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Unilateral Climate Policy, the Green Paradox, Coalition Size and Stability

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Abstract

We incorporate three important aspects of current climate policy, unilateralism, demand side approach and a climate target, in a multi-country model with flow dependent fossil fuel extraction costs and a backstop. It turns out that the optimal climate coalition should encompass all countries which are concerned about global warming and that the carbon tax increases initially to approach zero later on. While a fast increasing tax may cause an increase of early fossil fuel extraction (weak green paradox), a sufficiently large climate coalition can guarantee the adherence to the climate target. We present both a sufficient coalition size rule and the stable coalition size evolution path. It is shown that the results are robust to a stock dependence of extraction costs.

Keywords: Climate Change, Climate Target, Unilateral Climate Policy

JEL classification: Q41; Q42; Q54; Q58

1. Introduction

Due to its possibly extensive consequences, climate change belongs the most discussed subjects in the last decades. Current climate protection measures and the ongoing political negotiations feature three characteristics. Firstly, there is hardly a global attempt to limit climate change. Instead, sub-global coalitions or single nations follow their unilateral climate policy.[1] Secondly, the implemented measures focus on the demand side

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[1] The Kyoto Protocol, which can arguable be called the most ambitious political project of the last decades, was not signed by all nations of the world or at least all important ones but only by a sub-global coalition.

of fossil fuel markets.[2] And thirdly, the 2°C climate target, which allows for a maximal global temperature increase above the preindustrial level of 2°C, is regularly cited as the political goal.[3]

The first two characteristics are closely related to each other. The first concerns that a unilateral and demand side focused climate policy can harm the environment by accelerating fossil fuel extraction were raised by Sinn (2008a) and Sinn (2008b). Sinn refers to this phenomenon as the green paradox. Gerlagh (2011), Grafton et al. (2012), Van der Ploeg and Withagen (2012), and Hoel (2013) study the green paradox in one-country models under the assumptions of a climate cost (or damage) function. The aspect of unilateralism is incorporated by Eichner and Pethig (2011), Eichner and Pethig (2013), Eichner and Pethig (2014), Ritter and Schopf (2014), Hoel (2011), and Kollenbach (2014a) by applying two- or three-country models. The latter two contributions consider a model of steady time with a clean backstop but with exogenous climate policy changes. In contrast, the former four contributions make use of an two-period model without a clean backstop technology to study several political options to attain a climate target such as the 2°C target. A climate target is also discussed by Chakravorty et al. (2006a), Chakravorty et al. (2006b), Chakravorty et al. (2008), Chakravorty et al. (2012), Lafforgue et al. (2008), Henriet (2012), and Kollenbach (2014b). However, all mentioned studies assume an one-country model and focus either on the optimal depletion of fossil fuels or on economic growth.

The aim of our paper is to incorporate all three characteristics of current climate policy, i.e. unilateralism, demand side policies, and the 2° target. For this purpose we develop a multi-country model with polluting fossil fuels and a clean backstop (e.g. solar energy). Only a fraction of the countries is concerned about the environment (*environmental fraction*). These countries can form a *climate coalition* to attain a climate target, represented by a ceiling on the stock of emissions, by imposing a common quantity tax on fossil fuel consumption. We determine the optimal unilateral climate policy by describing

[2]For example, the EU emission trading scheme limits the CO_2 emissions of several economic sectors but does not directly affect the producers of fossil fuel. However, fossil fuel is the main source of CO_2 emissions. According to Hoel (2011) and Van der Ploeg and Withagen (2012), fossil fuels are responsible for 75% of greenhouse gas emissions.

[3]The climate target was endorsed by the United Nation Framework Conference on Climate Change in Cancun, cf. UNFCCC (2010).

the evolution of the tax and the optimal coalition size.[4] Furthermore, we study the stability of the climate coalition and under which conditions the unilateral climate policy may cause a *weak* and/or a *strong green paradox*. With respect to the weak green paradox, we apply Gerlagh's (2011) definition of an increase of early emissions. The strong green paradox is defined by Gerlagh (2011) as an increase of climate costs. As we abstain from a damage function, we redefine the strong green paradox as a violation of the climate target.

It turns out that the coalition should encompass the whole environmental fraction and that the tax increases as long as the ceiling is not binding, decreases to zero during the time period of a binding ceiling and equals zero for all following points in time. Thus, the evolution of the optimal unilateral tax mimics the results of Chakravorty et al. (2006a) and Kollenbach (2014b). If the tax increases sufficiently fast, it may give rise to a weak green paradox. However, a strong green paradox does not occur, if the coalition encompasses a sufficiently number of countries. To analyze the stability of the coalition, we use the concept of internal and external stability. According to d'Aspremont et al. (1983), a coalition is internally and externally stable, if no coalition country can increase its welfare by leaving the coalition and no non-coalition or *fringe* country can increase its welfare by joining the coalition. Under the assumption that the coalition can commit itself to a specific climate policy, i.e. a tax path, and only fossil fuel is used, the maximal coalition size is stable. In case that both fossil fuel and backstop are used simultaneously, we calculate a stable coalition size path which guarantees the adherence of the ceiling. We show also that a stock dependence of extraction costs does not alter the results considerably.

The paper is structured as follows. The model is described in section 2. In section 3 we determine the optimal unilateral climate policy and analyze whether this policy can cause a green paradox in section 4. The stable coalition size is scrutinized in section 5. Section 6 augments the model with stock dependent extraction costs, while section 7 concludes.

[4] Note that "optimal" refers here not to the social optimum (global coalition) but to the best solution given the size of the environmental fraction, i.e. the best small coalition size.

2. Model

Suppose an economy that consists of n independent nations. The identical quasi-linear utility function of the representative individual of each country i is given by

$$V(x_i, z_i) = U(x_i) + y_i. \tag{1}$$

x_i denotes the individual's consumption of energy and y_i the one of a tradable good. The price p_y of the latter is normalized to 1. The function $U(x_i)$ is well behaved, i.e. $U_x > 0$, $U_{xx} < 0$ and $U_{xxx} > 0$. Energy is either generated by fossil fuels r or a backstop b (solar energy) according to the simple one to one conversion $x_i = r_i + b_i$. Thus, backstop is a perfect substitute for fossil fuels. The world market consumer prices are denoted with p_r^c and p_b.[5] At every point in time, all individuals are exogenously endowed with an amount $\bar{y}_i(t)$ of the tradable good and the income $\bar{\xi}_i(t)$.[6] The representative individual of each country owns the shares of all companies located in its country, so that all profits are distributed to it. In similar manner, the tax yields Φ_i of the respective government i are distributed to the representative individual by a lump sum transfer.

We assume that backstop is available in all countries, while fossil fuels are only located in a subset q.[7] Nonetheless, the world markets of both fossil fuels and backstop are characterized by perfect competition. Thus, the representative fossil fuel and backstop firm possesses no market power. The costs of the representative backstop firm read cb, with $c > 0$. Thus, backstop unit costs are constant. The profit maximization of the backstop firm directly gives $p_b = c$ and $\Pi_b = 0$, with $\Pi_j,\ j = r, b$ denoting the profit of the representative firm in the fossil fuel and backstop sector respectively.
The representative fossil fuel firm owns the limited fossil fuel stock $s(0)$. The stock s decreases with extraction according to[8]

$$\dot{s} = -r. \tag{2}$$

The extraction costs $C(r)$ convexly increase in current extraction rate, i.e. $C_r > 0$ and

[5] As we do not need to distinguish between a producer and a consumer backstop price, the index c of p_b^c is omitted.

[6] For the sake of simplicity we suppress the time index t subsequently, if not needed for understanding.

[7] Subset labels, such as q, do not only indicate the respective subset but also the subset size, i.e. there are q countries with fossil fuel reserves.

[8] The notation $\dot{\nu}$ is used to indicate the change of the arbitrary variable ν in time, i.e. $\frac{d\nu}{dt}$. The growth rate $\frac{1}{\nu}\frac{d\nu}{dt}$ is denoted with $\hat{\nu}$.

$C_{rr} > 0$.[9] To ensure the competitiveness of fossil fuel, we set $C_r(0) < c$.

Despite the disparate distribution of fossil fuels, the nations differ in climate concerns but are identical in all other aspects. The environmental fraction $m < n$ is concerned about global warming, which is caused by the accumulation of carbon emissions in the atmosphere. According to

$$\dot{z} = r - \gamma z, \tag{3}$$

the emission concentration increases with fossil fuel extraction but decreases due to the natural regeneration rate γ. The countries belonging to the environmentally fraction agree that the emission concentration should not exceed the ceiling $\bar{z}$.[10] To realize this climate target, they could form a climate coalition that pursues a unified policy. The corresponding instrument is the fossil fuel quantity tax κ. The remaining $n-m$ countries do not agree with the climate target $\bar{z}$. While the concerns of these countries may be reflected by some higher critical value $\tilde{z}$, we assume $\tilde{z} = \infty$ to focus on the unilateral policy of the environmental fraction. Subsequently, all countries not belonging to the climate coalition are referred to as the fringe.

