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Gilbert Kollenbach

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Extraction costs matter**

Universität Siegen  
Fakultät III  
Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht  
Fachgebiet Volkswirtschaftslehre  
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# Unilateral climate policies and green paradoxes: Extraction costs matter

Gilbert Kollenbach<sup>a,\*</sup>

<sup>a</sup>*Department of Economics, University of Hagen, Germany & School of Economic Disciplines, University of Siegen, Germany*

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## Abstract

Under which conditions unilateral tightening of climate policy causes a weak or strong green paradox or even decreases social welfare has recently been studied by Hoel (2011). Hoel assumes that the costs of extracting fossil fuel are linear in output. We extend his model by allowing for progressively increasing and stock dependent extraction costs. Increasing unit costs imply the simultaneous utilization of fossil fuel and a clean backstop. This has a significant effect on the results, as the utilization of backstop by the country which tightens its climate policy always prevents a weak green paradox. As a consequence, the effect of a tighter climate policy on social welfare can be reversed. Due to the stock dependence of extraction costs the amount of fossil fuel left in situ may be increased by a tighter climate policy. This implies that social welfare may increase, even if a weak green paradox occurs.

*Keywords:* Climate change, green paradox, exhaustible resources, renewable energy

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JEL classification: Q41; Q42; Q54; Q58

## 1. Introduction

Natural non-renewable resources have been the subject of economic research at least since the seminal work of Hotelling (1931). Hotelling (1931) and later authors like Stiglitz (1974), Dasgupta and Heal (1974) and Barbier (1999) focus on the problem of exhaustibility and the optimal resource extraction path. However, during the last few decades the attention has shifted to the problem of pollution caused by non-renewable resources.<sup>1</sup>

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\*Department of Economics, University of Hagen, Universitätsstr. 41, 58097 Hagen, Germany, Telephone: +49 2331 987 - 2694, Fax: +49 2331 987 - 4143

*Email address:* Gilbert.Kollenbach@Fernuni-Hagen.de (Gilbert Kollenbach)

<sup>1</sup>Two ways of considering pollution can be distinguished in the literature. The straightforward one considers pollution explicitly in the form of a damage function. Examples of this approach are Farzin

Fossil fuel has attracted most attention, as it is responsible for about 75% of greenhouse gas emissions and therefore the main driving force of global warming.<sup>2</sup> The experience with the Kyoto Protocol and the discussion about a follow-up agreement have shown that a global response to climate change is unlikely. Rather, different countries or regions, e.g. the EU, act independently by introducing climate policies which affect mainly the demand side.<sup>3</sup> However, Sinn (2008a) and Sinn (2008b) have shown that ill-designed demand side climate policies may increase early CO<sub>2</sub> emissions instead of reducing them. This phenomenon, called the "green paradox", occurs because fossil fuel owners accelerate resource extraction as a response to a rapidly tightening climate policy to avoid selling their resources at heavily depressed prices.<sup>4</sup>

The green paradox concept has been studied in more detail by Gerlagh (2011), Grafton et al. (2012), Van der Ploeg and Withagen (2012) and Hoel (2013). That literature assumes a single economy, which can be interpreted as the whole world. Therefore, the analyzed climate policies are implicitly assumed to be the same across countries. As argued above, climate policies differ among countries. Recently, a two-country, two-period model has been developed by Eichner and Pethig (2011). Ritter and Schopf (2013) extend this model by stock dependent extraction costs. However, both studies do not consider a backstop technology. A model of continuous time with a backstop is used by Hoel (2011). Hoel analysis in his important work the effect of unilateral climate policy changes.<sup>5</sup> Hoel (2011) assumes that the countries do not grow and are identical in all respects but climate policy, i.e. fossil fuel tax and backstop subsidy. Appropriate climate policies are welfare enhancing, since the utilization of fossil fuel causes pollution which is ignored by

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(1996), Hoel and Kverndokk (1996), Tahvonen (1997) and Van der Ploeg and Withagen (2012). The second way dates back to Chakravorty et al. (2006a), who model environmental concerns implicitly by assuming a ceiling on the stock of pollution. See e.g. Chakravorty et al. (2006b), Chakravorty et al. (2008), Chakravorty et al. (2012), Henriot (2012) and Lafforgue et al. (2008).

<sup>2</sup>Cf. Hoel (2011) and Van der Ploeg and Withagen (2012).

<sup>3</sup>Some countries do not follow a specific climate policy at all. Others, like the EU and some US states are using a wide array of climate policy instruments. Examples are carbon taxes or renewable energy subsidies. E.g. the EU has implemented a CO<sub>2</sub> emission trading scheme. According to Hoel (2011) Norway has introduced a carbon tax of up to 50 euro per tonne, while Sweden charges 115 euro per tonne. Germany subsidizes energy generated by several renewable resources.

<sup>4</sup>Without abatement technologies like carbon capture and storage, an increase of early fossil fuel utilization will boost early emission and global warming. Throughout this paper abatement is not considered. The effects of abatement are analyzed by Farzin (1996), Smulders and Gradus (1996), Chakravorty et al. (2006a), Lafforgue et al. (2008) and Moreaux and Withagen (2013).

<sup>5</sup>The effects of internationally different climate policies have also been studied by Hoel (1991), Golombek and Hoel (2004), Ishikawa and Kiyono (2006) and Di Maria and Van der Werf (2008) but without linking pollution to an exhaustible resource.

all economic units. Both fossil fuel and a clean backstop are available at constant unit costs. Due to the constant unit costs of fossil fuel, the simultaneous use of both resources is not possible. Each country chooses a specific tax and subsidy rate which is then kept constant over time. Generally, it is assumed that one country levies a higher tax and/or grants a higher subsidy than the other. Hoel (2011) shows under which conditions a weak and/or a strong green paradox is caused by an unilateral increase of the tax or subsidy. Furthermore, he determines the welfare effect of the policy change.<sup>6</sup> In particular, a tax increase by the country which initially levies a low tax (low-tax country) speeds up fossil fuel use and therefore implies a strong green paradox, if energy demand is sufficiently price inelastic.<sup>7</sup> Furthermore, a decrease of backstop costs causes a strong green paradox, provided that climate policies do not differ among countries. An increase of the subsidy of one country will always cause a weak green paradox. A strong green paradox occurs, if the subsidy is increased by the country which has initially the lower subsidy.

However, Hoel (2011) states in his conclusion that he is "using an extremely simple model" and he acknowledges that his assumptions of linear extraction costs and their stock independence are restrictive. Consequently, he suggests the introduction of a stock dependence. Furthermore, in view of the knowledge about geology and extraction technology it seems realistic to expect that the extraction costs convexly increase in the current resource flow.<sup>8</sup> One or both assumptions are applied in numerous studies.<sup>9</sup> The present paper extends Hoel's (2011) model by an extraction cost function that increases progressively in extraction and is stock dependent. The objective is to investigate how the more realistic assumption on the extraction technology modifies Hoel's results.

The strictly convex increase of extraction costs turns out to have significant influence on the results and may even reverse the results Hoel (2011) obtained for the case of constant unit extraction costs. The convex increase implies the simultaneous use of both resources, a result non obtained by Hoel (2011), Eichner and Pethig (2011) and Ritter and Schopf

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<sup>6</sup>The terms "weak" and "strong green paradox" were introduced by Gerlagh (2011). He refers to a weak green paradox, when a tighter climate policy increases early fossil fuel utilization. If also the (discounted) damages, which are caused by fossil fuel utilization, increase, Gerlagh (2011) uses the term "strong green paradox".

<sup>7</sup>The importance of demand elasticity is also highlighted by Eichner and Pethig (2011) and Ritter and Schopf (2013).

<sup>8</sup>Cf. Farzin (1992), page 815.

<sup>9</sup>Cf. Farzin (1992), Farzin (1996), Grafton et al. (2012), Tsur and Zemel (2003) and Tsur and Zemel (2005).

(2013). We find that the utilization of backstop by the country which tightens its climate policy always prevents a weak green paradox. If the stock dependence of extraction costs is sufficiently weak, also a strong green paradox is ruled out. These results contrast with the mentioned findings of Hoel (2011). Initial backstop utilization and a weak stock dependence prevents also a strong green paradox, if the backstop costs are decreased and climate policy is identical among countries. This is of special interest, as the opposing result of Hoel (2011) is in line with the common theory of non-renewable resources.

The stock dependence of extraction costs adds two interesting outcomes to the analysis. The first case is characterized by a strong influence of the fossil fuel stock on extraction costs. Then, a tighter climate policy may increase the energy price ruling out a weak green paradox. In the second case a tighter policy of the low-tax country causes a strong increase of the stock left in situ. Thus, even if a weak green paradox occurs, the effect on welfare can be positive.

The outline of the paper is as follows. In section 2 we describe the model and deduce energy demand, fossil fuel and backstop supply as well as the equilibrium on the energy market. We analyze the effects of climate policy instruments in section 3. The discussion begins with a fossil fuel tax and then proceeds with a backstop subsidy. Section 4 concludes.

## 2. Model

We use the framework of Hoel (2011), which consists of two countries or alliances, respectively. The countries are alike in all aspects but climate policy. One country levies a higher fossil fuel tax and/or grants a higher subsidy for a clean backstop. If not stated otherwise, both policy instruments are fixed over time. We extend this framework by considering extraction costs that are progressively increasing in extraction and stock dependent.

In the following, the assumptions of the model are briefly discussed.<sup>10</sup> The utility  $U^i(x^i) + z^i$  of the representative individual depends in both countries  $i = 1, 2$  on energy  $x^i$  and on a tradable good  $z^i$ . The function  $U^i$  is well-behaved, i.e.  $U^i$  and both its first derivative  $U_x^i > 0$  and its second derivative  $U_{xx}^i < 0$  are continuous. In both countries the

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<sup>10</sup>For the sake of simplicity we omit the time index  $t$  when it is not necessary for understanding. Generally, fossil fuel supply  $R(t)$  is written  $R$ .

population size is fixed. At every point in time the individuals can trade their exogenously given endowment  $\bar{z}(t) = \bar{z}$  on a global market at the normalized price  $p_z = 1$ . To generate energy the individuals buy fossil fuel  $R$  and a backstop  $b$  (solar, wind or fusion power) on international markets. Both energy sources are converted into energy according to a simple one-to-one transformation of one unit fossil fuel or backstop, respectively, into one unit energy. Thus, in accordance with the literature, we assume that both energy sources are perfect substitutes.<sup>11</sup> Therefore, energy amount  $x^i$  of country  $i$  is given by the sum of fossil fuel and backstop  $x^i = R^i + b^i$ . As the two energy sources are perfect substitutes, their prices need to be equal. In the following the corresponding price is denoted with  $p_R$ . Fossil fuel and backstop are supplied by a large number of identical firms which own the rights of use of the backstop and the fossil fuel stock  $S$ . Both the extraction of fossil fuel and the utilization of the backstop cause costs. In case of the backstop the costs of the representative backstop firm are given by  $Nb$ , with  $N > 0$  and constant.  $N$  are the unit costs of backstop energy expressed in terms of good  $Z$ . The cost function of the representative fossil fuel firm reads  $M(R(t), S(t))$ , with  $S(t)$  denoting the resource stock at time  $t$ .<sup>12</sup> We assume that no fixed costs exist, i.e.  $M(0, S(t)) = 0$ . The cost function as well as its first and second derivative are continuous. Furthermore, extraction costs convexly increase in  $R$  and decrease in  $S$ , i.e.  $M_R > 0, M_{RR} > 0, M_S < 0, M_{SS} > 0$ .<sup>13</sup> The effect of a decreasing fossil fuel stock is the weaker the fewer fossil fuel units are used so that  $M_{SR} = M_{RS} < 0$ . The profits of all firms are distributed among the individuals, which own the firms.

