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Unilateral reduction of medium-term carbon emissions via taxing emissions *and* consumption*

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Abstract

Internalizing the global negative externality of carbon emissions requires flattening the extraction path of non-renewable fossil-fuel resources (= world carbon emissions). Following Eichner and Pethig (2011b) we set up a two-country two-period model in which one of the countries represents a sub-global climate coalition that implements a binding *ceiling* on the world's first-period emissions. The other country is the rest of the world and refrains from taking action. The climate coalition has at its disposal sign-unconstrained taxes on emissions in both periods, as in Eichner and Pethig (2011b), but in the present study it has the additional option of taxing consumption. The central question is whether and how the coalition makes use of consumption taxes along with emission taxes in its unilateral *cost-effective* ceiling policy. We identify cost-effective policies under various conditions and find that all consist of a (positive) tax on first-period consumption and of emission taxes whose rates are negative in the second period but may take on either sign in the first period.

JEL classification: H21, H23, Q54, Q58
Key words: carbon emissions, ceiling, unilateral, cost-effective regulation

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1 The problem

Global warming is caused by anthropogenic greenhouse gas emissions, notably by carbon dioxide emissions from burning fossil fuels,[1] and generates excessive worldwide climate damage in the absence of mitigation policies. Literature on internalizing such externalities (e.g. Sinclair 1992, Sinn 2008) finds that maximizing world welfare requires flattening the world carbon extraction path (= carbon emissions path). In the no-policy scenario too much carbon is emitted too early. As climate damage reduction is a global public good, global cooperation is a precondition for efficient action. Unfortunately, the prospects for cooperation are bleak in view of the poor progress made in international climate negotiations over the last decades. We therefore believe that if international cooperation will be achieved at all in the medium term, it will be incomplete and will go for pragmatic climate goals. A plausible course of events is that a *sub-global* climate coalition will be formed, rather than a global one, implementing a *ceiling* on cumulative medium-term world emissions that flattens the carbon extraction path.

Following Eichner and Pethig (2011b) the present paper aims to study such ceiling policies of a sub-global climate coalition,[2,3] in an effort to account for, in a very stylized way though, a likely outcome of the ongoing international climate negotiations. The political goal of keeping the world mean temperature from rising by 2° Celsius or more above preindustrial levels has been endorsed by numerous governments and most recently also by the UN Conference of the Parties in Cancun (UNFCCC 2010). We interpret that as an endorsement of the target to flatten the world carbon extraction path. As in Eichner and Pethig (2011b) we set up a two-country two-period model where one country, say country A, represents the sub-global climate coalition and the other country, say country B, stands for the rest of the world and is assumed to refrain from climate policy. Each country is endowed with a stock of fossil fuel which will be fully depleted at the end of the second period whether or not country A takes action. In that regard the model is in line with Sinclair (1992), Sinn (2008), Hoel (2010), Michielsen (2011) and others.[4] There are world markets for fossil fuel

[1] In the present paper we disregard greenhouse gases other than carbon dioxide.

[2] For further motivation of that problem see also Eichner and Pethig (2011b)

[3] Our approach is not about optimal policies whose study would require accounting for climate damage in the model. Implicitly, we presuppose that the coalition is capable of implementing a ceiling that enhances the coalition's welfare compared to the no-policy scenario.

[4] We take that route because we find plausible the argument of Sinn (2008, p. 376) that public policies are unlikely to succeed in leaving part of the world's total stock of fossil fuels in the ground forever. Note, however, that there is another strand of literature focussing on climate policy instruments capable to impact on the level of (total) cumulative extraction of fossil fuel resources. See e.g. van der Ploeg and Withagen (2010), Grafton et al. (2010), Kalkuhl and Edenhofer (2010) and Gerlagh (2011).

and for a consumption good in both periods. Country A makes use of its tax instruments as to prevent the first-period world emissions from exceeding some politically fixed level, called the *ceiling*. The key issue is then to characterize that particular unilateral climate policy which implements the predetermined ceiling at least cost for country A.

The present paper extends the analysis of Eichner and Pethig (2011b) by providing government A with the option of taxing consumption *in addition to* taxing emissions. Given the larger policy space the principal question to be answered is whether it is *cost-effective* for country A to choose a ceiling policy consisting of taxes on both emissions *and* consumption - and if so, what the role of consumption taxes is in the cost-effective policy mix. We will show that by taxing consumption country A can, indeed, reduce the costs of its policy below the level attainable through emission taxes only. More specifically, all cost-effective unilateral ceiling policies that we will determine under various parameter constellations consist of (i) a positive tax on first-period consumption and (ii) of emission taxes in both periods whose rate is negative in the second and may be positive or negative in the first period.