We assume that all fringe countries and the coalition consider themselves as small countries on the fossil fuel and backstop market, i.e. they take the prices as given. In absence of environmental concerns and market power, the fringe countries have no incentive to pursue climate policy. Thus, the fringe countries are inactive. Furthermore, we assume $q \in n - m$, so that there are no fossil fuel reserves in the countries of the environmental fraction. Consequently, the climate policy of the coalition can not directly affect the supply side of the fossil fuel market, i.e. it is a strictly demand side policy.

2.1. Fossil Fuel Supply

Fossil fuel supply is determined by the intertemporal profit maximization of the representative fossil fuel firm. With τ as the current value costate variable (shadow price, *scarcity rent*) of the resource stock and p_r as the (producer) fossil fuel price net of the tax κ, the current value Hamiltonian reads $H = p_r r - C(r) - \tau r$, with $\Pi_r(t) = p_r(t)r(t) - C(r(t))$

[9] A stock dependence of extraction costs is introduced in section 6.

[10] An endogenous explanation of the ceiling would be an augmented utility function which equals (1) as long as $z(t) \leq \bar{z}$ but reads $V(x, z) = -\infty$ for $z(t) > \bar{z}$. The ceiling could also be the result of international negotiations, as pointed out by Kollenbach (2014b) and Eichner and Pethig (2013). Further justifications for a ceiling are given by Chakravorty et al. (2006a), Chakravorty et al. (2008), Chakravorty et al. (2012) and Lafforgue et al. (2008).

denoting the profit for some arbitrary point in time. The conditions for an optimum are

$$p_r = C_r + \tau, \tag{4}$$

$$\hat{\tau} = \rho. \tag{5}$$

The transversality conditions read[11]

$$\tau(T) \geq 0, \ \tau(T)s(T) = 0 \text{ and } H(T) = 0, \tag{6}$$

with T denoting the point in time of fossil fuel extraction vanishes. At each point in time (4) determines fossil fuel supply $R(p_r, \tau)$ as a function of the fossil fuel producer price and the scarcity rent. The latter can be written as $\tau(t) = \tau_0 e^{\rho t}$, with $\tau_0 > 0$ as a constant of integration and initial scarcity rent. As we abstain here from a stock dependence of extraction costs, it is not optimal to leave some fossil fuel in situ, i.e. τ_0 is such that $\int_0^T r(t)dt = s(0)$ holds.

2.2. Fossil Fuel Demand

To determine the demand for fossil fuels, we have to solve the optimization problem of the representative individuals of the different countries. The countries can belong to the climate coalition or to the fringe. Furthermore, some fringe countries exhibit fossil fuel reserves. However, the individuals differ only in aspects which are exogenous to them, i.e. in $\bar{\xi}$, $\bar{y}$, Π_r and Φ.[12] Let Λ denote the sum of these variables. The budget constraint of an arbitrary individual reads $y = \Lambda - p_r r - \kappa r - p_b b = \Lambda - p_r^c r - p_b b$. Substituting in (1) gives $H = U(r + b) + \Lambda - p_r^c r - p_b b + \zeta_r r + \zeta_b b$, with ζ_r and ζ_b denoting the multipliers of the non-negative conditions $r \geq 0$ and $b \geq 0$. The first order conditions for an optimum and the complementary slackness conditions read

$$U_x(r + b) = p_r + \kappa - \zeta_r, \tag{7}$$

$$U_x(r + b) = p_b - \zeta_b, \tag{8}$$

$$\zeta_r \geq 0, \ \zeta_r r = 0, \ \zeta_b \geq 0, \ \zeta_b b = 0. \tag{9}$$

(7) and (8) determine energy demand $D(p_r^c)$ or $D(c)$. If the fossil fuel consumer price p_r^c falls short of the backstop price $p_b = c$, demand is only satisfied by fossil fuels, i.e.

[11] We follow Seierstad and Sydsaeter (1987), theorem 11 and Feichtinger and Hartl (1986), Satz 7.6. Due to $C_r(0) < c$, $T = 0$ is not possible. Since $\tau(t) + C_r(r)$ reaches c in finite time, $T = \infty$ is ruled out.

[12] To indicate that $\bar{\xi}$, $\bar{y}$, Φ, Λ, b, and r belong to a specific individual, we could use the index i. As the utility functions are identical, we abstain from the index to keep notation simple.

$U_x(D(p_r^c)) = p_r^c$. Furthermore, $D_p(p_r^c) = \frac{1}{U_{xx}} < 0$ and $D_{pp}(p_r^c) = -\frac{U_{xxx}}{U_{xx}^3} > 0$, so that demand is convexly decreasing in the consumer price. In case of $p_r^c = c$, the individual is indifferent between the two energy sources, as fossil fuels and backstop are perfect substitutes. Following Kollenbach (2014a), we assume that fossil fuels are used as long as they can be supplied at a marginal lower price than the backstop. Thus, a mix of both resources can be used. If the fossil fuel consumer price exceeds c, the demand for fossil fuel vanishes. Summarizing the latter two cases, we get $U_x(D(c)) = c$, if $p_r^c \geq c$. Hence, the fossil fuel demand function consists of the convexly decreasing part $D(p_r^c)$ and the constant one $D(c)$.

Suppose the individual is the representative citizen of a fringe country. With a degree sign (an asterisk) denoting values and functions of fringe (coalition) countries, fossil fuel demand reads

$$F^\circ(p_r) = \begin{cases} D^\circ(p_r), & \text{if } p_r < c, \\ 0 \leq F^\circ \leq D^\circ(c), & \text{if } p_r = c. \end{cases} \tag{10}$$

Otherwise, we get

$$F^*(p_r, \kappa) = \begin{cases} D^*(p_r + \kappa), & \text{if } p_r < c - \kappa, \\ 0 \leq F^* \leq D^*(c), & \text{if } p_r = c - \kappa. \end{cases} \tag{11}$$

Let k denote the number of coalition countries. Total fossil fuel demand is given by

$$A(p_r, \kappa, k) = \begin{cases} kD^*(p_r + \kappa) + (n-k)D^\circ(p_r), & \text{if } p_r < c - \kappa, \quad (i) \\ kF^* + (n-k)D^\circ(c-\kappa), & \text{if } p_r = c - \kappa, \quad (ii) \\ (n-k)D^\circ(p_r), & \text{if } c - \kappa < p_r < c, \quad (iii) \\ (n-k)F^\circ, & \text{if } p_r = c. \quad (iv) \end{cases} \tag{12}$$

2.3. Fossil fuel market equilibrium

At every point in time, the equilibrium of the fossil fuel market is described by $A(p_r(t), \kappa(t), k(t)) = R(p_r(t), \tau(t))$. Since the demand function (12) consists of four parts, labeled from top to bottom $(i), (ii), (iii)$ and (iv), we can distinguish between four equilibrium types depending on the position of the supply function and the tax κ. Fig. 1 illustrates the aggregated fossil fuel demand and the supply function. The equilibrium, denoted with E, is located on part (ii) of the demand function. If the tax or the scarcity rent are lower, the equilibrium could be located on part (i). In contrast, a higher scarcity rent or a higher tax may imply an equilibrium on part (iii). An equilibrium on part (iv) requires a sufficiently large scarcity rent. The junction point P^A of part (ii) and (iii) of the aggregated demand function plays an important part in the subsequent analysis. At time t it is defined by $\lim_{F^* \to 0}[kF^* + (n-k)D^\circ(c - \kappa(t))] = (n-k)D^\circ(c - \kappa(t))$.