Both countries have a fixed time-invariant fossil fuel tax  $\phi_i$  and backstop subsidy rate  $\sigma_i$ . The government budgets are always assumed to be balanced by lump sum transfers  $T_i \leq 0$ .

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<sup>11</sup>The assumption is used by Chakravorty et al. (2006a), Chakravorty et al. (2006b), Chakravorty et al. (2008), Lafforgue et al. (2008), Tahvonen (1997), Tsur and Zemel (2005) and Tsur and Zemel (2011).

<sup>12</sup>A similar function is used by Krautkraemer (1998) and in a less general sense by Van der Ploeg and Withagen (2012), Tahvonen (1997) and Tahvonen and Salo (2001). It is also common to assume  $M(R(t), A(t))$ , where  $A(t)$  denotes the cumulative amount of exploited resources - cf. Farzin (1992), Farzin (1996) and Hoel and Kverndokk (1996). However, the latter approach is better suited for an economically but not physically exhausted resource stock, which we are also going to consider.

<sup>13</sup>We are following Farzin (1996) concerning the stock dependence. Note that Farzin (1996) uses the cumulative output  $A(t)$  instead the stock  $S(t)$ . Therefore, his assumption  $M_A > 0$  translates into  $M_S < 0$ . Tsur and Zemel (2003), (2005) use similar assumptions with respect to the fossil fuel output  $R$ . According to Tsur and Zemel (2003) the extraction costs convexly increase in output, since the last unit used is supplied by the cheapest source operating below its capacity. Thus, the supply of more units requires the exploitation of more expensive sources.

Fossil fuels are not only problematic because they are non-renewable but also because their burning generates polluting emissions. Here we refer especially to CO<sub>2</sub> and global warming. This aspect is covered by the damage function  $C(S_E(t))$ , with  $S_E(t)$  denoting the emission stock at time  $t$ . Following Hoel (2011), the emission stock is split into one component which remains in the atmosphere forever and another one which decreases at the rate  $\gamma$ . Let  $\eta$  denote the share of the non-decreasing part of an emission unit, then the remaining amount at time  $\tau$  of one emission unit emitted at time  $t$  is  $\eta + (1 - \eta)e^{-\gamma(\tau-t)}$ . The damage caused by this amount at time  $\tau$  is  $[\eta + (1 - \eta)e^{-\gamma(\tau-t)}]C'(S_E(\tau))$ . Thus, by aggregating the damages for all  $\tau \geq t$ , we get the social costs of one additional emission unit at time  $t$

$$v(t) = \int_t^{\infty} e^{-\rho(\tau-t)} [\eta + (1 - \eta)e^{-\gamma(\tau-t)}] C'(S_E(\tau)) d\tau, \quad (1)$$

with  $\rho$  denoting the time preference rate. Following Hoel (2011) we assume that the marginal damage function is constant, i.e.  $C'' = 0$ . Therefore, (1) can be written as  $v = \left[ \frac{\eta}{\rho} + \frac{1-\eta}{\rho+\gamma} \right] C'$ . The social costs of the fossil fuel flow  $R(t)$  are then given by

$$\Phi = v \int_0^{\infty} e^{-\rho t} R(t) dt. \quad (2)$$

In the following we refer to  $\Phi$  as the social carbon costs.

### 2.1. The resource firms

As mentioned above, energy is generated by means of fossil fuel or backstop, both of which are supplied by a large number of firms. The representative backstop firm faces no intertemporal problem, since the backstop stems from a practically unlimited resource flow like solar radiation. Therefore, the firm maximizes its profits  $\pi_b = p_R b - N b$  at every point time. The first order condition reads  $\frac{\partial \pi_b}{\partial b} = p_R - N = 0$ . Thus, at the price  $p_R = N$  the firm is willing to supply any amount of backstop. If  $p_R < N$  holds, no backstop is supplied, as the profit is negative.  $p_R > N$ , no profit maximum exists.

The representative fossil fuel firm maximizes the present value of its profits  $\int_0^{\infty} e^{-\rho t} \pi_R(t) dt = \int_0^{\infty} e^{-\rho t} [p_R(t)R(t) - M(R(t), S(t))] dt$  subject to the resource stock  $S$ . The stock decreases with resource utilization according to  $\dot{S} = -R$ . Thus,  $\int_0^{\infty} R(t) dt \leq S(0)$  must hold. With  $\chi > 0$  denoting the costate variable of the resource stock, i.e. its shadow



price, the corresponding Hamiltonian is given by  $H_R = p_R R - M(R, S) - \chi R$ . The first order conditions are

$$\frac{\partial H_R}{\partial R} = p_R - M_R(R, S) - \chi = 0, \quad (3)$$

$$\frac{\partial H_R}{\partial S} = -M_S(R, S) = \rho\chi - \dot{\chi}. \quad (4)$$

The transversality conditions, which determine the point in time  $T_R$  the utilization of fossil fuel ends, read

$$\chi(T_R) \geq 0, \quad \chi(T_R)S(T_R) = 0, \quad H_R(T_R) = 0. \quad (5)$$

(3) can be written as  $p_R = \chi + M_R(R, S)$ . The equation implicitly determines the fossil fuel supply as a function of the fossil fuel price and the shadow price of the resource stock. The shadow price  $\chi$  is interpreted as the scarcity rent of the resource stock. Thus, the price of a marginal resource unit  $p_R(t)$  at time  $t$  covers not only the extraction costs but also reflects that extracting one resource unit increases the scarcity of the resource.<sup>14</sup> The initial scarcity rent  $\chi_0$  in combination with (4) needs to comply with  $\chi(T_R) + M_R(0, S(T_R)) = N$ . As the last economically usable fossil fuel unit is determined by  $M_R(0, S(T_R)) \leq N$ , the transversality condition (5) implies  $\chi(T_R) > 0$  in case of a physically exhausted stock and  $\chi(T_R) = 0$ , if the stock is economically but not physically exhausted.

## 2.2. The individuals

At every point in time the representative individuals of both countries use their income to buy fossil fuel, backstop and the good  $z$ .<sup>15</sup> The income is composed of the value of the net sales or purchases of good  $z$ , of aggregate firm profits  $E_\pi$  and the lump sum transfer  $T_i$ . Taking into account that the backstop is only supplied at  $p_R = N$ , the maximization problem of the representative individual of country  $i = 1, 2$  reads  $H = U^i(R^i + b^i) + z^i + \lambda^i [E_\pi + p_z(\bar{z} - z^i) + T_i - (N - \sigma_i)b^i - (p_R + \phi_i)R^i] + \zeta_b^i b^i + \zeta_R^i R^i$ , with  $\zeta_b^i$  and  $\zeta_R^i$  as Lagrange multipliers of the conditions  $b^i \geq 0$  and  $R^i \geq 0$ . The necessary

<sup>14</sup>This point was first elaborated by Hotelling (1931). Cf. also Krautkraemer (1998), p. 2065 - 2070.

<sup>15</sup>The possibility of saving income by lending it to other individuals is omitted here. If the possibility exists, the credit market clearing will imply that no individual lend and no individual borrow, since all individuals are identical. Thus, to keep the model as simple as possible, the credit market is ignored.

conditions for an optimum and the complementary slackness conditions are

$$\frac{\partial H}{\partial b^i} = U_x^i - \lambda^i(N - \sigma_i) + \zeta_b^i = 0, \quad (6)$$

$$\frac{\partial H}{\partial R^i} = U_x^i - \lambda^i(p_R + \phi_i) + \zeta_R^i = 0, \quad (7)$$

$$\frac{\partial H}{\partial z^i} = 1 - \lambda^i p_z = 0, \quad (8)$$

$$\zeta_b^i b^i = 0, \quad \zeta_b^i \geq 0, \quad (9)$$

$$\zeta_R^i R^i = 0, \quad \zeta_R^i \geq 0. \quad (10)$$

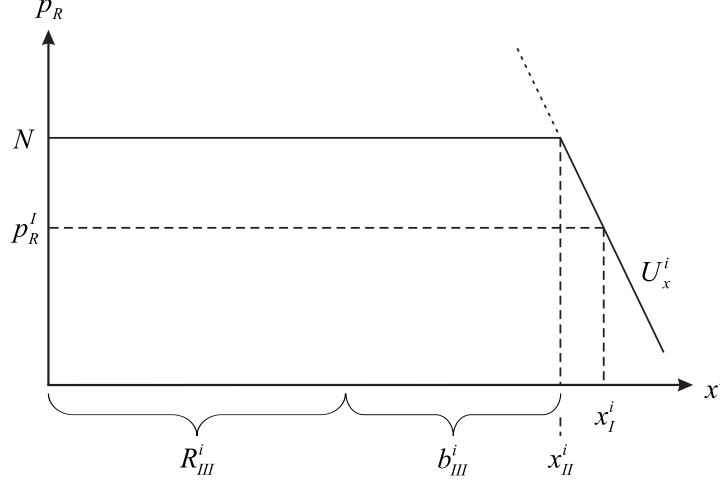
Since  $p_z$  is normalized to one, (8) implies  $\lambda^i = 1$ . Substituting into (6) and (7) gives

$$U_x^i - \phi_i = N - \sigma_i - \phi_i - \zeta_b^i = p_R - \zeta_R^i, \quad (11)$$

which determines both fossil fuel and backstop demand. We assume for the moment that  $\phi_i = \sigma_i = 0$  so that (11) reads  $U_x^i = N - \zeta_b^i = p_R - \zeta_R^i$ . Fig. 1 shows then that fossil fuel demand is implicitly given by  $U_x^i$  as long as fossil fuel can be bought at a price lower than  $N$ . If the price equals  $p_R^I$ , the energy demand equals  $x_I^i$  and is completely satisfied by fossil fuel, because backstop demand is zero. In that case (9), (10) and (11) imply  $\zeta_R^i = 0$  and  $\zeta_b^i > 0$ . On the other hand, no fossil fuel is bought and energy generation fully relies on backstop, if the fossil fuel price exceeds  $N$ . (9), (10) and (11) require then  $\zeta_b^i = 0$  and  $\zeta_R^i > 0$  while energy is given by  $x_{II}^i$ . If  $p_R$  equals  $N$  the individual's energy demand is also  $x_{II}^i$ . However, the individual is indifferent between the two energy sources. We assume that energy is generated by means of fossil fuel as long as it is supplied at a price marginally lower than  $N$ . Let be  $R_{III}^i$  the corresponding fossil fuel amount, then the energy mix is given by  $R_{III}^i$  and  $b_{III}^i = x_{II}^i - R_{III}^i$ . (9), (10) and (11) imply  $\zeta_b^i = \zeta_R^i = 0$  in that case. In all three cases backstop and fossil fuel demand and therefore the energy mix are determined by the resource price  $p_R$ . Thus,  $b^i = b^i(p_R)$ ,  $R^i = R^i(p_R)$  and  $x^i = x^i(p_R)$ , with  $\frac{\partial x^i}{\partial p_R} \leq 0$ . If the backstop is used,  $\frac{\partial R^i}{\partial p_R} < 0$  and  $\frac{\partial b^i}{\partial p_R} > 0$  hold; otherwise  $\frac{\partial R^i}{\partial p_R} < 0$  and  $\frac{\partial b^i}{\partial p_R} = 0$ .

