q))<m$ below the sufficient investment line. In the former case, (9) implies a positive mark-up $\omega$. In other words, the economy exhibits an under-capacity. The case of $F_x<m$ is ruled out by (9). Hence, the assumption of a binding constraint ($g=Q$) does not hold below the sufficient capacity line. Rather, $g<Q$ is necessary to guarantee a sufficient low energy input which is in line with (9). In other words, below the sufficient capacity line the economy is characterized by an over-capacity.

**Lemma 4** *Below (on, above) the sufficient capacity line the economy is characterized by an over- (under, just sufficient) capacity.*

**CCL**

Define $\Lambda_N$ as

$$\Lambda_N(k,s,Q):=F_k(k,x(k,s,Q))-\rho. \qquad \text{(A.9)}$$

According to the definition of the constant consumption line, $\Lambda_N(K^N(Q,s),s,Q)=0$. Differentiating $\Lambda_N$ with respect to capital gives $\frac{\partial \Lambda_N}{\partial k}=\frac{1}{M_{bb}-F_{xx}}\left[M_{bb}F_{kk}-J\right]<0$, so that

$F_k(k, x(k, s, Q)) > \rho$ below and $F_k(k, x(k, s, Q)) < \rho$ above the line. Due to the Ramsey-rule (17), the former connotes consumption growth below and the latter consumption decline above the constant consumption line.

**Lemma 5** *Consumption increases below and decreases above the constant consumption line.*

**SiL**

Define $\Lambda_S$ and $\varsigma$ as

$$\Lambda_S(k, s, Q) := F_k(k, x(k, s, Q)) - \omega(k, s, Q), \tag{A.10}$$

$$\varsigma := \lambda - \theta. \tag{A.11}$$

According to the definition of the singular line, $\Lambda_S(K^S(Q, s), s, Q) = 0$. Differentiating $\Lambda_S$ with respect to capital gives $\frac{\partial \Lambda_S}{\partial k} = \frac{1}{M_{bb} - F_{xx}} \left[M_{bb}(F_{kk} - F_{xk}) - J\right] < 0$. Thus, $\Lambda_S < 0$ above and $\Lambda_S > 0$ below the singular line.

Due to the definition of $\varsigma$ and (A.1), capacity investments $q$ are minimal if $\varsigma > 0$, singular if $\varsigma = 0$, and maximal if $\varsigma < 0$. (11) and (13) determine the evolution of $\varsigma$ as

$$\dot{\varsigma} = \rho\varsigma - \lambda\Lambda_S. \tag{A.12}$$

Consider an $(Q, k)$-process exhibiting maximal capacity investments below the long-run singular line, so that $\Lambda_S > 0$ and $\varsigma < 0$. According to (A.12), $\dot{\varsigma} < \rho\varsigma$. If the investment regime lasts forever, $\lim_{t\to\infty} e^{-\rho t}\theta(t) = \infty$. However, this contradicts transversality condition (18)(c). As the maximal capacity investment regime connotes a decreasing capital stock and the singular line increases in the $(Q, k)$-space, no $(Q, k)$ exhibiting maximal capacity investments below the long-run singular line can reach the line. Consequently, these $(Q, k)$-processes are not optimal.

**Lemma 6** *Maximal capacity investments below the long-run singular line are not optimal.*

If the capacity constraint is binding, net production $Y^n$ can be written as

$$Y^n(k, s, Q) = F(k, x(k, s, Q)) - M(b(k, s, Q)) - mQ. \tag{A.13}$$

Differentiating with respect to capital and capacity gives $Y^n_k = F_k + v\frac{\partial b}{\partial k}$ and $Y^n_Q = \omega + v\frac{\partial b}{\partial Q}$ so that

$$\frac{\partial Y^n}{\partial k} - \frac{\partial Y^n}{\partial Q} = \Lambda_S(k, s, Q) + v\left(\frac{\partial b}{\partial k} - \frac{\partial b}{\partial Q}\right). \tag{A.14}$$

The term in brackets is positive if fossil fuel is not exhausted, while $\Lambda_S > 0$ below and $\Lambda_S < 0$ above the singular line. Thus, below the singular line $Y_k^n - Y_Q^n > 0$, i.e. capital investments increase net production to a higher degree than capacity investments. For an exhausted fossil fuel stock $Y_k^n - Y_Q^n > 0$ below and $Y_k^n - Y_Q^n < 0$ above the long-run singular line.