This paper is related to the literature on carbon leakage that deals with the issue that one country's unilateral mitigation policy increases the emissions in other countries. The so-called green paradox or excessive leakage (Sinn 2008, Eichner and Pethig 2011a) is said to occur when unilateral emission reductions increase rather than reduce aggregate world emissions, as compared to their level in the no-policy scenario.[5] Hoel (1991), Bohm (1993), Golombek and Hoel (2004), Copeland and Taylor (2005), Di Maria and van der Werf (2005), Ishikawa and Kiyono (2006), van der Ploeg and Withagen (2010), Eichner and Pethig (2011a) have analytically explored various channels and determinants of carbon leakage and/or the green paradox. Although unilateral ceiling policies need to cope with leakage, it is a side aspect only of these policies because the coalition is able to implement the ceiling by assumption. That means that the coalition is assumed to succeed in avoiding excessive leakage by making intelligent use of its tax instruments. Yet ceiling policies may involve leakage at rates less than 100%, and that leakage increases the coalition's costs of implementing the ceiling. Our concept of ceiling policy is related to Chakravorty et al. (2006) and Kalkuhl and Edenhofer (2010) who study cost-effective (cooperative) so-called carbon budget policies. However, the common feature of these and most other studies referred to above is that they study *one-country* growth models.

The literature discussed in the preceding paragraph deals with emission taxes (prices) without addressing the impact of additional distortionary taxes, such as the consumption tax, on mitigation policies. Environmental policies with overlapping distortionary taxes are studied in the double-dividend literature initiated by Bovenberg and de Mooji (1994).

[5]For various related concepts of green paradox see Gerlagh (2011).

However, that literature is driven by the absence of neutral taxes and the government's revenue-raising requirement, while in our approach no such requirement applies and the government can refrain from imposing the distortionary consumption tax. Another strand of literature, e.g. Fullerton and Wolverton (2005), studies the regulation of pollution through a combination of consumption taxes and input subsidies in second-best scenarios in which emission taxes cannot be imposed due to enforceability or measurement problems. Yet in the present paper carbon emissions can be costlessly monitored and taxed and will indeed turn out to be part of the cost-effective policy. Closer to the topic of the present paper is the CGE approach of Elliot et al. (2010) who study various scenarios of taxing production and/or consumption of an energy-intensive good with international trade in final goods. Their focus is on leakage and they identify as crucial the differences in the incidence of taxes on production and consumption in scenarios of trade and unilateral taxation. However, they tax output rather than carbon emissions from burning fossil fuel, as we do, and they do not model the intertemporal allocation of fossil fuel resources.

Why is it that in the framework of our analysis the cost-effective unilateral ceiling policy consists of taxes on consumption as well as on emissions although the sub-global climate coalition neither faces a constraint on monitoring and enforcing emission taxes nor is forced to use the distortionary consumption tax? To answer this question, we first show that there are two alternative first-best 'cooperative' ceiling policies. One consists of a uniform tax on total first-period emissions and the other consists of a uniform tax on total first-period consumption. For the global coalition these two types of cost-effective policies are equivalent, and they cannot be improved upon by mixing consumption and emission taxes. However when we impose the constraint that country B does not engage in climate policy, country A's unilateral taxes on emissions and/or consumption (in both periods) cannot achieve the first-best solution anymore. The crucial feature of *unilateral* ceiling policy is that country A's goal is to reduce total first-period emissions, i.e. to meet the ceiling, without having (direct) control over country B's first-period emissions. Nonetheless, provided that the ceiling is not too low and country A is not too small we show that country A is capable of meeting the ceiling *either* via taxing emissions only *or* via taxing consumption only *or* via a mix of emission and consumption taxes. In all cases the challenge is to shift emissions from the first into the second period which requires country A to choose the rates of available taxes such that its own first-period emissions decline and that country B's incentives to expand its first-period emissions (leakage) remain low. If only emission taxes are applied, that shift is typically accomplished by discouraging fuel consumption in the first-period via an emission tax and by encouraging fuel consumption in the second period via an emission subsidy. With consumption taxes only the shift can be brought about cost-effectively by taxing first-period consumption as well as by subsidizing second-period consumption. In mixed policies the shift of emissions is secured by combining the incentives and disincentives of both one-tax-instrument policies.