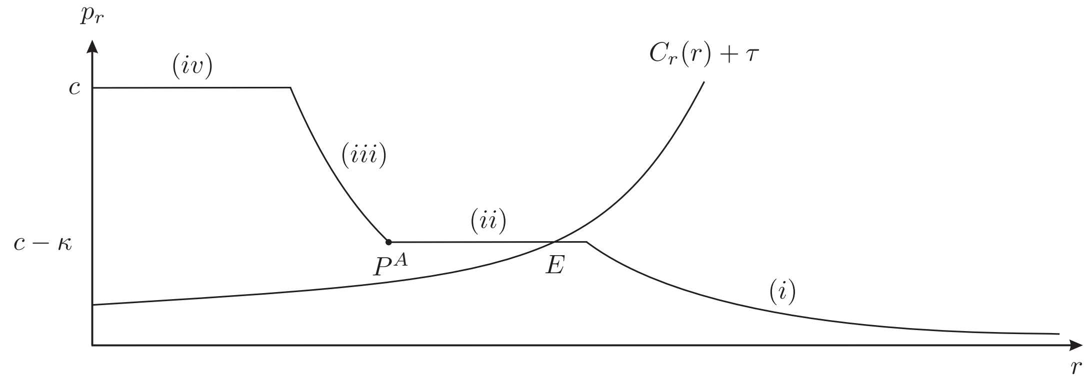

Figure 1: Fossil fuel demand and supply for an arbitrary tax rate κ

3. Optimal unilateral Climate Policy

To determine the optimal unilateral climate policy, we assume that a constrained planner maximizes the utility of the environmental fraction, given by a utilitarian welfare function, subject to the climate target.[13] As the tax yields Φ are distributed to the citizens of the coalition countries and no fossil fuel reserves are located in these countries, the corresponding budget constrain reads $y = \bar{\xi} + \bar{y} - p_r r^* - p_b b^*$. With θ denoting the shadow price of emissions, μ the Lagrange multiplier associated with the climate target, and $\zeta_i^*,\ i = r, b$ the multipliers of the non-negative conditions $r^* \geq 0$ and $b^* \geq 0$, the current-value Lagrangian is

$$\begin{aligned} L = & k\left[U(r^* + b^*) + \bar{\xi} + \bar{y} - p_r r^* - p_b b^*\right] + (m-k)\left[U(r^\circ + b^\circ) + \bar{\xi} + \bar{y} - p_r r^\circ - p_b b^\circ\right] \\ & + \theta[kr^* + (n-k)r^\circ - \gamma z] - \mu[kr^* + (n-k)r^\circ - \gamma z] + \zeta_r^* r^* + \zeta_b^* b^*. \end{aligned} \tag{13}$$

Since the fringe countries abstain from climate policy, r° and b° are given by (7) - (9) and exogenous to the constrained planner. The first order conditions of the optimization

[13]Recall that "optimal" refers here not to the social optimum but to the best solution giving the limited size of the environmental fraction.

problem and the complementary slackness conditions with respect to r^* and b^* are

$$U_x = p_r + \mu - \theta - \zeta_r^*, \tag{14}$$

$$U_x = p_b - \zeta_b^*, \tag{15}$$

$$\zeta_r^* \geq 0,\ \zeta_r r^* = 0,\ \zeta_b^* \geq 0,\ \zeta_b^* b^* = 0. \tag{16}$$

Comparing (14) with (7) shows that the optimal tax equals $\mu - \theta$. The growth rate of θ is given by

$$-\gamma\theta + \gamma\mu = \rho\theta - \dot{\theta}. \tag{17}$$

The complement slackness conditions with respect to the ceiling $\bar{z}$ read

$$\begin{aligned} &\frac{\partial L}{\partial \mu} = -kr^* - (n-k)r^\circ + \gamma z \geq 0, \qquad && \mu \geq 0, \qquad && \mu\frac{\partial L}{\partial \mu} = 0, \\ & && \bar{z} - z \geq 0, && \mu[\bar{z} - z] = 0, \\ & && \rho\mu - \dot{\mu} \geq 0, && [= 0 \quad \text{if} \quad \bar{z} - z > 0]. \end{aligned} \tag{18}$$

To determine the optimal tax path, we follow Kollenbach (2014b) and distinguish between three time periods. In *phase 1* the ceiling is non-binding but will bind at a later point in time, i.e. the ceiling is temporary non-binding. *Phase 2* is characterized a temporary binding ceiling and *phase 3* by a non-binding ceiling for all following points in time. The switching points from one phase to the next are called junction points.

During phase 2, the binding ceiling and (3) require $r(t) = \bar{r} = \gamma\bar{z}$, i.e. a constant fossil fuel extraction. If the fossil fuel market equilibrium is located on part (iii) or (iv) of the aggregated demand function, fossil fuel extraction decreases, since demand on the parts (iii) and (iv) is constant in time, while the scarcity rent continuously increases ($\hat{\tau} = \rho$). Thus, if phase 2 does not only last for one moment in time, the equilibria need to be located on the parts (i) or (ii) of the aggregated demand function, i.e. to the right of the point P^A in Fig. 1. On this two parts of the aggregated demand function, the climate coalition can guarantee the adherence of the ceiling by setting the appropriate tax rate. Thus, the optimal unilateral climate policy during phase 2 must guarantee fossil fuel market equilibria which coincide with $P^A(t)$ or are located to the right of it.[14]

During phase 1 (17) and (18) give $\theta(t) = \theta_0 e^{(\rho+\gamma)t}$, with θ_0 as constant of integration, and $\mu = 0$. Since an exogenous increase of the emission stock tightens the optimization

[14] P^A changes in time, if the tax changes.

problem of the constrained planner and because θ can be interpreted as the shadow price of emissions during phase 1, $\theta < 0$ and therefore $\theta_0 < 0$. As the tax equals $\mu - \theta$, $\kappa(t) = -\theta(t) > 0$ and $\frac{d\kappa}{dt} = \rho + \gamma > 0$. Thus, the optimal tax rate is positive and increases in time during phase 1.

The binding ceiling during phase 2 and (18) imply $\mu \geq 0$, so that (17) reads $\dot{\theta} = (\rho+\gamma)\theta - \mu\gamma$. According to Feichtinger and Hartl (1986), page 171 et seq., θ cannot be interpreted as the shadow price of emissions during phase 2, but equals the sum of μ and the shadow price.[15] As the latter is negative, the tax rate $\kappa(t) = \mu(t) - \theta(t)$ is positive. (17) and (18) do not provide unambiguous information about the growth rate of the tax. However, we know that the equilibria during phase 2 need to be located on part (i) or (ii) of the aggregated demand function. As the supply function continuously shifts upwards in a (r, p_r) diagram, the tax has to decrease to guarantee $r(t) = \gamma\bar{z}$ during phase 2. If the economy is in phase 3, $\kappa(t) = 0$, as $\mu = 0$ and $\theta = 0$. The latter follows from our omission of direct effects of pollution on utility.

In the Appendix we show that the tax and therefore the fossil fuel extraction path is continuous at the junction points and that the only sequence containing all three phases begins with a non-binding ceiling in phase 1, switches at $t = t_1$ into phase 2 for a limited time period and afterwards at $t = t_2$ into phase 3 for an unlimited period. Proposition 1, which summarizes the optimal tax path, follows directly.[16]

Proposition 1 *The optimal unilateral fossil fuel tax increases continuously, if the ceiling is non-binding for a limited time period. The tax reaches its maximum at the moment the ceiling becomes binding. During the time period of a binding ceiling the tax decreases and equals zero as of the moment the ceiling becomes non-binding.*

Our result is in line with the findings of Chakravorty et al. (2006a), Kollenbach (2014b) and other authors analyzing the effect of a ceiling in a closed economy. However, it contrasts with Van der Ploeg and Withagen (2012), who focus on the green paradox and find that the optimal tax increases continuously. On the one hand, they use a stock

[15] Further dynamic optimization text books are Chiang (1992), Seierstad and Sydsaeter (1987), and Kamien and Schwartz (2000).

[16] As mentioned above, the ceiling can be binding for only one moment in time, i.e. the emission path only touches or is tangent to the ceiling. The Appendix shows that in this special case $\theta(t) = 0$, $\mu(t) = 0$ and consequently $\kappa(t) = 0$ for all points in time. Therefore, the fossil fuel market equilibria during phase 2 can be located on part (iv) of the aggregated demand function. Without a tax, part (iii) of the aggregated demand function does not exist. We abstain from this case in the following, as the optimal climate policy is no climate policy.

dependent damage function instead of a ceiling. On the other, they abstain from natural regeneration, which gives rise to a non-decreasing tax path.

To determine the optimal coalition size, the constrained planner maximizes $H = k[U(r^* + b^*) + \bar{\xi} + \bar{y} - p_r r^* - p_b b^*] + (m-k)[U(r^\circ + b^\circ) + \bar{\xi} + \bar{y} - p_r R^\circ - p_b b^\circ] + \theta[kr^* + (n-k)r^\circ - \gamma z] - \mu[kr^* + (n-k)r^\circ - \gamma^z]$ with respect to k. We get

$$k = \begin{cases} 0, & \text{if } U(x^*) - U(x^\circ) - (p_r - \theta + \mu)(r^* - r^\circ) - p_b(b^* - b^\circ) < 0, \\ 0 \leq k \leq m, & \text{if } U(x^*) - U(x^\circ) - (p_r - \theta + \mu)(r^* - r^\circ) - p_b(b^* - b^\circ) = 0, \\ m, & \text{if } U(x^*) - U(x^\circ) - (p_r - \theta + \mu)(r^* - r^\circ) - p_b(b^* - b^\circ) > 0. \end{cases} \tag{19}$$

Due to $\kappa = \mu - \theta$ and (4) - (9), the condition on the right hand side of (19) reads $U(x^*) - U(x^\circ) - U_x(x^*)(x^* - x^\circ) \lesseqgtr 0$. As the tax and the scarcity rent increase continuously during phase 1 and the equilibria during phase 2 need to be located on part *(i)* or *(ii)* of the aggregated demand function, they must be located there also during phase 1. Therefore, we concentrate on these two parts, so that $x^\circ > x^*$ and $U(x^\circ) > U(x^*)$. Then, the condition gives

$$\frac{U(x^\circ) - U(x^*)}{U_x(x^*)} \frac{x^*}{x^\circ - x^*} < \epsilon(x^*), \tag{20}$$

with $\epsilon(x^*)$ as the elasticity of utility evaluated at $x = x^*$.[17] Consequently, $k = m$.