Consider a $(Q, k)$-process which exhibits capacity investments between the current and the long-run singular line. As the singular line shifts downwards in the $(Q, k)$-space the $(Q, k)$-process may reach the line, so that transversality condition (18)(c) is not violated. However, according to (A.14), the reallocation of a marginal investment unit from capacity to capital for one moment in time increases net production. As the additional production can be used for consumption, capital or capacity accumulation, the original investment plan is not optimal. The argument can be repeated till there are no capacity investment below the singular line.

**Lemma 7** *Capacity investments below the singular line are not optimal.*

Consider a $(Q, k)$-process with minimal capacity investments above the singular line, so that $\varsigma > 0$, $\Lambda_S < 0$, and, according to (A.12), $\dot{\varsigma} = \rho\varsigma - \lambda\Lambda_S > \rho\varsigma$. If the minimal capacity investment regime last forever, $\lim_{t\to\infty} e^{-\rho t}\lambda(t) = \infty$, which contradicts transversality condition (18)(a). As a steady state requires $q = 0$, we can conclude as follows.

**Lemma 8** *A steady state above the long-run singular line is not optimal.*

According to lemma 8, a $(Q, k)$-process with minimal capacity investments cannot evolve above the singular line forever. Consequently, the capital stock decreases until a steady state on the long-run constant consumption line, which needs to be located below the long-run singular line, is reached. A switch to singular capacity investments on the singular line is not possible. If it were possible, the $(Q, k)$-process would reach an arbitrary point on the long-run singular line. However, the same point could be reached by maximal capacity investments above the singular line. If the latter is not optimal, the former cannot be optimal.

**Lemma 9** *If minimal capacity investments are optimal above the singular line, the $(Q, k)$-process converges against the steady-state line by means of capital stock adjustments.*

As $\frac{dK^C}{dQ} > \frac{dK^S}{dQ} > \frac{dK^N}{dQ}$, the sufficient capacity line is located above the singular line for large capacity values, and the singular line is located above the constant consumption line. Suppose a $(Q, k)$-process that evolves along the singular line in the long-run, so that capacity investments are positive. Due to the lower position of the singular line, the $(Q, k)$-process evolves below the sufficient capacity line. According to lemma 4 and proposition 1, this connotes an over-capacity and therefore the non-optimality of capacity investments. Consequently, a long-run evolution of the $(Q, k)$-process along the singular line cannot be optimal. The argument holds in a similar way for a $(Q, k)$-process with maximal capacity investments.

**Lemma 10** *Positive long-run capacity investments are not optimal.*

Suppose the $(Q, k)$-process is located on the singular line, so that $\Lambda_S = 0$ and $\varsigma = 0$. If the $(Q, k)$-process diverges from the singular line upwards, $\Lambda_S < 0$. According to (A.12), $\varsigma > 0$, i.e. minimal capacity investments are optimal above the singular line. However, lemma 9 implies that the previous capacity investments were not optimal. Thus, an upward deviation of the $(Q, k)$-process from the singular line is not optimal.

If the $(Q, k)$-process diverges downwards, $\Lambda_S > 0$, so that $\varsigma < 0$. In other words, capacity investments are maximal below the singular line. However, the corresponding investment plan contradicts lemma 7. Consequently, a downward divergence is not possible

**Lemma 11** *The singular line exerts a bounding force on the $(Q, k)$-process.*

Lemma 4 proves property (a) and lemma 5 property (b). Property (c) follows from the definition of the singular line, lemma 7, and lemma 9. Property (d) is given by lemma 11.

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