The reason why there are unilateral ceiling policies with mixed instruments that are less costly than policies without such a mix is related to the allocative distortions generated by these taxes. Emission taxes distort world production and consumption taxes distort world consumption. If both kinds of taxes are applied to meet the ceiling, tax rates can be smaller than in the case of policies applying only one kind of taxes. Since the distortions tend to increase progressively in tax rates, an appropriately chosen policy mix minimizes the climate coalition's total costs of implementing the ceiling.

The paper is organized as follows. Section 2 outlines the model. Section 3 begins with briefly investigating the benchmark of the fully cooperative cost-effective ceiling policy and then characterizes unilateral ceiling policies. Here the focus is first on the feasibility of unilateral ceiling policies before the core issue of unilateral cost-effective ceiling policies is studied. In the last part of Section 3 we add to insights of our analytical approach into the characteristics of cost-effective ceiling policies by discussing simple numerical examples for various parameter constellations. Section 4 concludes.

2 The model

Consider a two-period model[6] with two (groups of) countries A and B. In period $t = 1, 2$ each country $i = A, B$ produces the output x_{it}^s of the consumption good X with the input e_{it} of fossil fuel according to the increasing and strictly concave production function[7]

$$x_{it}^s = X^i\left(e_{it}\right) \qquad i = A, B. \tag{1}$$

Country i is endowed with the share $\alpha_i \bar{e}$ of the world stock of fossil fuel, $\bar{e} > 0$, where $\alpha_A = 1 - \alpha_B \in]0, 1[$. To economize on notation we envisage a single price taking fossil-fuel extracting firm jointly owned by both countries that sells the fuel to the producers of good

[6]Essentially, the model is the same as in Eichner and Pethig (2011a), even though we now deal with 2 rather than 3 countries. In Eichner and Pethig (2011a) the third country owns the entire stock of fossil fuel and lives on the revenues of exporting its fuel. In the present paper the residents of each of the 2 countries own a share of the world stock of fossil fuel. The two-period time horizon is chosen for reasons of analytical tractability. Eichner and Pethig (2011a) show that the model is robust with respect to extending the number of periods.

[7]In (1) the superscript s indicates quantities supplied. Upper case letters denote functions and subscripts attached to them indicate first partial derivatives. Note that the production functions (1) can be interpreted as being linear homogeneous in fossil fuel and in a domestic production factor, e.g. labor, which is in fixed supply.

X in both countries and transfers to their residents the shares α_A and α_B, respectively, of the resource rent.

The representative consumer of country i derives utility from consumption x_{i1} in period 1 and from x_{i2} in period 2 according to the intertemporal utility function

$$u_i = U^i\left(x_{i1}, x_{i2}\right) \qquad i = A, B, \tag{2}$$

which is increasing in both arguments and quasi-concave.

In each period, good X and fossil fuel are traded on perfectly competitive world markets (consisting of the countries A and B) at prices p_{xt} and p_{et}, respectively. For $t = 1, 2$ the market clearing conditions are

$$x^s_{At} + x^s_{Bt} = x_{At} + x_{Bt}, \tag{3}$$

$$e_t = e_{At} + e_{Bt}, \tag{4}$$

where e_t is the supply of the fossil-fuel extracting firm in period t. Obviously, the supplies e_t for $t = 1, 2$ need to satisfy the intertemporal constraint

$$\bar{e} = e_1 + e_2. \tag{5}$$

Carbon emissions result from burning fossil fuel. We take them to be proportional to fossil fuel and we use the letter 'e' for both carbon and fuel (after an appropriate choice of units). To regulate carbon emissions in both periods, the governments A and B have at their disposal two sets of instruments: (i) taxes on the consumption of good X and (ii) carbon emission taxes, where all taxes can be levied in both periods at sign-unconstrained rates.[8]

We denote by p_{xt} and p_{et} the prices on the world markets for the consumption good and fossil energy, respectively, in period $t = 1, 2$. Due to taxation, the consumer price of the consumption good is $p_{xt} + \tau_{it}$ and the producer price of fossil fuel is $p_{et} + \pi_{it}$. Thus, country i's fiscal policy is fully described by the tax rates $(\pi_{it}, \tau_{it})_{i=A,B;t=1,2}$.