Proposition 2 *The optimal coalition consists of all countries of the environmental fraction.*

According to proposition 2, the gains one country could realize by leaving the coalition are outweighed by the additional costs the remaining countries have to bear in terms of less fossil fuel use. By forming a maximal coalition these costs are minimized for the individual members. In light of the used utilitarian welfare function, the result is not surprising. It is also in line with the tax competition literature, e.g. Hoyt (1991) shows that the efficient coalition size is a grant coalition.

4. The green paradox

Unilateral climate policies have given rise to concerns that they may harm and not protect the environment. Sinn (2008a,b) was the first one to express these concerns and

[17] Due to the concavity of the utility function, $U(x^\circ) - U(x^*)$ grows at a lower pace than $x^\circ - x^*$, with $\lim\limits_{x^\circ \to x^*} \frac{U(x^\circ) - U(x^*)}{U(x^*)} \frac{x^*}{x^\circ - x^*} = \epsilon(x^*)$.

has labeled the related phenomenon the green paradox. Gerlagh (2011) distinguishes between a weak and a strong green paradox. He refers to a weak green paradox, if the introduction or tightening of a climate policy increases early emissions, i.e. if early fossil fuel extraction is increased. A strong green paradox is defined by an increase of cumulative environmental damage. As we abstain from damages caused by emissions to concentrate on the unilateral climate target (ceiling), we redefine the strong green paradox as a violation of the target.

4.1. Weak green paradox

To analyze if the optimal unilateral climate policy can cause a weak green paradox, we apply the method of Kollenbach (2014a). If a weak green paradox occurs, early fossil fuel extraction is higher after the implementation (announcement) of climate policy than without it. As the fossil fuel stock is not altered and the area below the extraction path needs to equal the stock $\left(\int_0^T r(t)dt = s(0)\right)$, a weak green paradox requires that the fossil fuel extraction path valid after the introduction intersects the extraction path valid before the introduction from above at some point in time t. Following Hoel (2011) and Kollenbach (2014a), we refer to the situation before (after) the implementation of climate policy as the old (new) one, indicated by the index O (N). Thus, the intersection of the extraction paths is characterized by $r^O(t) = r^N(t)$ and $\frac{dr^O(t)}{dt} > \frac{dr^N(t)}{dt}$, with $r^O(t)$ and $r^N(t)$ representing the old and new fossil fuel market equilibria.

In Fig. 2 we illustrate the old (solid line) and new (dahed line) aggregated fossil fuel demand function.[18] Obviously, an arbitrary positive fossil fuel tax κ lowers demand for

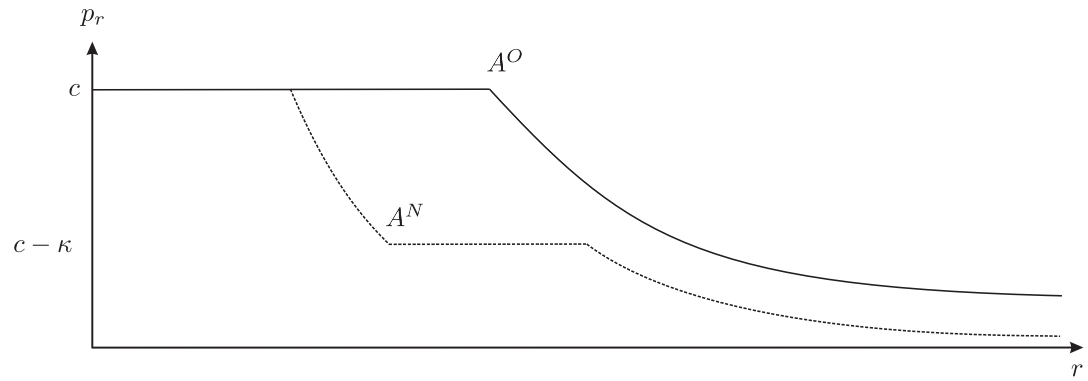

Figure 2: Fossil fuel demand and supply for an arbitrary tax rate κ

[18]The function without climate policy follows from (12) with $\kappa = 0$.

all producer prices $p_r \leq c$. Assume that τ_0 is such that $\int_0^T r^O(t)dt = s(0)$ holds before the implementation of the unilateral climate policy. Ceteris paribus, the market equilibria $r^N(t)$ are lower than $r^O(t)$, so that $\int_0^T r^N(t) < s(0)$. Consequently, the initial scarcity rent decreases to ensure $\int_0^T r^N(t)dt = s(0)$, i.e. $\tau^O(0) > \tau^N(0)$. According to (5), $\hat{\tau}^O = \hat{\tau}^N = \rho$, which implies that at every point in time the old scarcity rent increases faster than the new one. By summarizing the requirements for a weak green paradox, i.e. the requirements for the intersection of the old and new fossil fuel extraction path we are interested in, we get[19]

$$r^O(t) = r^N(t), \tag{21}$$
$$\frac{dr^O(t)}{dt} > \frac{dr^N(t)}{dt}, \tag{22}$$
$$\frac{d\tau^O(t)}{dt} > \frac{d\tau^N(t)}{dt}. \tag{23}$$

Hereafter, we check if (21) - (23) may hold during phase 1 or 2. In phase 3, (21) - (23) can never hold.[20] Recall that the new fossil fuel market equilibria during phase 1 and 2 need to be located on part (i) or (ii) of the new aggregated demand function. Taking (21) into account, Fig. 2 shows that $r^O(t)$ can be located either on the falling part (part (i)) of the old aggregated demand function or on the horizontal part (part (ii)). Hence, (21) may hold for the equilibria combination $((i)^O, (i)^N)$, $((i)^O, (ii)^N)$ and $((ii)^O, (ii)^N)$, with the first element of each pair referring to $r^O(t)$ and the second one to $r^N(t)$.

As shown in the Appendix, (21) and (22) imply for the three equilibria combinations $((i)^O, (i)^N)$, $((i)^O, (ii)^N)$ and $((ii)^O, (ii)^N)$

$$\frac{d\tau^N}{dt} > \Omega_1 \frac{d\tau^O}{dt} - \Omega_2 \frac{d\kappa}{dt}, \tag{24}$$
$$\frac{d\tau^N}{dt} > \Omega_3 \frac{d\tau^O}{dt} - \frac{d\kappa}{dt}, \tag{25}$$
$$\frac{d\tau^N}{dt} > \frac{d\tau^O}{dt} - \frac{d\kappa}{dt}, \tag{26}$$

with $\Omega_1 := \frac{C_{rr} - \left[kD_p^*(p_r^N+\kappa)+(n-k)D_p^\circ(p_r^N)\right]^{-1}}{C_{rr}-[nD_p(p_r^O)]^{-1}} > 0$, $\Omega_2 := \frac{kD_p^*(p_r^N+\kappa)}{kD_p^*(p_r^N+\kappa)+(n-k)D_p^\circ(p_r^N)} > 0$ and $\Omega_3 := \frac{C_{rr}}{C_{rr}-[nD_p(p_r^O)]^{-1}} > 0$ depending on the price elasticity of the new and old aggregated demand

[19]For a more detailed discussion we refer to Kollenbach (2014a).

[20]Due to $\kappa(t) = 0$, $\forall t \geq t_2$, the old and new aggregated demand function coincide. Thus, (21) requires also the coincidence of the supply function, i.e. $\tau^N(t) = \tau^O(t)$ and therefore $\frac{d\tau^N}{dt} = \frac{d\tau^O}{dt}$, which contradicts (23).

function, respectively. Obviously, all three conditions hold, if the tax increases sufficiently fast.