### 2.3. The energy market

The equilibrium on the energy market is determined by the supply functions of the energy firms and the demand functions of the individuals. Fossil fuel supply is implicitly given by  $\chi + M_R(R, S)$ . The market is cleared at the price  $p_R$ . If  $p_R = N$  holds, global energy demand is  $\sum_{i=1}^2 x_{II}^i$  and is composed of both resources or only backstop. In case



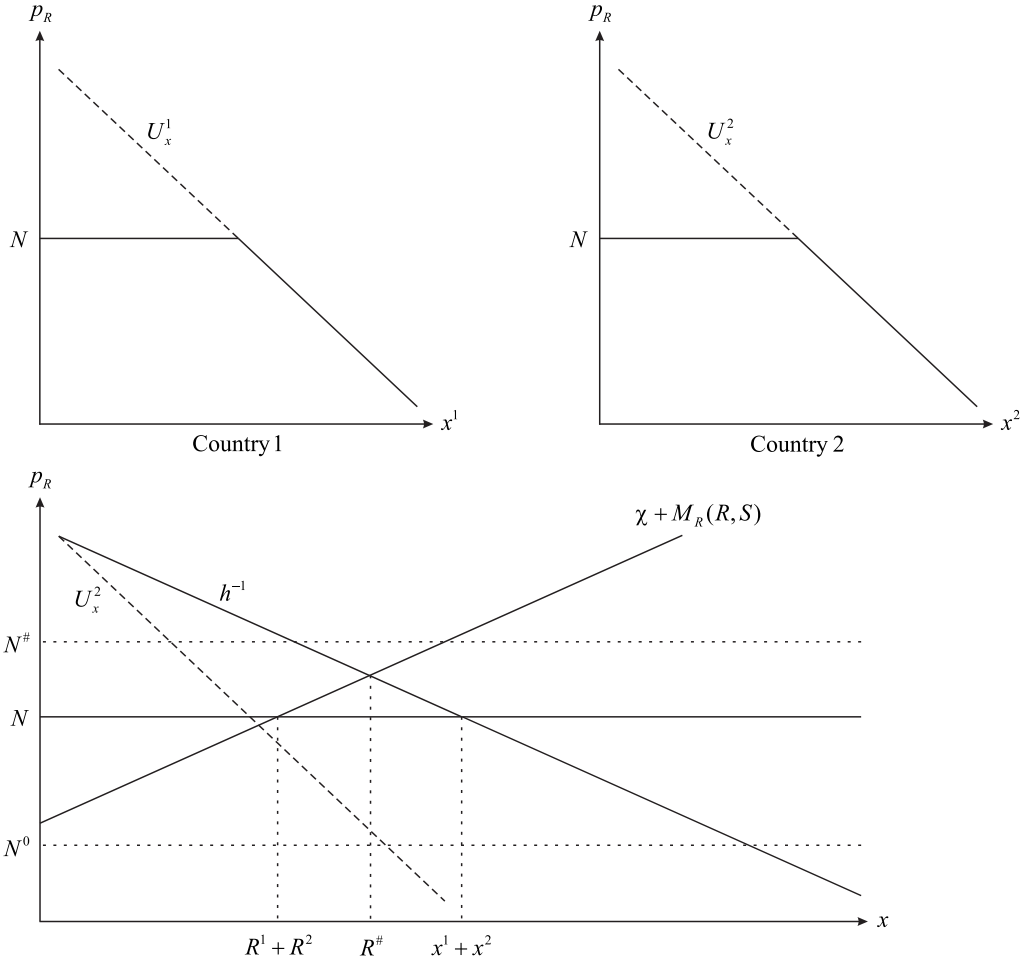
**Figure 1:** Fossil fuel demand of the representative individual without tax and subsidy

of  $p_R < N$ , no backstop is supplied and energy demand reads  $h(p_R) = \sum_{i=1}^2 g^i(p_R)$ , with  $h' < 0$  and  $g^i(p_R)$  denoting the inverse function of  $U_x^i(x^i)$ . In other words,  $g^i(p_R)$  satisfies  $g^i(U_x^i(x^i)) = x^i$ .<sup>16</sup> Fig. 2 illustrates the market. In the upper panel the figure shows the demand functions of the individuals. The aggregated demand function, the fossil fuel and backstop supply functions and the equilibrium are depicted in the lower panel. The backstop is not used, if the unit backstop costs  $N$  exceed  $\chi + M_R(R^\#, S)$ , as in case of  $N^\#$ , implying that total energy equals  $R^\#$ . If the unit backstop costs are lower than  $\chi + M_R(0, S)$ , as  $N^0$ , fossil fuel is not used. In Fig. 2 the equilibrium fossil fuel price equals  $N$  so that both energy sources are used and total energy given by  $x = x^1 + x^2$ . Fossil fuel utilization is determined by  $\chi + M_R(R, S) = N$  and equals  $R = R^1 + R^2$ .

Notice that the assumption  $M_{RR}(R, S) > 0$  implies the simultaneous utilization of both energy sources before  $t = T_R$  and a smooth transition to exclusive backstop utilization. Furthermore, the fossil fuel utilization path is continuous.<sup>17</sup> The simultaneous utilization of both energy sources contrasts with Hoel (2011), who assumes that the marginal fossil fuel costs  $M_R(R, S)$  are independent of  $R$ . In this case, the fossil fuel supply curve in Fig. 2 is a horizontal straight line. Fossil fuel is used exclusively as long as  $\chi(t) + M_R(S(t)) < N$ . From the point in time at which  $\chi(t) + M_R(S(t)) = N$  holds, fossil fuel demand is

<sup>16</sup> $p_R > N$  is ruled out, since energy demand is satisfied at  $p_R = N$ .

<sup>17</sup>This follows from (3), (4) and the continuity of all cost and utility functions. (4) implies a continuous development of the scarcity rent  $\chi$  and therefore together with (3) and the continuity of all cost and utility functions a continuous fossil fuel price path. Since the model includes no market distortions, the proof from Tsur and Zemel (2003), Appendix A.1. - A.2. can be applied here with respect to the smooth transition at  $t = T_R$ .



**Figure 2:** Energy market equilibrium

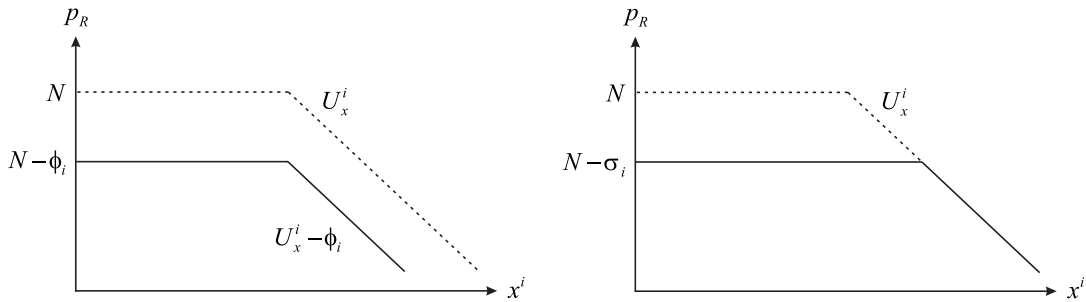
completely replaced by backstop use so that the fossil fuel utilization path exhibits a jump to zero at  $T_R$ . However, with either assumption the area below the fossil fuel utilization path equals (is smaller than) the resource stock, if  $M_R(0,0) \leq (>) N$ . Note that the extraction path strictly decreases, if  $M_S(R, S) = 0$  and  $h' < 0$ , since the aggregated demand function decreases in the fossil fuel price and is time-invariant by assumption while the supply curve is shifting upwards due to  $\hat{\chi} = \rho$  and  $M_R(0) = 0$ . If  $M_S(R, S) < 0$ , the growth rate of  $\chi$  can be negative. This implies that increasing fossil fuel utilization over time is possible.<sup>18</sup>

### 3. Unilateral Changes of Climate Policy

Having described the model, we now turn to the analysis of unilateral changes of climate policy. Consequently, we return to the case of positive tax and subsidy rates.

<sup>18</sup>The development of the scarcity rent in case of stock dependence has been analyzed by Farzin (1992).

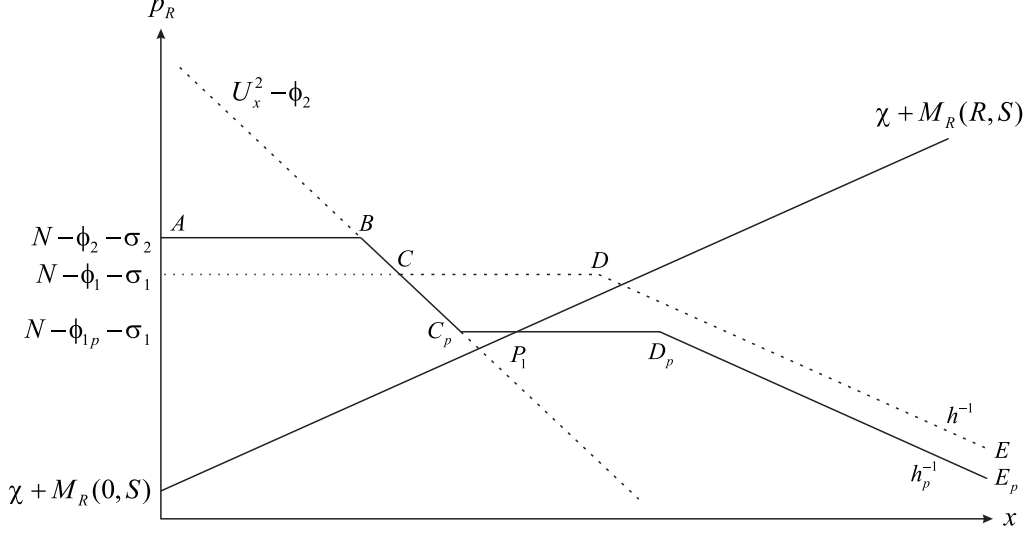
As mentioned above, the two countries differ in respect to their climate policy. One country, say country 1, wants to postpone the utilization of fossil fuels to later periods and therefore to reduce the social costs of carbon  $\Phi$  to a higher degree than country 2. Without discussing possible reasons for that difference in climate policies we follow Hoel (2011) and assume that initially the countries have fixed time-invariant tax and subsidy rates which differ across countries. Throughout the paper we assume that  $\phi_1 + \sigma_1 > \phi_2 + \sigma_2$  and refer to country 1 (2) as the high-tax (low-tax) country. As per (11) the fossil fuel tax increases the costs of fossil fuel, while the backstop subsidy decreases the costs of the backstop. The effects of the tax and the subsidy are illustrated in Fig. 3 by comparing countries' energy demand with and without a fossil fuel tax (left-hand side) or subsidy (right-hand side), respectively. In case of the tax, fossil fuel demand is lowered by the tax