In the competitive two-country economy (1) - (5) the profits of the price-taking firms are[9]

$$\Pi^i := \sum_t \left[p_{xt}X^i(e_{it}) - (p_{et} + \pi_{it})e_{it}\right] \qquad \text{for } i = A, B, \tag{6}$$

$$\Pi^F := \sum_t p_{et}e_t, \tag{7}$$

[8] As negative tax rates (=subsidy rates) will turn out to become relevant, carbon taxes are not equivalent to an emission-cap-and-trade system in the model at hand because prices on the market for emission permits are bound to be non-negative.

[9] The extraction of fossil fuel is assumed to be costless and the interest rate is zero. For the rationale (and admissibility) of setting equal to zero the interest rate see Eichner and Pethig (2011b).

and the first-order conditions of maximizing (6) and (7) read, respectively,

$$\pi_{it} = p_{xt} X^i_{e_{it}} - p_{et} \geq 0 \qquad \text{for } i = A, B \quad \text{and} \quad t = 1, 2, \tag{8}$$

$$p_{e1} = p_{e2} =: p_e. \tag{9}$$

The consumer maximizes utility (2) subject to her budget constraint

$$(p_{x1} + \tau_{i1}) x_{i1} + (p_{x2} + \tau_{i2}) x_{i2} = y_i, \quad \text{where } y_i := \Pi^{i*} + \alpha_i \Pi^{F*} + \sum_t (\pi_{it} e_{it} + \tau_{it} x_{it}). \tag{10}$$

In (10), Π^{i*} is the maximum profit of the firm in country $i = A, B$ and Π^{F*} is the maximum profit of the fossil-fuel extracting firm. The budget constraint can be rearranged to $\sum_t \left[p_{xt} \left(x^s_{it} - x_{it} \right) + p_e (\alpha_i \bar{e} - e_{it}) \right] = 0$ which turns out to be country i's intertemporal trade balance. Utility maximization yields

$$\frac{U^i_{x_{i2}}}{U^i_{x_{i1}}} = \frac{p_{x2} + \tau_{i2}}{p_{x1} + \tau_{i1}} \quad \text{for } i = A, B. \tag{11}$$

So far the policies (π_{it}, τ_{it}) for $i = A, B$ and $t = 1, 2$ have been introduced without specifying policy targets. As explained in the introduction and motivated in more detail in Eichner and Pethig (2011b) the climate policy goal is to keep total first-period emissions from exceeding some ceiling $\bar{e}_1$ where $\bar{e}_1$ is understood to be lower than total first-period emissions in the absence of regulation. The constraint

$$e_{A1} + e_{B1} = \bar{e}_1 \tag{12}$$

is supposed to be observed not only when both countries cooperate - a case that we briefly consider in the next section for benchmark purposes - but also when country B is inactive $(\pi_{Bt} = 0, \tau_{Bt} = 0)_{t=1,2}$ and country A goes it alone. In that case country A's challenge is to satisfy (12) although it has no (direct) control over e_{B1}.

3 Unilateral carbon ceiling regulation

Before we are going to study country A's unilateral ceiling policy it is useful to characterize the fully cooperative cost-effective ceiling policy for the purpose of later comparison.[10]

[10] The proofs of the Propositions 1(ia) and 1(ii) are due to Eichner and Pethig (2011b) where governments have at their disposal emission taxes but no consumption taxes. It is straightforward to prove Proposition (ib) by comparing the first-order conditions of the social planner's solution (Eichner and Pethig 2011b, Appendix B) with the first-order conditions of the market agents in Section 2 of the present paper.

Proposition 1 .

(i) The cooperative tax policy $(\pi_{it}, \tau_{it})_{i=A,B;t=1,2}$ *implements the ceiling* $\bar{e}_1$ *cost-effectively,*

(a) either if $\tau_{A1} = \tau_{B1} = \tau_{A2} = \tau_{B2} = 0$, $\pi_{A2} = \pi_{B2} = 0$ *and* $\pi_{A1} = \pi_{B1} = \bar{\mu}$, *where* $\bar{\mu} > 0$ *is the shadow price of the ceiling constraint* $e_{A1} + e_{B1} = \bar{e}_1$

(b) or, alternatively, if $\pi_{A1} = \pi_{B1} = \pi_{A2} = \pi_{B2} = 0$, $\tau_{A2} = \tau_{B2} = 0$ *and* $\tau_{A1} = \tau_{B1} = \frac{\bar{\mu}}{p_e - \bar{\mu}} > 0$.