Proposition 3 *Suppose the unilateral climate policy is announced and implemented at time ω. A weak green paradox can occur, if the fossil fuel quantity tax increases sufficiently fast, regardless if only fossil fuel or both energy sources are simultaneously used before ω.*

Our result contrasts with Kollenbach (2014a), who found that a weak green paradox is only possible, if no backstop is used before the implementation of climate policy, i.e. if the old equilibrium is initially located on part (i) of the old aggregated demand function. However, Kollenbach (2014a) follows Hoel (2011) and focuses on the effects of exogenous climate policy changes, whereas we have endogenized the optimal unilateral fossil fuel tax. Therefore, our emphasis is the effect of the complete tax path. Nonetheless, our result also shows the importance of demand elasticity, which is stressed by Hoel (2011) and Kollenbach (2014a). If the old aggregated demand function is sufficiently price inelastic, i.e. $|D_p(p_r^O)|$ small, (24) and (25) may hold for a constant tax.
With proposition 3 we seize on Sinn's (2008b) original idea that an increasing ad valorem, cash flow or qunatity tax may cause an increase of early emissions. Sinn (2008b) suggests a constant quantity tax as an alternative to prevent a weak green paradox. However, we have shown that the optimal unilateral quantity tax increases in early periods to decrease later on. This raises the question, if a weak green paradox gives also rise to a strong green paradox.

4.2. Strong green paradox

At the beginning of section 4 we have redefined a strong green paradox as a violation of the ceiling $\bar{z}$. To prevent a strong green paradox, the climate coalition must guarantee $\bar{r} = \gamma\bar{z}$ during phase 2 and $z(t) < \bar{z}$ during phase 1. As stated above, this requires that not only $\bar{r}$ but all fossil fuel market equilibria during phase 1 and 2 are located on the parts (i) and (ii) of the (new) aggregated fossil fuel demand function.[21] In other words, the climate coalition must be powerful enough to control the equilibrium extraction rate, i.e. at least the last marginal traded fossil fuel unit must be used by the climate coalition. Since the net of tax demand and the marginal extraction cost function $C_r(r)$ are time invariant, all market equilibria are determined by the scarcity rent path $\tau(t)$, the tax path $\kappa(t)$ and the coalition size path $k(t)$. Recall that the scarcity rent path is determined by

[21]Cf. Fig. 1.

τ_0, since $\tau(t) = \tau_0 e^{\rho t}$. Given a time invariant coalition size k and the tax path $\kappa(t)$, τ_0 must be such that $\int_0^T r(t)dt = s(0)$. On the other hand, the tax path must guarantee the adherence to the ceiling, given the coalition size k and initial scarcity rent τ_0. Thus, if the coalition size is time invariant and given, the appearance of a strong green paradox depends on whether there exists a simultaneous solution to both problems or not. If the coalition size is not fixed, it may vary in time, so that the solution consists also of the coalition path $k(t)$.

Proposition 4 *If there exists a combination $(\tau_0, k(t), \kappa(t))$ such that $\int_0^T r(t)dt = s(0)$, $r(t) > \gamma\bar{z}$ but $z(t) < \bar{z}$ if the ceiling is not yet binding ($t \in [0, t_1[$) and $r(t) = \bar{r} = \gamma\bar{z}$ if the ceiling is binding ($t \in [t_1, t_2[$), the ceiling is not violated, i.e. no strong green paradox occurs.*

Considering our rather general assumptions, it is not possible to calculate a solution of the stated problem. Even by using specified, e.g. quadratic or linear, functions, numerical simulations are hard to compute. Therefore, we are going to establish a condition which does not include all solutions of proposition 4 but is easier to verify. For this purpose, we assume an unlimited fossil fuel stock. Obviously, this is the worst case from the climate coalition's point of view, as $\tau(t) = 0\ \forall t$, so that the fossil fuel supply function reaches its lowest possible position in a (r, p_r) diagram. In other words, for all p_r fossil fuel supply exceeds the level valid for a limited fossil fuel stock. Fig. 3 illustrates the supply function without a scarcity rent and the aggregated demand function for an arbitrary but time invariant coalition size (k_1) and an arbitrary fossil fuel tax κ_1 (solid line). Suppose the tax is at its maximum, i.e. $\kappa_1 = \kappa_1(t_1)$ and the illustrated equilibrium gives $r(t_1) = \bar{r}$. Then, there also exists a tax path $\kappa_1(t)$ that enables the climate coalition to control the traded fossil fuel amount during phase 1 and 2, so that $z(t) < \bar{z}$ for all $t \in [0, t_1[$ and $z(t) = \bar{z}$ for all $t \in [t_1, \infty[$.[22]

The dashed line in Fig. 3 together with Fig. 4 visualize the adaption process triggered by a reduction of the coalition size from k_1 to k_2. The ceteris paribus effect of the size reduction on aggregated demand is illustrated in Fig. 3 by the dashed line. With more fringe countries the parts (iii) and (iv) of the aggregated demand function represent more and the parts (i) and (ii) less countries. Graphically, part (iv) is lengthened by the same extend part (ii) is shortened, as the corresponding demand $(k_1 - k_2)D(c)$ moves from the coalition

[22] t_2 equals ∞, since the economy will be stuck at the ceiling without an increasing scarcity rent.

to the fringe. Furthermore, both part (iii) and (i) are flattened.[23] The new aggregated demand function intersects the supply function to the right of $\bar{r}$. The resulting equilibrium extraction rate $\tilde{r}$ violates the binding ceiling, since $\tilde{r} > \bar{r} = \gamma \bar{z}$. To reduce equilibrium extraction to $\bar{r}$, the fossil fuel tax κ_1 has to be increased to the level κ_2. The corresponding aggregated demand function is illustrated in Fig. 4 by the dashed line. The solid line depicts the aggregated demand function before the tax adjustment, i.e. the dashed line of Fig. 3. Furthermore, P_1^A and P_2^A show that the distance between $P^A(t_1)$ and the market

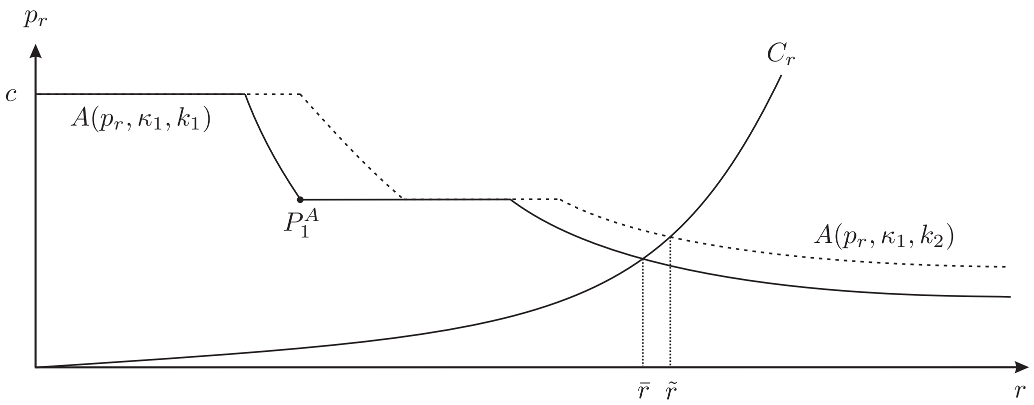

Figure 3: Fossil fuel supply for an unlimited resource stock and aggregated demand before and after a reduction of coalition size

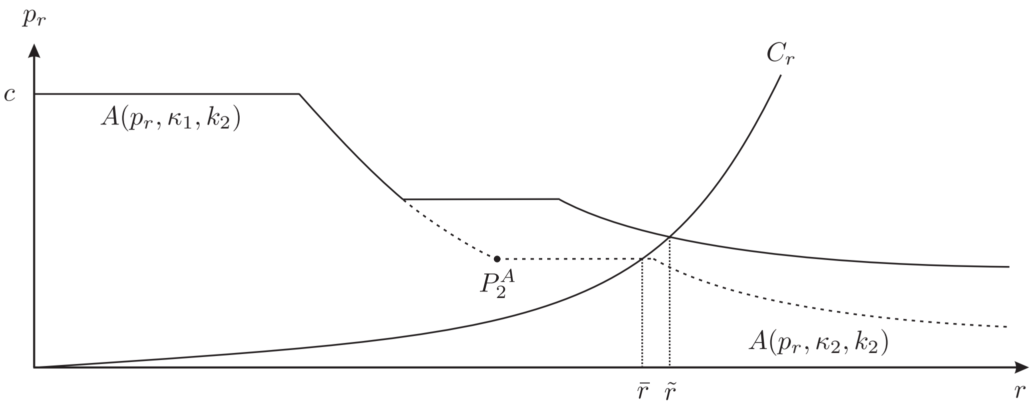

Figure 4: Fossil fuel supply for an unlimited resource stock and aggregated demand before and after a tax adjustment

equilibrium point is reduced. The argumentation can be repeated until the equilibrium