**Figure 3:** Impact of taxes and subsidies on fossil fuel demand

for any price  $p_R$  implying that the termination price  $N - \phi_i$  at which fossil fuel demand vanishes is reduced, as fossil fuel demand is only positive for  $p_R \leq N - \phi_i$ . The subsidy reduces fossil fuel demand by reducing the termination price from  $N$  to  $N - \sigma_i$ . However, it does not affect the fossil fuel demand function for lower price levels. Obviously, a unilateral increase of the tax or subsidy rate reduces fossil fuel demand at some point in time. To derive the corresponding adaption of the initial scarcity rent  $\chi_0$  to changes in energy demand, we take a closer look at the case of a tax increase by the high-tax country. The energy market before and after the tax hike is illustrated in Fig. 4, with the subscript  $p$  denoting curves and variables after the tax increase.<sup>19</sup> The aggregate fossil fuel demand curve before the tax increase is given by  $ABCDE$ , while the demand curve after the tax increase is  $ABC_p D_p E_p$ . If the supply curve intersects the demand curve to the right of

<sup>19</sup>Notice that the straight lines with negative slopes are only a graphical simplification that does not affect the results. Especially, an intersection of the pre-tax hike and the post-tax hike demand curve is not possible, since the sum of the two pre-tax hike demand curves is larger for every price  $p_R$  than the sum of the post-tax hike demand curves.



**Figure 4:** Energy market before and after an increase of the tax rate  $\phi_1$

$C$  ( $C_p$ ), total energy utilization and the energy mix can be derived in the same manner as in Fig. 2. On the other hand, an intersection to left of  $C$  ( $C_p$ ) implies that country 1 only uses backstop and that its backstop demand is given by  $CD$  ( $C_p D_p$ ), while the energy mix of country 2 is determined by the intersection corresponding to the remarks made with respect to Fig. 1.

Obviously, the tighter policy of country 1 can influence the market equilibrium only if the equilibrium before the tax hike lies in the segment  $CDE$  of the demand curve. Otherwise, country 1 already uses only backstops and  $\frac{d\phi_1}{dt} > 0$  has no effect. If the market equilibrium is affected by the tax increase,  $\chi_0$  has to adapt to the new policy. This is shown in the following for every decrease of energy demand caused by an unilateral climate policy change.

Suppose the initial scarcity rent remains unchanged after the decrease of fossil fuel demand caused by a tighter unilateral climate policy. Then in either early or late periods less fossil fuel is used, which implies a higher stock  $S(t)$  and a lower flow  $R(t)$ . According to (4) the equation of motion of the scarcity rent reads  $\dot{\chi} = \rho\chi + M_S(R, S)$ . Thus,  $\chi$  grows at a higher rate after the policy change than before. At any point in time fossil fuel is supplied, if and only if its price equals or exceeds  $\chi(t) + M_R(0, S(t))$ . This critical price level is composed of the scarcity rent  $\chi(t)$  and the marginal costs of the first fossil fuel unit at time  $t$ :  $M_R(0, S(t)) > 0$ . Since  $M_{RS} < 0$ ,  $M_R(0, S(t))$  is smaller for a higher fossil fuel stock  $S(t)$ . Thus, the reduced fossil fuel demand affects  $\chi(t) + M_R(0, S(t))$  in two opposite ways. However,  $\chi(t) + M_R(0, S(t))$  needs to develop so that it reaches the

fossil fuel termination price  $N - \phi_l - \sigma_l$ , with  $N - \phi_l - \sigma_l \geq N - \phi_i - \sigma_i$ ,  $i, l = 1, 2$ , in the moment the fossil fuel stock becomes (economically) exhausted, i.e. in the moment in which  $M_R(0, 0) \leq N - \phi_l - \sigma_l$  ( $M_R(0, S(T_R)) = N - \phi_l - \sigma_l$ ) holds. Thus, only if the two opposing effects neutralize each other or if the reduced demand exactly offsets the reduction of the economically usable fossil fuel stock, the initial scarcity rent needs no change. However, it seems more realistic that the critical level  $\chi(t) + M_R(0, S(t))$  grows either too fast or too slowly to ensure  $M_R(0, 0) \leq N - \phi_l - \sigma_l$  ( $M_R(0, S(T_R)) = N - \phi_l - \sigma_l$ ). In the first case, the termination price is exceeded at a point in time at which  $S(t) > S(T_R)$  holds. Then the response to the tighter climate policy is  $\chi_{0p} < \chi_0$ . In the opposite case, the fossil fuel stock is (economically) exhausted before the termination price is reached implying a rise of the initial scarcity rent. Consequently, a weak green paradox is ruled out.<sup>20</sup>

Suppose next the fossil fuel extraction costs are stock-independent, i.e. that  $M_S = 0$  and  $\dot{\chi} = \rho\chi$ . A decrease of global fossil fuel demand then unambiguously requires a decrease of the initial scarcity rent. Otherwise, the termination price is reached at a point in time at which  $S(t) > 0$  holds. Thus, if the fossil fuel stock influences the costs only to a small degree, i.e. if  $|M_S|$  and  $|M_{RS}|$  are sufficiently small, the initial scarcity rent decreases to adapt to a lower global fossil fuel demand. In this case, fossil fuel utilization may be increased by a unilaterally tightened climate policy, i.e. the policy change may cause a weak green paradox. To illustrate this, we refer again to Fig. 4. Before the tax hike, the energy market equilibrium is given at the point  $P_0$ . If the initial scarcity is decreased sufficiently by a policy change, the new equilibrium is located to the right of  $P_0$ . Hence,  $R_p > R$ , i.e. a weak green paradox occurs.

**Lemma 1** *Suppose one of the countries tightens its climate policy unilaterally.*

- (a) *Then a weak green paradox may not occur, if the stock dependence of extraction costs is strong.*
- (b) *Then a weak green paradox occurs, if the initial scarcity rent declines sufficiently strongly. The initial scarcity rent unambiguously declines, if the stock dependence of extraction costs is sufficiently weak.*

A weak green paradox is of interest for two reasons. On the one hand, it is a counter-intuitive result, as one would expect that a tighter climate policy reduces fossil fuel use. On the other hand, a tighter climate policy may not only cause a weak green paradox but

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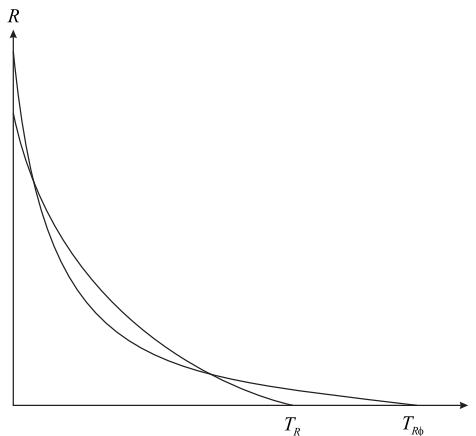
<sup>20</sup>See footnote 6 for the definition of weak and strong green paradox.

also a strong green paradox, i.e. an increase of social carbon costs. Therefore, welfare may be reduced by a unilaterally tightened climate policy. To discuss this possibility, we follow Hoel (2011) in describing the welfare of country  $i = 1, 2$  as

$$\begin{aligned}
W_i = & \int_0^{\infty} e^{-\rho t} [U^i(b^i + R^i) + z^i - Nb^i + p_z[\bar{z} - z^i] + p_R(\beta_i R - R^i)] dt \\
& - \int_0^{\infty} e^{-\rho t} \beta_i M(R, S) dt - \alpha_i \Phi,
\end{aligned} \tag{12}$$

where  $\alpha_i$  is country  $i$ 's share of social carbon costs and  $\beta_i$  is its share of ownership of the fossil fuel stock, with  $\sum_i \alpha_i = \sum_i \beta_i = 1$ . The first three terms in the square brackets indicate utility from energy and good  $z$  net of backstop costs. The fourth and fifth term are the trade balances of good  $z$  and fossil fuel, respectively. The terms in the second line of (12) represent the fossil fuel and social carbon costs, respectively, assigned to country  $i$ .

As mentioned in section 2.3, fossil fuel extraction is continuous with a smooth transition to exclusive backstop utilization. Furthermore, the economically usable fossil fuel stock is either not affected by tighter unilateral climate policy or decreased. Since the weak green paradox implies higher early fossil fuel utilization by definition, the extraction path after the policy change needs to intersect the pre-policy change extraction path at least once, as depicted in Fig. 5. Otherwise, the post-policy change extraction path



**Figure 5:** Extraction paths exhibiting a weak green paradox

violates either the condition  $\int_0^{\infty} R_p(t) dt \leq S(0)$  in case of a fully exhausted fossil fuel stock or the condition  $\int_0^{\infty} R_p(t) dt \leq S(0) - S(T_R)$  in case of an economically but non physically exhausted stock. Fig. 5 shows that at the first intersection the negative slope



of the post-policy change extraction path needs to be steeper than that of the pre-policy change path. In other words, the growth rate of  $R_p(t)$  needs to be smaller than the one of  $R(t)$ . This observation can be generalized, as it also holds for increasing segments of the utilization paths. Thus, if a weak green paradox is caused by a unilaterally tightened climate policy, there needs to be at least one intersection point of two utilization paths characterized by

$$\frac{dR_p(t)}{dt} < \frac{dR(t)}{dt} \quad (13)$$

and

$$R_p(t) = R(t). \quad (14)$$

Furthermore, a weak green paradox implies  $S_p(t) < S(t)$  and  $R_p(t) > R(t)$  for all points in time before the first intersection and requires  $\chi_{0p} < \chi_0$ . Therefore, both  $\chi_{0p}$  and  $M_S(R_p, S_p)$  are lower than without the policy change. Thus, we get from (4) that

$$\frac{d\chi_p}{dt} < \frac{d\chi}{dt} \quad (15)$$

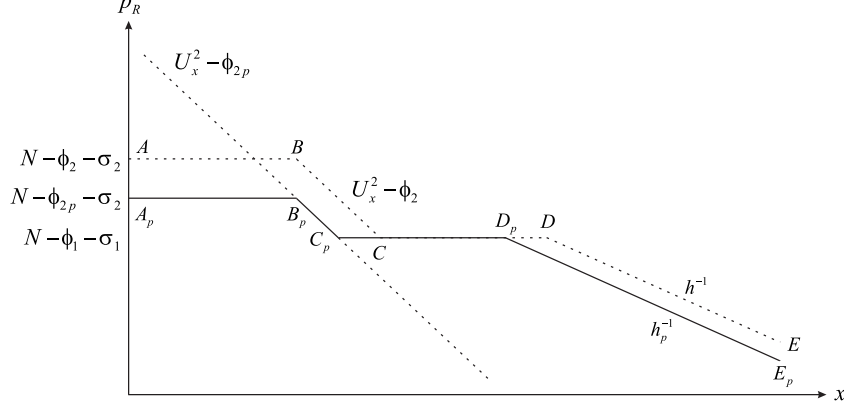
holds during the time interval before and at the first intersection. Based on these observations, we investigate in the following whether the conditions for a weak green paradox are met and analyze the effect on welfare. Consequently, we assume throughout the following analysis that the initial scarcity rent is decreased by unilateral policy changes.