(ii) The implementation of the ceiling by means of the policies of Proposition 1(i) distorts the allocation (compared to the no-policy equilibrium) by driving a wedge between the marginal rates of intertemporal substitution in production and consumption:

$$\frac{U^i_{x_{i2}}}{U^i_{x_{i1}}} - \frac{X^i_{e_{i1}}}{X^i_{e_{i2}}} = -\frac{\bar{\mu} p_x}{p_e} \quad \textit{for} \quad i = A, B. \tag{13}$$

According to Proposition 1(ia) the cost-effective implementation of the ceiling is secured by a policy that leaves consumption as well as second-period emissions unregulated and levies a tax on first-period emissions that is uniform across countries and reflects the stringency of the ceiling. Conceptually, Proposition 1(ia) conforms with the findings of Sinclair (1994), Sinn (2008) and others in one-country growth models that flattening the extraction path can be secured by declining emission taxes over time. Surprisingly, the same result can be attained by the policy of Proposition 1(ib) that leaves emissions and second-period consumption unregulated and levies a uniform tax on total first-period consumption. That tax creates incentives for expanding second-period consumption and also increases the relative producer price of the second-period consumption good which in turn shifts some production from the first to the second period.[11] It is clear that the tax policies of Proposition 1(i) create a distortion of the laissez-faire allocation. As Proposition 1(ii) shows that distortion comes in form of a wedge between the marginal rates of intertemporal subsitution in consumption and production.[12]

Suppose now, the cooperative solution characterized in Proposition 1 cannot be attained because country B abstains from emission regulation while country A implements the ceiling $\bar{e}_1$ unilaterally. To ease the notation we write in the following $\pi_{At} = \pi_t$ and

[11] To keep focused we mention only in passing that 'convex combinations' of the first period taxes on emissions and consumption also do the job as well as policies that consist of emission taxes [consumption taxes] only that differ from the policies of Proposition 1(ia) [1(ib)] in that the tax rate in each period is shifted up or down by a constant (see Eichner and Pethig 2011b). The crucial feature of all these hybrid policies is that they sustain one and the same equilibrium allocation.

[12] As the wedge is the same across countries, the distortion is smaller than in the case of unilateral ceiling policy to be studied later. Note also that $\bar{\mu} = 0$ if the ceiling is too high to be binding.

$\tau_{At} = \tau_t$ for $t = 1, 2$. In Section 3.1 we aim to characterize those policies $(\pi_1, \pi_2, \tau_1, \tau_2)$ of country A which succeed in bringing down e_1 from its laissez-faire level e_1^o to $\bar{e}_1$. In Section 3.2 we analyze unilateral ceiling policies that are cost effective for country A.

3.1 Feasible unilateral ceiling policies

Suppose country B does not engage in climate policy and country A has at its disposal a tax policy $(\pi_t, \tau_t)_{t=1,2}$, denoted *ceiling policy*, for short, that implements the politically fixed ceiling $\bar{e}_1 (\leq e_1^o)$. The competitive equilibrium corresponding to that unilateral ceiling policy is denoted *ceiling-policy equilibrium.* Such an equilibrium clearly exists, if country A is large (or 'rich') enough relative to country B and if the required total emission reduction, $e_1^o - \bar{e}_1$, is not too large. It is important to know whether country A can choose among different ceiling policies because that is a precondition for the opportunity - and challenge - to select that particular tax policy which implements the ceiling at least cost for country A.

Technically speaking, the ceiling is implemented if $e_{A1} \geq 0$, $e_{B1} \geq 0$ and $e_{A1} + e_{B1} = \bar{e}_1$. Hence if e_{A1} is part of the allocation of a ceiling-policy equilibrium, then $e_{A1} \in [0, \bar{e}_1]$. Let $E \subset [0, \bar{e}_1]$ be the set of emissions e_{A1} which belong to some ceiling-policy equilibrium. Unfortunately, a full analytical characterization of the set E with the general functional forms (1) and (2) turns out to be impossible. We therefore proceed by investigating three different scenarios with parametric functional forms and/or with constraints imposed on the use of fiscal instruments.