[23]Let $G^{(iii)}(r)$ be the inverse function of $(n-k)D^{\circ}(p_r)$ and $G^{(i)}(r)$ the inverse function of $kD^*(p_r+\kappa)+(n-k)D^{\circ}(p_r)$ for a given k and κ. We get $G_r^{(iii)} = \frac{1}{(n-k)D_p^{\circ}(p_r)} < 0$, $G_r^{(i)} = \frac{1}{kD_p^*(p_r+\kappa)+(n-k)D_p^{\circ}(p_r)} < 0$, $\frac{dG_r^{(iii)}}{dk} = \frac{D_p^{\circ}(p_r)}{[(n-k)D_p^{\circ}(p_r)]^2} < 0$, and $\frac{dG_r^{(i)}}{dk} = \frac{D_p^{\circ}(p_r)-D_p^*(p_r+\kappa)}{[kD_p^*(p_r+\kappa)+(n-k)D_p^{\circ}(p_r)]} < 0$. Thus, both part (iii) and (i) of the aggregated demand function become flatter, if the coalition size is reduced.

point coincide with $P^A(t_1)$. A further reduction of the coalition size is impossible, as the remaining coalition countries would not be able to control equilibrium extraction. Thus, with an unlimited fossil fuel stock, the coincidence of $P^A(t_1)$ and market equilibrium determines the minimal necessary coalition size to prevent a strong green paradox. As stated above, $P^A(t_1)$ is defined by $\lim_{F^* \to 0}[kF^* + (n-k)D^\circ(c-\kappa(t_1))] = (n-k)D^\circ(c-\kappa(t_1))$. The intersection of the aggregated demand and the supply function at $P^A(t_1)$ requires $\bar{r} = R(p_r) = (n-k)D^\circ(c-\kappa(t_1))$, with $c-\kappa(t_1) = p_r$ and, according to (4), $p_r = C_r(\bar{r})$. Thus, the minimal coalition size is given by

$$k^{min} = n - \frac{\bar{r}}{D^\circ(C_r(\bar{r}))}. \tag{27}$$

In other words, for all $k(t) \geq k^{min}$ there exists a tax path $\kappa(t)$ that fulfills the requirements of proposition 4 for an unlimited fossil fuel stock. With a limited fossil fuel stock the scarcity rent shifts the supply function upwards in a (r, p_r) diagram, i.e. supply is lower for all p_r. Thus, if a tax path exits which is in line with proposition 4 for an unlimited fossil fuel stock, there will also be a tax path for a limited stock, i.e. $k(t) \geq k^{min}$ is a sufficient conditions for averting a strong green paradox.

Proposition 5 *If the coalition size path $k(t)$ exceeds or equals k^{min} for all points in time, a pair $(\tau_0, \kappa(t))$ exists such that the ceiling is not violated and the fossil fuel stock exhausted in finite time.*

With proposition 3 we have shown that a sufficiently fast increasing unilateral fossil fuel tax can boost early emissions. However, if the ceiling reflects environmental concerns or the dangers of climate change correctly, it is more important if the ceiling is violated. According to proposition 5, the ceiling is not violated, if the climate coalition encompass enough countries and the tax path is set accordingly. In this case, the optimal unilateral climate policy may very well cause an increase of early emissions but ensures the adherence to the ceiling.

5. Stability of a climate coalition

Section 4 has shown that the size of the climate coalition is crucial for the adherence of the ceiling. This directly raises the question of the stability of a climate coalition. To verify the stability, we apply the concept of internal and external stability, which harks back to d'Aspremont et al. (1983). According to this concept, a coalition is stable, if no fringe country has an incentive to join the coalition (external stability) and no coalition country has an incentive to leave the coalition (internal stability). We assume that the governments of the environmental fraction m are benevolent. Thus, they maximize the intertemporal utility of their representative inhabitant under the constraint of the climate target. As the fossil fuel tax κ distorts fossil fuel demand, no country has an incentive to join the coalition, if the coalition is large enough to ensure the adherence of the ceiling. Due to proposition 5, it follows directly that every coalition with $k(t) \geq k^{min}$ is externally stable.

Lemma 1 *Every coalition which can guarantee the adherence of the ceiling, especially every coalition with $k(t) \geq k^{min}$, is externally stable.*

For the analysis of the internal stability we assume at first that the coalition can commit itself to the tax path $\kappa(t)$. In light of the tedious and complicated climate negotiations, this assumption seems not unrealistic. By the commitment to the tax path the climate coalition commits itself implicitly to a specific extraction path that is in line with proposition 4. As illustrated in Fig. 3, the resignation of one coalition country can increase fossil fuel demand ceteris paribus, so that the scarcity rent has to adapt to guarantee the exhaustion of fossil fuel in finite time, i.e. $\int_0^T r(t)dt = s(0)$. If the adaption of the scarcity rent does not exactly offset the demand changes, which seems quite realistic, either at early or late periods fossil fuel extraction is boosted. To guarantee the adherence to the ceiling, the tax path need to adapt. However, a tax change is not possible, as the coalition is committed to the tax path implying the internal stability of the coalition.

Thus, we have to analyze under which conditions one country can leave the coalition without altering $r(t)$, given that the scarcity rent adaption does not offset the demand effect of a smaller coalition. For a start, suppose the fossil fuel market equilibrium is located on part (i) of the aggregated demand function, i.e. that the equilibrium is given by $R(p_r, \tau) = kD^*(p_r + \kappa) + (n-k)D^\circ(p_r)$. As illustrated by Fig. 3 a reduction of the coalition size increases the equilibrium extraction. As the necessary tax path adaption is ruled out by assumption, the climate coalition is internally stable.

The result changes, if the equilibrium is located on part (ii) of the aggregated demand function, i.e. if it is determined by $R(p_r, \tau) = kF^* + (n-k)D^\circ(c-\kappa)$, with $p_r = c - \kappa$. Suppose the tax equals its maximum $\kappa(t_1)$ but the equilibrium does not coincide with $P^A(t_1)$, such as the equilibrium $(\bar{r}, c - \kappa_2)$ of Fig. 4. In this case, a ceteris paribus reduction of the coalition size shifts $P^A(t_1)$ to the right but does not alter the equilibrium.

Thus, some coalition countries can leave the coalition without jeopardizing the climate target. The argument holds also for all $t < t_1$. In other words, countries have no incentive to stay in the climate coalition during phase 1 as long as $k(t) > \tilde{k}^{min}(t_1)$, with $\tilde{k}^{min}(t_1)$ determined by the coincidence of the fossil fuel market equilibrium and $P^A(t_1)$, i.e.[24]

$$\tilde{k}^{min}(t_1) = n - \frac{\bar{r}}{D^\circ(\tau(t_1) + C_r(\bar{r}))}. \tag{28}$$

Since $\tilde{k}^{min}(t_1)$ is in line with lemma 1, the stable coalition size during phase 1 is given by (28).

During phase 2 the tax decreases to compensate the increase of the scarcity, so that $r(t) = \bar{r}$ holds. At the end of phase 2, i.e. at $t = t_2$, the tax reaches zero. Therefore, $P^A(t)$ shifts to the left in the (r, p_r) diagram while $r(t) = \bar{r}$ holds. In other words, if the coalition size remains at $\tilde{k}^{min}(t_1)$, the fossil fuel market equilibrium is located to the right of $P^A(t)$. Consequently, at each $t \in [t_1, t_2[$ some countries can leave the coalition without violating the ceiling. The stable coalition size for all points in time of phase 2 is given by

$$\tilde{k}^{min}(t) = n - \frac{\bar{r}}{D^\circ(\tau(t) + C_r(\bar{r}))}, \ t \in [t_1, t_2[. \tag{29}$$

Taking lemma 1 into account, we can conclude as follows.

Proposition 6 *Suppose a climate coalition can commit itself to a tax path $\kappa(t)$, its size $k(t)$ is in line with lemma 1, and that the effect of a smaller coalition size on aggregated demand is not exactly offset by an adaption of the scarcity rent. If the coalition only uses fossil fuels during the time interval $[0, t_2[$, it is stable. If the coalition uses both energy sources simultaneously, its stable size is given by $k(t) = \tilde{k}^{min}(t_1), \ \forall t \in [0, t_1[$ and $k(t) = \tilde{k}^{min}(t), \ \forall t \in [t_1, t_2[$.*

As almost all industrialized nations use both fossil fuels and renewable energies, it seems to be realistic to expect the climate coalition to rely on both energy sources. In this case, the stable coalition size equals the optimal size, defined by proposition 2, during phase 1, if $k^{min}(t_1) = m$. However, if $k^{min}(t_1) < m$, the optimal coalition is not stable during phase 1. In phase 2, the stable coalition size shrinks. Consequently, the optimal coalition cannot be stable in this phase.