### 3.1. Increasing fossil fuel tax

First we turn to the case of a unilateral increase of the fossil fuel tax of either country 1 or 2. The first case was already illustrated in Fig. 4. The latter is depicted in Fig. 6, which can be interpreted in the same way as Fig. 4. Table 1 lists the combinations of market equilibria and the corresponding market clearing conditions of both the case that the high tax country and that the low tax country raises its fossil fuel tax.<sup>21</sup> Combinations characterized by an identical position of the pre-tax hike and post-tax hike demand curve, like the segment  $ABC$  in Fig. 4, are omitted, as the required conditions  $\chi_{0p} < \chi_0$  and  $\frac{d\chi_p}{dt} < \frac{d\chi}{dt}$  rule out the validity of (14).

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<sup>21</sup>The capital letter combinations, like  $DE$ , indicate the position of the corresponding market equilibrium on the aggregated demand curve of Fig. 4 and Fig. 6, respectively. The Roman numerals are used to differentiate the different equilibria and to assign the post-tax hike equilibria to the pre-tax hike equilibria. Thus, the number  $(I_p.A)$  indicate that the corresponding equilibrium is the first post-tax hike equilibrium assign to the pre-tax hike equilibrium ( $I$ ).



**Figure 6:** Energy demand before and after an increase of the rate  $\phi_2$

	pre-tax hike	post-tax hike
$d\phi_1$	$DE \Rightarrow \chi + M_R(R(t), S(t)) = h^{-1}(R(t))$ (I)	$D_p E_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = h_p^{-1}(R_p(t))$ ( $I_p.A$ ) $C_p D_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_{1p} - \sigma_1$ ( $I_p.B$ ) $CC_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = U_x^2(R_p(t)) - \phi_2$ ( $I_p.C$ )
	$CD \Rightarrow \chi + M_R(R(t), S(t)) = N - \phi_1 - \sigma_1$ (II)	$C_p D_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_{1p} - \sigma_1$ ( $II_p.A$ ) $CC_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = U_x^2(R_p(t)) - \phi_2$ ( $II_p.B$ )
$d\phi_2$	$DE \Rightarrow \chi + M_R(R(t), S(t)) = h^{-1}(R(t))$ (III)	$D_p E_p \Rightarrow \chi_p + M_R(R_p(t)) = h_p^{-1}(R_p(t))$ ( $III_p$ )
	$CD \Rightarrow \chi + M_R(R(t), S(t)) = N - \phi_1 - \sigma_1$ (IV)	$D_p E_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = h_p^{-1}(R_p(t))$ ( $IV_p.A$ ) $C_p D_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_1 - \sigma_1$ ( $IV_p.B$ )
	$BC \Rightarrow \chi + M_R(R(t), S(t)) = U_x^2(R(t)) - \phi_2$ (V)	$D_p E_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = h_p^{-1}(R_p(t))$ ( $V_p.A$ ) $C_p D_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_1 - \sigma_1$ ( $V_p.B$ ) $B_p C_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = U_x^2(R_p(t)) - \phi_{2p}$ ( $V_p.C$ )
	$AB \Rightarrow \chi + M_R(R(t), S(t)) = N - \phi_2 - \sigma_2$ (VI)	$A_p B_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_{2p} - \sigma_2$ ( $VI_p$ )

**Table 1:** Comparative statics of unilateral tax increases

To verify (13), (14) and (15), the derivatives of the market clearing conditions are substituted into (13). Table 2 summarizes the corresponding results for the market equilibrium combinations listed in Table 1. Notice that all inequalities are evaluated at  $R(t) = R_p(t)$ .

### Tax increase in the high-tax country

First we consider an increase of the tax rate  $\phi_1$  in the high-tax country. If one of the combinations of (I) and ( $I_p.i$ ),  $i = A, B, C$  holds, no backstop is used before the tax hike. The corresponding conditions for a weak green paradox are listed in the first row of Table 2, i.e. in the cells 1x1, 1x2 and 1x3.<sup>22</sup> Suppose fossil fuel costs do not depend on the stock. Then the last term in all three inequalities equals zero. Since  $\frac{d\chi_p}{dt} < \frac{d\chi}{dt}$ , the inequality in

<sup>22</sup>The notation used for the denomination of table cells follows the concept used for matrix elements. Thus,  $NxM$  is the cell in the  $N$ th row of the  $M$ th column

	post-tax hike		
	$(I_p.A), (III_p), (IV_p.A),$ $(V_p.A)$	$(I_p.B), (II_p.A), (IV_p.B),$ $(V_p.B), (VI_p)$	$(I_p.C), (II_p.B), (V_p.C)$
(I), (III)	$\frac{d\chi_p}{dt} > \frac{M_{RR} - (h_p^{-1})' d\chi}{M_{RR} - (h^{-1})' dt}$ $+  M_{RS}  R \left[ \frac{M_{RR} - (h_p^{-1})'}{M_{RR} - (h^{-1})'} - 1 \right]$	$\frac{d\chi_p}{dt} > \frac{M_{RR}}{M_{RR} - (h^{-1})'} \frac{d\chi}{dt}$ $+  M_{RS}  R \left[ \frac{M_{RR}}{M_{RR} - (h^{-1})'} - 1 \right]$	$\frac{d\chi_p}{dt} > \frac{M_{RR} - U_{xx}^2}{M_{RR} - (h^{-1})'} \frac{d\chi}{dt}$ $+  M_{RS}  R \left[ \frac{M_{RR} - U_{xx}^2}{M_{RR} - (h^{-1})'} - 1 \right]$
(II), (IV), (VI)	$\frac{d\chi_p}{dt} > \frac{M_{RR} - (h_p^{-1})' d\chi}{M_{RR}} dt$ $+  M_{RS}  R \left[ \frac{M_{RR} - (h_p^{-1})'}{M_{RR}} - 1 \right]$	$\frac{d\chi_p}{dt} > \frac{d\chi}{dt}$	$\frac{d\chi_p}{dt} > \frac{M_{RR} - U_{xx}^2}{M_{RR}} \frac{d\chi}{dt}$ $+  M_{RS}  R \left[ \frac{M_{RR} - U_{xx}^2}{M_{RR}} - 1 \right]$
(V)	$\frac{d\chi_p}{dt} > \frac{M_{RR} - (h_p^{-1})' d\chi}{M_{RR} - U_{xx}^2} dt$ $+  M_{RS}  R \left[ \frac{M_{RR} - (h_p^{-1})'}{M_{RR} - U_{xx}^2} - 1 \right]$	$\frac{d\chi_p}{dt} > \frac{M_{RR}}{M_{RR} - U_{xx}^2} \frac{d\chi}{dt}$ $+  M_{RS}  R \left[ \frac{M_{RR}}{M_{RR} - U_{xx}^2} - 1 \right]$	$\frac{d\chi_p}{dt} > \frac{d\chi}{dt}$
	pre-tax hike		

**Table 2:** Conditions for a weak green paradox

cell 1x1 cannot hold unless  $\frac{M_{RR} - (h_p^{-1})'}{M_{RR} - (h^{-1})'} < 1$ , which implies  $|(h_p^{-1}(R(t)))'| < |(h^{-1}(R(t)))'|$ . Hence, the pre-tax hike demand function needs to be sufficiently inelastic. However, in this case the term in square brackets is negative. Thus, the right-hand side of the inequality is smaller in the case of  $|M_{RS}| > 0$  than in case of  $|M_{RS}| = 0$ . If the term in square brackets is positive and  $|M_{RS}| > 0$ , the right-hand side is larger. Though, even without stock dependence  $|(h_p^{-1}(R(t)))'| > |(h^{-1}(R(t)))'|$  implies that the inequality is not satisfied. Thus, reintroducing the stock dependence of extraction costs does not change the result that the inequality holds, if the pre-tax hike demand function is sufficiently inelastic. However, with stock dependence the inequality holds for larger values of the elasticity than without. Notice that  $(h_p^{-1}(R(t)))'$  and  $(h^{-1}(R(t)))'$  are generally not equal.<sup>23</sup> The second inequality in cell 1x2 holds under the same qualitative condition as the inequality in cell 1x1. In contrast, the third inequality in cell 1x3 is not satisfied, since  $U_{xx}^2 = \frac{1}{(g^2)'}$  and  $(h^{-1})' = \frac{1}{h'} = \frac{1}{(g^1)' + (g^2)'}$ . Thus, to allow  $\frac{M_{RR} - U_{xx}^2}{M_{RR} - (h^{-1})'} < 1$  the demand function of country 1  $g^1(p_R)$ , with  $(g^1)' = \frac{1}{U_{xx}^1}$ , needs to increase, which contradicts the assumption  $U_{xx}^1 < 0$ .

In the case of the pre-tax hike equilibrium (II), the high-tax country uses both resources. A weak green paradox cannot occur, since  $\frac{dX_p}{dt} < \frac{dX}{dt}$  and  $U_{xx}^2 < 0$  implying that the corresponding inequalities in the cells 2x2 and 2x3 never hold. Thus, backstop utilization before the tax increase prevents the occurrence of a weak green paradox. In other words:

**Lemma 2** *Suppose the high-tax country increases its fossil fuel tax unilaterally. Then no weak green paradox occurs, if the backstop is used before the tax hike.*

This result cannot be obtained in Hoel (2011), since in his model both energy sources are not used simultaneously at any time.

The welfare effects of the tax increase by the high-tax country are analyzed by taking the derivation of (12) with respect to tax rate. Using  $b^i = 0$  for  $p_R < N$  and  $R^i = 0$  for

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<sup>23</sup>We obtain  $(h_p^{-1}(R(t)))' = \frac{U_{xx}^1(R_p^1(t))U_{xx}^2(R_p^2(t))}{U_{xx}^1(R_p^1(t)) + U_{xx}^2(R_p^2(t))}$  and  $R_p^2(t) > R^2(t)$  due to the lower fossil fuel price after the tax hike. Since  $R(t) = R_p(t)$ , this implies  $R_p^1(t) < R^1(t)$ . Thus,  $U_{xx}^1(R_p^1(t))$  and  $U_{xx}^2(R_p^2(t))$  equal their equivalents before the tax hike, only if the marginal utility functions are linear.