Scenario I. In the first scenario country A refrains from taxing consumption ($\tau_1 = \tau_2 = 0$). Under the additional simplifying assumptions that production functions are quadratic, the same across periods and countries, and that utility functions are Cobb-Douglas and identical across countries, Eichner and Pethig (2011b) establish[13]

Proposition 2 . *Suppose country A applies emission taxes only. Under the assumptions stated above, there are unilateral ceiling policies consisting of emission tax rates* $\begin{Bmatrix} \pi_1 > 0, & \pi_2 > 0 \\ \pi_1 > 0, & \pi_2 < 0 \\ \pi_1 < 0, & \pi_2 < 0 \end{Bmatrix}$ *iff e_{A1} is* $\begin{Bmatrix} \textit{small} \\ \textit{intermediate} \\ \textit{large} \end{Bmatrix}$ *in the corresponding ceiling-policy equilibrium.*

The key message of Proposition 2 is that in the absence of consumption taxes government A can choose from a variety of ceiling policies that, in general, consist of non-zero tax rates in both periods and may even exhibit a negative rate in the second period or in both periods.

[13]The special cases $(\pi_1 > 0, \pi_2 = 0)$ and $(\pi_1 = 0, \pi_2 < 0)$ are also feasible. For more analytical detail and rigor see Eichner and Pethig (2011b).

All these ceiling policies differ with respect to the allocation of the corresponding ceiling-policy equilibrium implying that, in general, the costs for country A of implementing the ceiling varies with the ceiling policy chosen.

Scenario II. Here we take as given the production functions (1) and consider the CES utility function

$$U(x_{i1}, x_{i2}) = (\gamma_1 x_{i1}^{\frac{\sigma_i - 1}{\sigma_i}} + \gamma_2 x_{i2}^{\frac{\sigma_i - 1}{\sigma_i}})^{\frac{h\sigma_i}{\sigma_i - 1}} \quad \text{for} \quad i = A, B \tag{14}$$

with $\gamma_1, \gamma_2, h, \sigma_A, \sigma_B > 0$ and $\sigma_A, \sigma_B \neq 1$. Country A is now assumed to refrain from taxing emissions ($\pi_1 = \pi_2 = 0$). For that case we prove in the Appendix A

Proposition 3. *Suppose country A applies consumption taxes only and functional forms (1) and (14) are given. Then the allocation of the ceiling-policy equilibrium is invariant with regard to all combinations* (τ_1, τ_2) *satisfying* $\tau_1 = F(\tau_2)$*, where the properties of the function* $F : \mathbb{R} \to \mathbb{R}$ *are given by*

(i) $F_{\tau_2} > 0$ *for* $\sigma_A \geq 1$ *and* F_{τ_2} *constant* [14] *for* $\sigma_A = 1$*,*

(ii) $[F(\tau_2), \tau_2 = 0]$ *and* $[F(\tau_2) = 0, \tau_2]$ *are ceiling policies*

The economic interpretation of the multiplicity of (τ_1, τ_2) combinations sustaining the cost-effective ceiling-policy equilibrium is straightforward. Given the consumer's non-transfer income $X(e_{A1}) + p_x X(e_{A2}) + p_e \Delta e_A$, she can be induced to demand the bundle of consumption goods (x^*_{A1}, x^*_{A2}) by any tuple (τ_1, τ_2) satisfying $\tau_1 = F(\tau_2)$ provided that the tax revenue $\tau_1 x^*_{A1} + \tau_2 x^*_{A2}$ is returned to the consumer as a lumpsum transfer. If, for example, the tuple $[\tau_1 = F(0) > 0, \tau_2 = 0]$ is replaced by $[\tau_1 = 0, \tau_2 = F^{-1}(0) < 0]$ it is easy to see that the former policy discourages the first-period consumption directly whereas the latter policy discourages first-period consumption indirectly by encouraging second-period consumption.

The Propositions 2 and 3 show that in both cases the ceiling can be implemented via multiple combinations of tax rates (π_1, π_2) and (τ_1, τ_2), respectively. The important difference is, however, that all ceiling policies (π_1, π_2) and (π'_1, π'_2), $(\pi_1, \pi_2) \neq (\pi'_1, \pi'_2)$ differ in the allocation of the corresponding ceiling-policy equilibrium (except that $\bar{e}_1 = \bar{e}'_1$ by assumption). In contrast, the allocations of all ceiling-policy equilibria attainable through consumption taxes only are the same for all ceiling policies $[\tau_1 = F(\tau_2), \tau_2]$. In other words, the incidence of all consumption-tax ceiling policies is the same, while emission-tax ceiling policies differ with regard to the tax incidence and thus also with regard to the costs of the policy accruing to country A. That intriguing difference makes us wondering what ceiling policies are like that consist of a mix of taxes on consumption and emissions.