If we relax the assumption of a tax path commitment, the stable coalition size is given by the lowest path $\tilde{k}^{min}(t)$ which fulfills the conditions of proposition 4. Thus, all other paths $k(t)$ which fulfill proposition 4 are characterized by $k(t) \geq \tilde{k}^{min}(t), \ \forall t$ and

[24] Note that $\tilde{k}(t_1)^{min} < k^{min}$, as $\tau(t_1) + C_r(\bar{r}) > C_r(\bar{r})$.

$k(t) > \tilde{k}^{min}(t)$ for at least one point in time. Due to the interdependences of τ_0, $\kappa(t)$ and $k(t)$, not much can be said about the stable coalition size path.

6. Stock dependent extraction costs

In the previous analysis we have abstained from a stock dependence of fossil fuel extraction costs. This assumption is relaxed now by introducing the flow and stock dependent cost function $C(r,s)$, with $C_r > 0$, $C_{rr} > 0$, $C_s < 0$, $C_{ss} > 0$, $C_{rs} = C_{sr} < 0$ and $C_{hij} = 0,\ h,i,j = r,s$.[25] The optimization problem of the representative fossil fuel firm is solved by

$$p_r = C_r(r,s) + \tau, \tag{30}$$

$$\dot{\tau} = \rho\tau + C_s(r,s) \tag{31}$$

and (6). Apart from the price p_r, the fossil fuel supply function $R(p_r, \tau, s)$ depends on both the scarcity rent and the remaining fossil fuel stock. Consequently, fossil fuel market equilibria are determined by $A(p_r(t), \kappa(t), k(t)) = R(p_r(t), \tau(t), s(t))$. Graphically, the p_r intercept of the supply function in a (r, p_r) diagram equals the sum of the scarcity rent and the marginal costs of the first fossil fuel unit $C_r(0, s(t))$.

The stock dependence directly affects only the supply but not the demand side. Thus, the conditions for the optimal unilateral climate policy remain the same. The optimal fossil fuel tax increases in phase 1 with the rate $\rho + \gamma$ and equals zero at the end of phase 2. However, the stock dependence can affect the evolution of the tax during phase 2. Due to $C_s < 0$ and (31), the supply function may not constantly shift upwards in a (r, p_r) diagram. Therefore, the tax may also increase for some time during phase 2 to compensate a downward shifting supply function, so that $r(t) = \bar{r}$ is guaranteed. In the following we denote the point in time the tax reaches its maximum with t^{max}, i.e. $\kappa(t^{max}) > \kappa(t),\ \forall t$ and $t_1 \leq t^{max} < t_2$.

Due to the temporary character of the fossil fuel tax, the equation determining the last economically usable fossil fuel unit $\tau(T) + C_r(0, s(T)) = c$ is not altered. Thus, total fossil fuel extraction is independent from the unilateral climate policy. This result contrasts with Kollenbach (2014a) and Van der Ploeg and Withagen (2012), who show that a more

[25] Recall that r refers to the current extraction and s to the stock of fossil fuels. Kollenbach (2014a) uses a similar extraction cost function.

ambitious climate policy may reduce total fossil fuel extraction. As in case of proposition 1, the difference is explained by our endogenization of the optimal tax path and our application of both a ceiling and a natural regeneration rate.

A weak green paradox requires $r^N(t) > r^O(t)$ and therefore $s^N(t) < s^O(t)$ at early points of time. Consequently, the new fossil fuel extraction path intersects the old one from above, if a weak green paradox occurs. Thus, (21) and (22) need to hold at the intersection. (23) holds also, as $\tau_0^N < \tau_0^O$ is necessary for a weak green paradox, $s^N(t) < s^O(t)$ and $r^N(t) > r^O(t)$ before the intersection, and $C_{ss} > 0$ and $C_{sr} < 0$. Therefore, the right hand side of (31) is smaller for the new extraction path than for the old one before the intersection. In the Appendix we show that with a stock dependence of extraction costs (24), (25), and (26) read

$$\frac{d\tau^N}{dt} > \Omega_1 \frac{d\tau^O}{dt} - \Omega_2 \frac{d\kappa}{dt} + [\Omega_1 - 1]\,|C_{rs}|r, \tag{32}$$

$$\frac{d\tau^N}{dt} > \Omega_3 \frac{d\tau^O}{dt} - \frac{d\kappa}{dt} + [\Omega_3 - 1]\,|C_{rs}|r, \tag{33}$$

$$\frac{d\tau^N}{dt} > \frac{d\tau^O}{dt} - \frac{d\kappa}{dt}. \tag{34}$$

While (26) and (34) are identical, an additional term is added in case of (32) and (33). The sign of the term is ambiguous for (32) and negative for (33). However, if the tax increases sufficiently fast, proposition 3 still holds.

The sufficient condition (27) which prevents a strong green paradox was deduced for an unlimited resource stock. Consequently, neither (27) nor proposition 5 or lemma 1 are affected by a stock effect.

For the derivation of (28), (29) and proposition 5 we have used that the tax reaches its maximum at $t = t_1$ without a stock dependence of extraction costs. With the stock dependence, the tax may also increase during phase 2. However, the tax increases if and only if the sum of the scarcity rent and the marginal costs of the first fossil fuel unit ($\tau(t) + C_r(0, s(t))$) decreases, so that $r(t) = \bar{r}$ holds. Consequently, $\tau(t) + C_r(r(t), s(t))$ reaches its lowest position in a (r, p_r) diagram at $t = t^{max}$. By modifying the argumentation of section 5 accordingly, we find that before $t = t^{max}$ no country has an incentive to stay in a climate coalition which uses both resources simultaneously, if $k(t) > \tilde{k}^{min}(t^{max})$, with $\tilde{k}^{min}(t^{max}) = n - \frac{\bar{r}}{D^\circ(\tau(t^{max}) + C_r(\bar{r}, s(t^{max})))}$. For all following points in time we get $\tilde{k}^{min}(t) = n - \frac{\bar{r}}{D^\circ(\tau(t) + C_r(\bar{r}, s(t)))}$, $t \in [t^{max}, t_2[$. Thus, proposition 6 still holds, if t_1 is replaced by t^{max}.

7. Conclusion

This paper incorporates three important aspects of current climate policy, unilateralism, a demand side approach, and a climate target such as the frequently cited 2° target. By using a multi-country model with flow dependent fossil fuel extraction costs and a backstop, we point out that the optimal climate coalition, which levies a fossil fuel quantity tax, should consists of all countries concerned about global warming. If the coalition is sufficiently large, it can ensure that the emission stock does not exceed a critical value (ceiling) which reflects the climate target of the coalition. In line with the results of Chakravorty et al. (2006a) and Kollenbach (2014b), we can distinguish between three time periods. During the first limited period (phase 1) the emission stock approaches the ceiling, which is reflected by an increasing fossil fuel tax. The second period (phase 2) is characterized by a binding ceiling. Due to the decreasing fossil fuel stock, the tax can be lowered during this period until it equals zero at the end of the period. In the third everlasting period a sufficiently low fossil fuel stock renders the tax redundant.

By taking up Sinn's (2008a,b) argumentation, we show that a sufficiently fast increasing tax can cause a weak green paradox, i.e. an increase of early emission. However, if the ceiling reflects the damages of climate change correctly and the coalition is large enough, the occurrence of a weak green paradox is not worrying. Rather, it is the indication of a policy that ensures the adherence of the ceiling. In other words, the coalition size matters. This result has not been stressed in the literature, e.g. by Hoel (2011), Eichner and Pethig (2013) or Van der Ploeg and Withagen (2012), which does not focus on coalition size or use one-country models. Due to the importance of the coalition size, we determine sufficient size rule. Every coalition which complies with this rule is large (or powerful) enough to prevent a violation of the ceiling, i.e. a strong green paradox. Under the assumption that a coalition can commit itself to a tax path, which is not unrealistic given the ongoing complicated international climate policy negotiations, we determine the stable size of a climate coalition. At least if the ceiling is binding and the coalition simultaneously uses fossil fuels and backstop, the stable size is not identical with the optimal one. However, the coalition is still large enough to guarantee the adherence of the ceiling.

Furthermore, we show that a stock dependence of extraction costs does not alter our results considerably. However, in contrast to Kollenbach (2014a) and Van der Ploeg and Withagen (2012), we find that the climate policy does not alter total fossil fuel extraction.

The difference is explained by the temporary character of the our fossil fuel tax.

We use several simplifying assumption such as a uniform fossil fuel resource and constant backstop unit costs. Relaxing the former assumption allows the analysis of optimal unilateral climate policy with differently polluting resources. Chakravorty et al. (2008) conducts such an analysis in a closed economy. Supply side climate policy can be examined by reallocating some fossil fuel resources towards the environmental fraction.