$p_R > N$  we get

$$\begin{aligned} \frac{dW_i}{d\phi_l} = & \phi_i \int_0^\infty e^{-\rho t} \frac{dR^i(t)}{d\phi_l} dt - \sigma_i \int_0^\infty e^{-\rho t} \frac{db^i(t)}{d\phi_l} dt + \int_0^\infty e^{-\rho t} (\beta_i R - R^i) \frac{dp_R(t)}{d\phi_l} dt - \alpha_i \frac{d\Phi}{d\phi_l} \\ & + \beta_i \int_0^\infty e^{-\rho t} \left[ \chi(t) \frac{dR(t)}{d\phi_l} + |M_S(R(t), S(t))| \frac{dS(t)}{d\phi_l} \right] dt, \quad i, l = 1, 2. \end{aligned} \quad (16)$$

The first line establishes the result of Hoel (2011), with the first two terms reflecting the distortions in the economy caused by the tax and the subsidy if climate effects are ignored. The third term is a pure terms-of-trade effect. The sum of the terms-of-trade terms of the high-tax and the low-tax country equal zero, since  $\sum_i \beta_i = 1$ . Following Hoel (2011), we ignore the terms-of-trade term in the subsequent analysis. The fourth term gives the change of the climate costs of country  $i$ . The term in the second line of (16) reflects the stock dependence of extraction costs. The sign of the term in brackets is ambiguous. E.g. if the extraction utilization path is flattened (steepened) by the tax increase, it follows that  $\frac{dR(t)}{d\phi_l} < (>) 0$  in early periods and  $\frac{dR(t)}{d\phi_l} > (<) 0$  in later periods, whereas  $\frac{dS(t)}{d\phi_l} > (<) 0$ . However, if fossil fuel costs are independent of the stock, one gets  $M_S = 0$  and  $\chi(t) = \chi_0 e^{\rho t}$  so that the term in the second line equals zero. Therefore, if the stock dependence of extraction costs is sufficiently weak, the term in the second line can be ignored. Furthermore, two other important implications follow from a minor influence of the stock on extraction costs. As already stated in Lemma 1, the initial scarcity is decreased by the tax hike. Secondly, a small  $|M_S|$  implies that the growth rate of the scarcity rent  $\hat{\chi} = \rho + \frac{M_S}{\chi}$  approximates  $\rho$  with and without the tax hike. Taking into account that the global termination price  $N - \phi_2 - \sigma_2$  is not affected by a unilateral tax hike of country 1, we conclude that the extraction period is extended.

If the stock dependence of extraction costs is weak, the first term in the upper line of (16) is positive for the low-tax country, since the fossil fuel price at  $t = 0$  decreases, while the country's termination price  $N - \phi_2 - \sigma_2$  is not affected. As a consequence, more fossil fuel is used. For the high-tax country the sign of the first term of (16) depends on the reaction of early fossil fuel use. It is positive in the case of sufficiently increasing early utilization (weak green paradox). If no weak green paradox occurs, the term is unambiguously negative. The effect of the tax hike on backstop utilization is reciprocal to the one on fossil fuel utilization for the low-tax country, since earlier and more fossil fuel utilization implies later and lower backstop utilization. Thus, the second term has the same sign as

the first one. For the high-tax country the sign is ambiguous. E.g. even if a weak green paradox occurs, backstop utilization may begin at an earlier point in time, as the critical price  $N - \phi_1 - \sigma_1$  is reduced. The last term in the upper line of (16) reflects the effect of a tax increase on social carbon costs. If the conditions (13) and (14) do not hold at any point in time, the fossil fuel utilization path is flattened, which decreases social carbon costs. If the stock dependence of extraction costs is sufficiently weak, (15) needs to hold, too. Since the conditions in the cells 2x2 and 2x3 of Table 2 contradict (15), backstop utilization by the high-tax country before the tax hike prevents a rise of social carbon costs, i.e. a strong green paradox. On the other hand, a weak green paradox is possible, if the high-tax country uses only fossil fuel before the tax hike so that early extraction is boosted and social carbon costs may increase. Thus, the welfare of the low-tax country is increased by a higher tax of the high-tax country, if no weak green paradox occurs. However, the welfare effect for the high-tax country is unclear. The welfare effect is also unclear, if either a weak green paradox occurs or the stock dependence of the extraction costs is significant.

By using (2) and ignoring the stock dependence of extraction costs we can replicate the effect found by Hoel (2011) of an tax increase on total welfare  $W_1 + W_2$ . It reads

$$\begin{aligned} \frac{d(W_1 + W_2)}{d\phi_l} &= \int_0^{\infty} e^{-\rho t} \left[ \phi_1 \frac{dR^1(t)}{d\phi_l} - \sigma_1 \frac{db^1(t)}{d\phi_l} \right] dt + \int_0^{\infty} e^{-\rho t} \left[ \phi_2 \frac{dR^2(t)}{d\phi_l} - \sigma_2 \frac{db^2(t)}{d\phi_l} \right] dt \\ &\quad - v \int_0^{\infty} e^{-\rho t} \frac{dR(t)}{d\phi_l} dt, \quad l = 1, 2. \end{aligned} \quad (17)$$

However, as it is ambiguous how the welfare of the high-tax country reacts to an increase of  $\phi_1$ , not much can be said about the change of total welfare. Nevertheless, the positive welfare effect of  $d\phi_1 > 0$  for the low-tax country without a weak green paradox implies that total welfare may increase and that the low-tax country can (partly) offset the potential welfare losses of the high-tax country.

**Proposition 1** *Suppose the high-tax country increases its fossil fuel tax unilaterally.*

(a) *Then a weak green paradox occurs, if only fossil fuel is used, the pre-tax hike global fossil fuel demand function is sufficiently inelastic and the increased tax does not completely drive out fossil fuel from the high-tax country.*

(b) *Then a strong green paradox does not occur, if backstop is used by the high-tax country before the tax hike and the stock dependence of extraction costs is sufficiently weak.*

(c) *Then welfare of both countries declines, if the social carbon costs are sufficiently high, the stock dependence of extraction costs is sufficiently weak and early emissions increase sufficiently under the conditions of Proposition 1(a).*

The above analysis is easily applied to the case of a common tax rate and a common subsidy. Assume that  $ABCDE$  in Fig. 4 is the pre-tax hike energy demand curve. Then the post-tax hike demand is given by a downward shifted curve as on the left-hand side of Fig. 3. Thus, the relevant combinations of market clearing conditions are given by  $(I), (I_p.A), (II), (II_p.A), (V), (V_p.C)$  and  $(VI), (VI_p)$  in Table 1. The inequality associated with the combination  $(I), (I_p.A)$  reduces to  $\frac{dX_p}{dt} > \frac{dX}{dt}$ , since a higher common tax does not affect the allocation of resources among the countries. Therefore,  $U_{xx}^i(R^i(t)) = U_{xx}^i(R_p^i(t))$ ,  $i = 1, 2$  and consequently  $(h^{-1}(R(t)))' = (h_p^{-1}(R_p(t)))'$ . The inequality for the other three combinations also reads  $\frac{dX_p}{dt} > \frac{dX}{dt}$ . Thus, none of the inequalities hold, as  $\frac{dX_p}{dt} < \frac{dX}{dt}$ . This implies that a common tax hike cannot cause a weak green paradox. The length of the extraction period is affected in two ways. On the one hand, the higher common tax decreases the termination price  $N - \phi - \sigma_2$ , which reduces the extraction period. On the other hand, a lower initial scarcity rent implies a longer period. Together with the impossibility of a weak green paradox, the lower initial scarcity rent connotes a flatter extraction path and therefore lower social carbon costs, if the extraction paths intersect once. If the utilization period is shortened, the post-tax hike extraction path may be completely below the pre-tax hike path or intersect it at least twice. In the first case less fossil fuel is used and the stock left in situ increased. Consequently, social carbon costs are lower. In the second case extraction is lower in early and late periods but higher in the mid-term. Thus, social carbon costs increase, if the mid-term emissions are increased sufficiently. However, at the last intersection of the extraction paths the post-tax hike path needs to intersect the pre-tax hike path from above. Provided that the stock dependence of extraction costs is sufficiently weak, this requires (13), (14) and (15) to hold. As the above analysis shows, this is never the case.

Under the assumption of a weak stock dependence of extraction costs and no backstop subsidies the total welfare effect of a common tax increase reads

$$\frac{d(W_1 + W_2)}{d\phi} = (\phi - v) \int_0^{\infty} e^{-\rho t} \frac{dR(t)}{d\phi} dt. \quad (18)$$

Thus, a higher tax increases welfare as long as the tax falls short of its Pigovian level  $v$ .

**Proposition 2** *Suppose a common fossil fuel tax is increased.*

- (a) *Then a weak green paradox cannot occur.*
- (b) *Then a strong green paradox does not occur, if the stock dependence of extraction costs is sufficiently weak.*

(c) Then welfare in both countries increases, if the common fossil fuel tax falls short of its Pigovian level, no backstop subsidies are granted and the stock dependence of extraction costs is sufficiently weak.

Proposition 2 shows that the similar result of Hoel (2011) does not only hold for linear extraction costs without a stock dependence but can be generalized to the case of progressively increasing costs with a sufficiently weak stock dependence.

### Tax increase in the low-tax country

We turn now to the case of a tax increase by the low-tax country. We begin our analysis again with exclusive fossil fuel utilization before the tax hike. The corresponding condition for a weak green paradox - the combination (III) and (III<sub>p</sub>) in Table 1 - is the same as for the combination (I) and (I<sub>p</sub>.A) already discussed. I.e. a weak green paradox does not occur, unless the pre-tax hike global demand is sufficiently inelastic.

If the high-tax country uses both energy sources before the tax hike, the conditions for a weak green paradox are  $\frac{dX_b}{dt} > \frac{M_{RR} - (h_p^{-1})'}{M_{RR}} \frac{dX}{dt} + |M_{RS}| \left[ \frac{M_{RR} - (h_p^{-1})'}{M_{RR}} - 1 \right]$  and  $\frac{dX_p}{dt} > \frac{dX}{dt}$ . As pointed out above, the latter inequality never holds. This is also true for the former inequality, since  $(h_p^{-1})' < 0$ .

The next case to be analyzed is given by the combinations of (V) and (V<sub>p</sub>.i),  $i = A, B, C$ . Here the low-tax country completely relies on fossil fuel before the tax hike, whereas the high-tax country uses only backstop. The first inequality for a weak green paradox is satisfied, if  $|(h_p^{-1})'(R(t))| < |U_{xx}^2(R^2(t))|$ , i.e. if the demand function of country 2 is steeper than the global demand function after the tax hike. If the fossil fuel demand function of country 2 is sufficiently inelastic, the second inequality holds, too. However, the third inequality can never be fulfilled, as it requires  $\frac{dX_p}{dt} > \frac{dX}{dt}$ . Since  $\frac{dX_p}{dt} < \frac{dX}{dt}$  is true, a weak green paradox can be ruled out, too, if the backstop is utilized by the low-tax country before the tax hike.

For the welfare analysis we assume again that the influence of the fossil fuel stock on extraction costs is small. In this case, the unilateral tax increase of the low-tax country decreases the initial scarcity rent. However, the global termination price  $N - \phi_2 - \sigma_2$  decreases, too. Therefore, it is unclear whether the extraction period is extended or shortened and if the social carbon costs rise or fall. However, the lower termination price may leave a higher stock unexploited, which has a decreasing effect on social carbon costs no matter whether a weak green paradox occurs. If the post-tax hike extraction path



is completely below the pre-tax hike path, the social carbon costs are unambiguously lowered. In case of a non-affected stock left in situ the result concerning the social carbon costs is still ambiguous. On the one hand, there cannot be a weak green paradox, if backstop is used by the low-tax country before the tax hike, which may lead to  $T_{Rp} \geq T_R$  and therefore to lower social carbon costs. On the other hand, a possible weak green paradox and a shortened extraction period imply higher social carbon costs. Furthermore, the utilization paths can intersect more than once, connoting an ambiguous social carbon costs effect. However, the uncertainty about the impact of a tax hike by the low-tax country contrasts with proposition 1 from Hoel (2011), which requires only a sufficiently inelastic fossil fuel demand function for increased social carbon costs. Hoel's result is partly replicated by our results, since a weak green paradox requires a sufficiently inelastic pre-tax hike fossil fuel demand function. Nonetheless, we find that backstop utilization by the low-tax country before the tax hike prevents a weak green paradox. If the stock dependence of extraction costs is sufficiently weak, a strong green paradox is also ruled out, as (13), (14) and (15) never hold which implies either a longer extraction period or lower total extraction. This disparity highlights the impact of extraction costs which progressively increases in the resource flow.

Obviously, since the effect of the low-tax country's tax hike on social carbon costs is ambiguous not much can be said about the effect of the tax hike on both the welfare of individual countries and total welfare, as the sign of the last term in the upper line of (16) is unclear. However, Hoel's (2011) result that the increased tax will decrease welfare of both countries, if social carbon costs are sufficiently high and a weak green paradox occurs, can be confirmed here only under certain conditions. On the one hand, a weak green paradox increases early emissions. On the other hand, the tax hike may boost the amount of the stock left in situ, which decreases social carbon costs. Thus, if the total amount of extracted fossil fuel is decreased sufficiently by the tax increase and social carbon cost are sufficiently high, welfare of both countries increases even in the presence of a weak green paradox. Therefore, the consideration of stock dependent extraction costs may reverse Hoel's (2011) result.

**Proposition 3** *Suppose the low-tax country increases its fossil fuel tax.*

(a) *Then the weak green paradox occurs,*

(a1) *if both countries use only fossil fuel before the tax hike and the pre-tax hike global demand function is sufficiently inelastic.*

(a2) *if only the low-tax country uses fossil fuel before the tax hike but both countries*

afterwards and the pre-tax hike demand function of the low-tax country is sufficiently inelastic.

(b) Then a strong green paradox cannot occur, if the low-tax country uses backstop before the tax hike and the stock dependence of extraction costs is sufficiently weak.

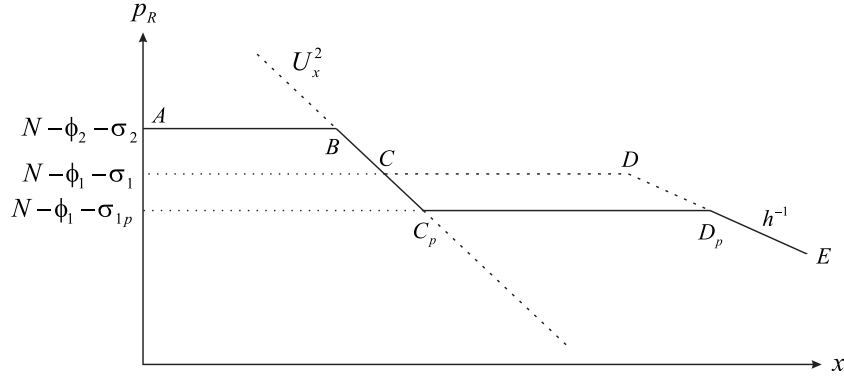
(c) Then total welfare

(c1) declines, if the conditions of Proposition 3 (a1) or (a2) hold, early emissions increase sufficiently, the social carbon costs are sufficiently high and the stock dependence of extraction is sufficiently weak.

(c2) increases, if the fossil fuel stock left in situ increases sufficiently and social carbon costs are sufficiently high.

### 3.2. Increasing backstop subsidy

While a carbon tax is a well founded climate policy instrument, backstop subsidies are also often justified with climate concerns.<sup>24</sup> Therefore, this section focuses on the subsidy  $\sigma_i$ ,  $i = 1, 2$  which is paid for every backstop unit supplied. Fig. 7 and Fig. 8 illustrate the effect of  $d\sigma_i > 0$  on global fossil fuel demand, if the subsidy is increased by the high-tax country or the low-tax country, respectively. Table 3 summarizes the combinations of market equilibria which may lead to a weak green paradox.

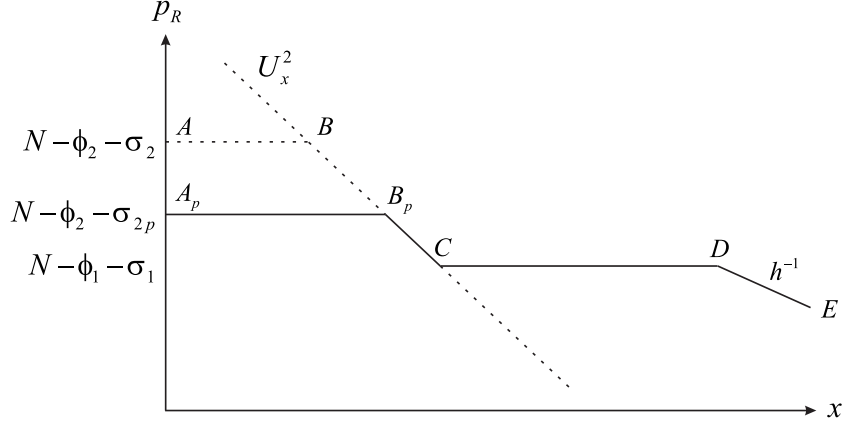


**Figure 7:** Energy demand before and after an increase of the subsidy rate  $\sigma_1$

If the high-tax country increases its subsidy, the conditions for (13) and (14) to hold are given by the cells 1x2, 1x3, 2x2 and 2x3 of Table 2. For the corresponding conditions of a subsidy hike by the low-tax country we get the cells 3x2 and 2x2.

As discussed above, only the first condition for a subsidy hike by the high-tax country holds, if the pre-subsidy hike demand function is sufficiently inelastic. The other three conditions never hold. Thus, if the high-tax country increases its subsidy, a weak green

<sup>24</sup>For example, the German "Erneuerbare Energien Gesetz" (EEG), which stipulates subsidies for suppliers of renewable energies, explicitly refers to climate protection as a motivation for the law (§1 EEG).



**Figure 8:** Energy demand before and after an increase of the subsidy rate  $\sigma_2$

	pre-tax hike	post-tax hike
$d\sigma_1$	$DD_p \Rightarrow \chi + M_R(R(t), S(t)) = h^{-1}R(t)$ ( $I^\sigma$ )	$C_pD_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_1 - \sigma_{1p}$ ( $I_p^\sigma.A$ ) $CC_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = U_x^2(R_p(t)) - \phi_2$ ( $I_p^\sigma.B$ )
	$CD \Rightarrow \chi + M_R(R(t), S(t)) = N - \phi_1 - \sigma_1$ ( $II^\sigma$ )	$C_pD_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_1 - \sigma_{1p}$ ( $II_p^\sigma.A$ ) $CC_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = U_x^2(R_p(t)) - \phi_2$ ( $II_p^\sigma.B$ )
$d\sigma_2$	$BB_p \Rightarrow \chi + M_R(R(t), S(t)) = U_x^2(R(t)) - \phi_2$ ( $III^\sigma$ )	$A_pB_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_2 - \sigma_{2p}$ ( $III_p^\sigma$ )
	$AB \Rightarrow \chi + M_R(R(t), S(t)) = N - \phi_2 - \sigma_2$ ( $IV^\sigma$ )	$A_pB_p \Rightarrow \chi_p + M_R(R_p(t), S_p(t)) = N - \phi_2 - \sigma_{2p}$ ( $IV_p^\sigma$ )

**Table 3:** Comparative statics of unilateral subsidy rate increases

paradox can only occur, if the high-tax country does not use the backstop before the subsidy hike and the increased subsidy does not completely drive out fossil fuel of the high-tax country. Notice that a weak green paradox necessarily occurs, if the pre-subsidy hike equilibrium is located to the right of  $D_p$  in Fig. 7. Since a weak green paradox is ruled out for all equilibria to the left of  $D$ ,  $DD_p$  is the only segment where the occurrence of a weak green paradox is ambiguous. As the termination price of the low-subsidy country  $N - \phi_2 - \sigma_2$  remains unchanged, the total amount of fossil fuel used is not affected by the increased subsidy. If the stock dependence of the extraction costs is sufficiently weak,  $\chi_{0p} < \chi_0$  implies that the extraction period is extended. Thus, a higher backstop subsidy of the high-tax country decreases social carbon costs, i.e. prevents a strong green paradox, provided that no weak green paradox is caused.

If the low-tax country increases its subsidy rate, the cells  $3 \times 2$  and  $2 \times 2$  of Table 2 contain the conditions for a weak green paradox. Obviously, the condition in cell  $2 \times 2$  contradicts  $\frac{d\chi_p}{dt} < \frac{d\chi}{dt}$ . However, the condition in cell  $3 \times 2$  holds, if the demand function of country 2 is sufficiently inelastic. Thus, a weak green paradox does not occur, if the low-tax country uses backstop before increasing its subsidy, but may occur, if the low-tax

country uses only fossil fuel. Furthermore, Fig. 8 reveals that a weak green paradox occurs, if the market equilibrium before the subsidy hike is located to the right of  $B_p$ . Thus, the area of uncertainty is here the segment between  $B$  and  $B_p$ .

As in the case of a tax hike by the low-tax country, the effect on the social carbon costs is ambiguous, since both the termination price and the initial scarcity rent decrease. Thus, social carbon costs are lower, if the conditions (13) and (14) do not hold, which implies either a longer extraction period and/or a reduction of the aggregated extraction. This is the case, if the backstop is used before the subsidy hike and stock dependence of extraction costs is weak. If there is only one intersection between the extraction paths, the stock left in situ is the same and a weak green paradox occurs, social carbon costs increase and the extraction period is shorter. With two or more intersections, the effect of  $d\sigma_2 > 0$  on social carbon costs is unclear. Since only the second case is in line with the unambiguous result of Hoel (2011), our assumptions on extraction costs that are more general than those of Hoel lead to a considerable degree of ambiguity.

A higher subsidy by the low-tax country always causes an increase of early emissions, if the high-tax country uses fossil fuel before the subsidy hike. Nonetheless, even in this case, the subsidy hike lowers the social carbon costs, if it increases the stock left in situ sufficiently.