A. Appendix

Smooth development

Hereafter, we give a brief version of the proofs of Kollenbach (2014b). Following Kollenbach (2014b), we refer to Feichtinger and Hartl (1986), page 164 et seq. for technical details.[26]

The jump conditions of θ at an entry point and a boundary point j is[27]

$$\theta^-(j) = \theta^+(j) + J\frac{\partial[\bar{z} - z]}{\partial z} = \theta^+(j) - J, \; J \geq 0, \tag{A.1}$$

with J as a jump parameter, and $^-$ and $^+$ denoting the values directly before and after the junction point. We use the indirect method to solve the optimization problem. According to Feichtinger and Hartl (1986), page 172 we can write the jump condition at an entry point and an boundary point as

$$\theta^+(j) = \theta^-(j) + \mu^+(j) + \bar{J}, \tag{A.2}$$

$$\theta^+(j) = \theta^-(j) + \bar{J}, \tag{A.3}$$

with $\bar{J} \geq 0$ denoting the jump parameter of the costate variable of the emission stock from the direct optimization approach. Due to the natural regeneration rate, fossil fuel extraction can only jump downwards at an entry or boundary point. As both $C_r(r)$ and $\tau(t)$ are continuous functions, it is clear from Fig. 1 that an downward jump is only possible if the tax $\kappa = \mu - \theta$ jumps upwards, i.e. $-\theta^-(j) \leq \mu^+(j) - \theta^+(j)$ at an entry and $-\theta^-(j) \leq -\theta^+(j)$ at a boundary point. By substituting (A.2) and (A.3) we get for both cases $\bar{J} \leq 0$. As $\bar{J} \geq 0$ needs to hold, $\bar{J} = 0$ and therefore $-\theta^-(j) = \mu^+(j) - \theta^+(j)$ at an entry and $\theta^+(j) = \theta^-(j)$ at a boundary point, i.e. a continuous development of fossil fuel

[26] See also Chiang (1992), page 298 et seq. and Seierstad and Sydsaeter (1987), page 357 et seq.

[27] At an entry point the ceiling becomes binding, while it becomes non-binding at an exit point. A boundary point is characterized by ceiling that bounds only at this point in time.

extraction and tax. As the ceiling will never be reached after a boundary point, $\theta^+(j) = 0$ and therefore $\theta(t) = 0 \; \forall t$.[28]

At an exit point θ is continuous, while fossil fuel extraction may jump downwards. Thus, with j denoting the exit point

$$\theta^-(j) = \theta^+(j) \text{ and } \mu^-(j) - \theta^-(j) \leq -\theta^+(j) \tag{A.4}$$

need to hold, which gives $\mu^-(j) = 0$, as $\mu \geq 0$, implying a continuous fossil fuel extraction path. If the economy switches into phase 3 at the exit point, $\theta^+(j) = \theta^-(j) = \mu^-(j) = 0$.

Lemma 2 *Both fossil fuel extraction and the optimal unilateral fossil fuel tax are continuous at the junction points.*

Phase sequence

Let t_1 denote an entry point, t_2 an exit point from phase 2 to phase 3, and t_3 an exit point from phase 2 to phase 1. Furthermore, the index $i = 1, 2, 3$ indicates the phase a costate variable belongs to. At the junction points

$$-\theta_1(t_1) = \mu_2(t_1) - \theta_1(t_1), \tag{A.5}$$

$$-\theta_2(t_2) = 0, \tag{A.6}$$

$$-\theta_2(t_3) = -\theta_1(t_3) \tag{A.7}$$

hold. Solving (17) and (18) for θ_1, θ_2 and μ_2 gives $\theta_1(t) = \theta_{01}e^{(\rho+\gamma)t}$, $\theta_2(t) = \theta_{02}e^{(\rho+\gamma)t} - \gamma\mu_{02}e^{(\rho+\gamma)t}\int e^{-(\rho+\gamma)t+\rho\int\chi(t)dt}dt$ and $\mu_2(t) = \mu_{02}e^{\rho\int\chi(t)dt}$, with θ_{01}, θ_{02} and μ_{02} as constant of integration, and $\chi(t) \leq 1$ reflecting the limited information (18) provides about the growth rate of μ_2. (A.5) and (A.7) read

$$\frac{\theta_{02} - \theta_{01}}{\mu_{02}} = e^{-(\rho+\gamma)t_1+\rho\int\chi(t_1)dt_1} + \gamma\int e^{-(\rho+\gamma)t_1+\rho\int\chi(t_1)dt_1}dt_1, \tag{A.8}$$

$$\frac{\theta_{02} - \theta_{01}}{\mu_{02}} = \gamma\int e^{-(\rho+\gamma)t_3+\rho\int\chi(t_3)dt_3}dt_3. \tag{A.9}$$

We denote the right hand side (RHS) of (A.8) with $\Gamma_1(t)$ and the RHS of (A.9) with $\Gamma_3(t)$. It is $\frac{d\Gamma_1}{dt} < 0$, if $\chi(t) < 1$; $\frac{d\Gamma_1}{dt} = 0$, if $\chi(t) = 1$; and $\frac{d\Gamma_3}{dt} > 0$. Obviously, $\Gamma_1(t) > \Gamma_3(t)$. The left hand sides (LHS) of (A.8) and (A.9) are constant and identical.

If there is a junction point t_3 from phase 2 to phase 1, there needs to be also a later junction point from phase 1 to phase 2, i.e. $t_1 > t_3$. As $\frac{d\Gamma_1}{dt} \leq 0$ and $\frac{d\Gamma_3}{dt} > 0$, we get $\Gamma_1(t_1) < \Gamma_3(t_1)$, which contradicts $\Gamma_1(t) > \Gamma_3(t)$. Lemma 3 follows directly.

Lemma 3 *The only sequence containing all three phases is phase 1, phase 2, phase 3.*

[28] Reaching the ceiling after the boundary point requires an increase of fossil fuel extraction for some time, which contradicts $\hat{\theta} = \rho + \gamma$ during phase 1 and $\hat{\tau} = \rho$.

Conditions for a weak green paradox

For the following proof we assume a stock dependence of extraction costs. The results of section 4 are obtained by setting $C(r,s)=C(r)$ and therefore $C_{rs}=0$.

The supply side of the fossil fuel market is given by (30). Differentiation with respect to time gives

$$\frac{dp_r^i}{dt}=C_{rr}\frac{dr^i}{dt}-C_{rs}r^i+\frac{d\tau^i}{dt},\ \ i=O,N. \tag{A.10}$$

The relevant parts of the new aggregated demand function are (i) and (ii), so that either $A(p_r^N,\kappa)=kD^*(p_r^N+\kappa)+(n-k)D^\circ(p_r^N)$ or $p_r^N=c-\kappa$ holds. By differentiating with respect to time we get

$$\frac{dr^N}{dt}=kD_p^*(p_r^N+\kappa)\left[\frac{dp_r^N}{dt}+\frac{d\kappa}{dt}\right]+(n-k)D_p^\circ(p_r^N)\frac{dp_r^N}{dt}, \tag{A.11}$$

$$\frac{dp_r^N}{dt}=-\frac{d\kappa}{dt}. \tag{A.12}$$

Substituting (A.10) and reorganizing gives

$$\frac{dr^N}{dt}=-\left[C_{rr}-\frac{1}{kD_p^*(p_r^N+\kappa)+(n-k)D_p^\circ(p_r^N)}\right]^{-1}$$

$$\left[\frac{d\tau^N}{dt}+\frac{kD_p^*(p_r^N+\kappa)}{kD_p(p_r^N+\kappa)+(n-k)D_p^\circ(p_r^N)}\frac{d\kappa}{dt}+|C_{rs}|r^N\right], \tag{A.13}$$

$$\frac{dr^N}{dt}=-\frac{1}{C_{rr}}\left[\frac{d\tau^N}{dt}+\frac{d\kappa}{dt}\right]-\frac{|C_{rs}|}{C_{rr}}r^N. \tag{A.14}$$

A similar argumentation for the old aggregated fossil fuel demand function yields

$$\frac{dr^O}{dt}=-\left[C_{rr}-\frac{1}{nD_p(p_r^O)}\right]^{-1}\frac{d\tau^O}{dt}-\left[C_{rr}-\frac{1}{nD_p(p_r^O)}\right]^{-1}|C_{rs}|r^O, \tag{A.15}$$

$$\frac{dr^O}{dt}=-\frac{1}{C_{rr}}\frac{d\tau^O}{dt}-\frac{|C_{rs}|}{C_{rr}}r^O \tag{A.16}$$

for fossil fuel market equilibria on part (i) and part (ii), respectively.

By substituting (A.13), (A.14), (A.15) and (A.16) in (22) for the equilibria combinations $((i)^O,(i)^N)$, $((i)^O,(ii)^N)$ and $((ii)^O,(ii)^N)$ we get (24), (25), and (26) without a stock dependence of extraction costs and (32), (33), and (34) with a stock dependence.

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