**Proposition 4** *Suppose country  $i = 1, 2$  increases its backstop subsidy.*

(a) *A weak green paradox occurs,*

(a1) *if country  $i$  uses only fossil fuel before and after the subsidy hike.*

(a2) *if country  $i$  uses only fossil fuel before the subsidy hike and the relevant pre-subsidy hike demand function is sufficiently inelastic.*

(b) *Then a strong green paradox cannot occur, if country  $i$  uses backstop before the subsidy hike and the stock dependence of extraction costs is sufficiently weak.*

To analyze the effect of higher subsidies on welfare, we differentiate (12) with respect to  $\sigma_i$ ,  $i = 1, 2$ . By ignoring the terms-of-trade effect and assuming a sufficiently weak stock dependence of extraction costs, we get

$$\frac{dW_i}{d\sigma_l} = \phi_i \int_0^{\infty} e^{-\rho t} \frac{dR^i(t)}{d\sigma_l} dt - \sigma_i \int_0^{\infty} e^{-\rho t} \frac{db^i(t)}{d\sigma_l} dt - \alpha_i \frac{d\Phi}{d\sigma_l}, \quad i, l = 1, 2. \quad (19)$$

Since a unilateral subsidy hike of either the high-tax or low-tax country can decrease or increase social carbon costs, the effect on the welfare of both countries is ambiguous. However, the first two terms are positive for the passive country, which is the country

whose subsidy remains constant, as the lower initial scarcity rent and the constant termination price of the passive country imply earlier and higher overall fossil fuel utilization by this country. Consequently, the backstop utilization is delayed. Thus, even if the social carbon costs rise, welfare of the passive country may increase. In contrast, given a sufficiently small stock effect on extraction costs, the second term is negative for the subsidy increasing country.<sup>25</sup> The sign of the first term depends on the occurrence of a weak green paradox. These results are in line with those of Hoel (2011).

**Proposition 5** *Suppose country  $i = 1, 2$  increases its backstop subsidy.*

(a) *Then welfare net of social carbon costs increases in the passive country.*

(b) *Then welfare declines in both countries, if the social carbon costs are sufficiently high, the stock dependence of extraction costs sufficiently weak and early emissions increase sufficiently.*

To apply the analysis to the case of identical subsidies across countries the changes displayed in the figures 7 and 8 have to be combined. Thus, all market clearing combinations listed in Table 3 are relevant. Consequently, the conditions for a weak green paradox are given by the cells 1x2, 1x3, 2x2, 2x3 and 3x2 of Table 2. As shown above, only the first and last condition can hold. Since both the initial scarcity rent and the termination price  $N - \phi_2 - \sigma$  decrease, the effect on social carbon costs is ambiguous.

If both the fossil fuel tax and the backstop subsidy are the same across countries, Fig. 7 illustrates the changes of the demand curve, where  $(N - \phi_1 - \sigma_1)DE$  is the demand before and  $(N - \phi_1 - \sigma_{1p})D_pE$  is the demand after the subsidy hike. Since the combinations of the market clearing conditions are given by  $(I^\sigma)$ ,  $(I_p^\sigma.A)$  and  $(I^\sigma)$ ,  $(I_p^\sigma.B)$ , the result replicates the result for  $d\sigma_1 > 0$ . Provided that the stock left in situ is not affected by the reduced termination price, this result has significant consequences for the length of the extraction period. A shorter period is then only possible, if the conditions (13) and (14) hold at least once. Consequently, backstop utilization before the subsidy hike and a sufficiently weak stock dependence of extraction costs imply that an increase of a common subsidy extends the extraction period and decreases social carbon costs.

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<sup>25</sup>To see this, assume the opposite, i.e. that early backstop utilization declines. Due to the assumption of a small stock effect  $M_R(R(t), S(t))$  can be written as  $M_R(R(t))$ . Therefore, the reduced backstop utilization after the subsidy hike implies that  $\chi_p(t_q) < \chi(t_q)$  and  $\chi(t_q) + M_R(R(t_q)) = N - \phi_i - \sigma_i$  hold, at some point in time  $t_q$ . This implies  $R_p(\tau) > R(\tau)$ ,  $\forall \tau \leq t_q$ . To comply with  $\int_0^\infty R_p(t)dt \leq S(0) - S(T_R)$ , the conditions for a weak green paradox need to hold after  $t_q$ . However, at  $t_q$  the backstop would be used by the subsidy increasing country without the subsidy hike. Thus, according to proposition 4, a weak green paradox is ruled out.

Under the assumptions of a common fossil fuel tax and a common backstop subsidy, the effect of a higher subsidy on total welfare is given by

$$\frac{d[W_1 + W_2]}{d\sigma} = -\sigma \int_0^{\infty} e^{-\rho t} \left[ \frac{db^1(t)}{d\sigma} + \frac{db^2(t)}{d\sigma} \right] dt + (\phi - v) \int_0^{\infty} e^{-\rho t} \frac{dR(t)}{d\sigma} dt. \quad (20)$$

The first term of (20) is negative, while the sign of the last term depends on the tax rate and the development of emissions. Provided the conditions (13) and (14) do not hold, the last integral is negative. Thus, if the tax rate falls short of the Pigovian tax rate  $v$ , a higher subsidy may increase social welfare. This result contrasts with Hoel's (2011). Furthermore, our result is attributed to the simultaneous use of both resources and highlights therefore the impact of extraction costs that are increasing and strictly convex in the resource flow. If a weak green paradox occurs and is sufficiently strong to offset for decreasing late emissions, the higher subsidy decreases total welfare. As already stressed by Hoel (2011) and Van der Ploeg and Withagen (2012), a tax on the backstop increases welfare in this case.

**Proposition 6** *Suppose a common backstop subsidy is increased and the common fossil fuel tax is below the Pigovian level. Then welfare in both countries increases, if the social carbon costs are sufficiently high, the stock dependence of extraction costs sufficiently weak and the conditions for a weak green paradox do not hold.*

### 3.3. Lower backstop costs

Notice that the cells 1x2, 1x3, 2x2, 2x3 and 3x2 of Table 2 also contain the conditions for a weak green paradox caused by a reduction of backstop costs, i.e.  $dN < 0$ . If both taxes and subsidies are identical across countries, the last two cells are not relevant any more. It follows that backstop utilization before the backstop cost reduction prevents a weak green paradox. If in this case the stock dependence of extraction costs is sufficiently weak, either the extraction period is extended or the stock left in situ increased. In both cases social carbon costs are decreased. This result contrast with Hoel's (2011). This seems to be of special interest, as Hoel's (2011) result is in line with the common theory of non-renewable resources.<sup>26</sup> The result concerning welfare is also affected. By adding up  $\frac{dW_1}{dN}$  and  $\frac{dW_2}{dN}$  under the assumptions of a sufficiently weak stock dependence of extraction costs and  $\sigma = 0$  we get

$$\frac{d[W_1 + W_2]}{dN} = - \int_0^{\infty} e^{-\rho t} [b^1(t) + b^2(t)] dt + [\phi - v] \int_0^{\infty} e^{-\rho t} \frac{dR(t)}{dN} dt. \quad (21)$$

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<sup>26</sup>Cf. Hoel (2011), page 857.

The first term is clearly negative, while the sign of the last term depends on the tax rate and on the effect of lower backstop costs on fossil fuel utilization. With  $\phi < v$  welfare declines when backstop costs decrease provided that a weak green paradox occurs. That result is in line with Hoel (2011). However, if backstop is used before the drop of  $N$  and stock dependence of extraction costs weak, the last integral is clearly negative and welfare may increase.

**Proposition 7** *Suppose backstop costs decline, the common fossil fuel tax is below the Pigovian level and no subsidies are granted. Then welfare may increase in both countries, if the stock dependence of extraction costs is sufficiently weak and backstop is used before the drop of backstop costs.*

#### 4. Conclusion

This paper focuses on how unilateral climate policies are affecting the fossil fuel extraction path and social welfare. For this purpose we have extended the model of Hoel (2011) by fossil fuel extraction costs that are progressively increasing in the resource flow and stock-dependent. The flow dependence of extraction costs leads to simultaneous utilization of both resources, which makes results considerably more ambiguous compared to Hoel's (2011) and may even reverse them. Provided that the low-tax country does not use backstop, we can support Hoel's (2011) result that a tax or subsidy increase by the low-tax country causes a weak green paradox, if the demand function is sufficiently inelastic. However, the utilization of the backstop by the low-tax country before the tax/subsidy hike prevents a weak green paradox. Similarly, a weak green paradox cannot be caused by a tax/subsidy hike of a common tax/subsidy or of the high-tax country, if backstop is used before the tax/subsidy hike. If the stock dependence of extraction costs is sufficiently weak, also a strong green paradox is ruled out in all mentioned cases.

Consequently, a higher subsidy of the low-tax country may increase welfare in this country, if backstop is used before the subsidy hike. This result is in contrast to the one of Hoel (2011), who finds that welfare will decrease. Concerning the increase of a common tax, our result supports Hoel's (2011), which says that the tax increase boosts welfare, provided that the tax falls short of its Pigovian level. In contrast to Hoel's (2011) findings a higher common subsidy can also increase welfare, if no weak green paradox occurs. A drop of the backstop costs will increase welfare, if backstop is used before the drop. This result contrasts with the theory of non-renewable resources.

The stock dependence of extraction costs complicates the analysis considerably and renders the welfare effects ambiguous. With respect to the weak green paradox, the results are not altered substantially. The stock dependence makes the paradox more likely. However, stock dependence gives rise to two interesting outcomes, which cannot occur in Hoel's (2011) model. Firstly, due to a strong effect of the resource stock on extraction costs the scarcity rent may increase rather than decrease following a tighter unilateral climate policy such that a weak green paradox is ruled out. Secondly, a tighter climate policy of the low-tax country can decrease the aggregated extraction. Provided that reduction is sufficiently large, welfare increases even if the tighter climate policy causes a weak green paradox. Thus, extraction costs which are progressively increasing in the resource flow and stock dependent give rise to a more optimistic view of a tighter unilateral climate policy of countries with currently rather lax policies.

However, there are numerous industrialized countries that already use renewable energy sources and follow a strict climate policy. Thus, the most realistic setting seems to be that of a high-tax country using both fossil fuel and backstop combined with a low-tax country using fossil fuel only. In this case, an increase of the fossil fuel tax by the low-tax country may cause a weak green paradox, while a higher backstop subsidy definitely causes the paradox. Thus, despite the fact that our result rests on the assumption of linear backstop costs, it suggests that high-tax countries should react to global warming to prevent a weak green paradox. Furthermore, we show that welfare of the low-tax country increases, while the welfare effect for the high-tax country is ambiguous. However, the higher welfare of the former country implies that the latter can be at least partly compensated for potential welfare losses.

While this paper has assumed more complex extraction costs, it adopts the other simplifying assumptions from Hoel (2011). These are in particular linear backstop costs, perfect substitutability of backstop with fossil fuel and only a single type of fossil fuel. Relaxing these assumptions may alter our results considerably. For instance, Chakravorty et al. (2008) show how the utilization order of different fossil fuels is affected by a climate target. Considering progressively increasing, flow dependent backstop costs or the backstop as an imperfect substitute for fossil fuel may boost extraction and increase the energy price.<sup>27</sup> Furthermore, our assumptions rule out discontinuous extraction paths.

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<sup>27</sup>Gerlagh (2011) shows in a one country model that a weak as well as a strong green paradox may



Furthermore, it seems promising to use parametric extraction costs functions, as these may yield more informative results.

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