[14] For an illustration see Figure 1 in Section 3.2.2 below.

Scenario III. We now aim to characterize ceiling policies in which $\tau_2 \equiv 0$ while π_1, π_2 and τ_1 are allowed to be non-zero. We apply production functions (1) and CES utility functions (14) with $\sigma_A = \sigma_B \equiv \sigma$. For the subsequent analysis the following notation is useful. If in a ceiling-policy equilibrium with policy $(\pi_1, \pi_2, \tau_1, \tau_2 = 0)$ the emissions $e_{A1} = \bar{e}_{A1}$ and $e_{A2} = \bar{e}_{A2}$ prevail[15] we denote that ceiling policy by $(\pi_1, \pi_2, \tau_1, \tau_2 = 0; \bar{e}_{A1}, \bar{e}_{A2})$.

Proposition 4. *Suppose country A applies emission taxes and a tax on first-period consumption. If the functional forms (1) and (14) are given, for every* $(\bar{e}_{A1}, \bar{e}_{A2}) \in [0, \bar{e}_1] \times [0, \bar{e} - \bar{e}_1]$ *there exist* $\tau_1 \in \mathbb{R}$ *and a set* $S \in \mathbb{R}_{++}$ *with non-empty interior such that the policy* $(\pi_1, \pi_2, \tau_1, \tau_2 = 0; \bar{e}_{A1}, \bar{e}_{A2})$ *is a ceiling-policy in all economies with* $\sigma \in S$*. In general,* $\tau_1 \neq 0$.

Proposition 4 clarifies that whether or not $\bar{e}_1$ can be implemented by a fiscal policy resulting in $(\bar{e}_{A1}, \bar{e}_{A2}) \in [0, \bar{e}_1] \times [0, \bar{e} - \bar{e}_1]$ depends on the size of the elasticity of substitution in demand, σ, and on an appropriate level of the tax rate τ_1. The proof of Proposition 4 in the Appendix B highlights the important role of a non-zero consumption tax τ_1 to implement the ceiling $\bar{e}_1$ in a way that is not viable in case of $\tau_1 = \tau_2 = 0$. To further clarify the relevance of the consumption tax as a regulatory device in such policies we find it useful to consider the following special case of Scenario III:[16]

Proposition 5. *Suppose that under the conditions of Proposition 4 country A seeks to implement the ceiling while keeping its laissez-faire emissions unchanged,* $\bar{e}_{At} = e^o_{At}$ *for* $t = 1, 2$*. There exist tax rates* $\pi_1 < 0, \pi_2 > 0$ *and* $\tau_1 > 0$ *such that* $(\pi_1, \pi_2, \tau_1, \tau_2 = 0, e^o_{A1}, e^o_{A2})$ *is a ceiling policy. In the corresponding ceiling-policy equilibrium the prices* p_e *and* p_x *are higher than their counterparts in laissez faire.*

The remarkable feature of the ceiling policy of Proposition 5 is that country A leaves its first and second-period emissions at their laissez-faire levels. As a consequence, the burden of reducing first-period emissions from e^o_1 to $\bar{e}_1$ is entirely on country B(!).[17] Country A induces country B to reduce its first-period emissions by raising p_e. If p_e goes up, p_x must increase as well, because the necessary expansion of country B's second-period production requires the price ratio (p_e/p_x) to decline. The rise in p_x shifts (some) consumption from the second to the first period which would create an excess demand for consumption in period 1, ceteris paribus. To prevent that from happening, country A makes use of the consumption tax. The rationale of $\tau_1 > 0$ is to discourage first-period consumption in country A or, in

[15] Instead of treating the emission tax rates π_1, π_2 as policy instruments one can also treat the tax rates π_1, π_2 as endogenous and take the target emission levels $\bar{e}_{A1}$, $\bar{e}_{A2}$ as policy instruments.

[16] The proof of Proposition 5 is delegated to the Appendix C.

[17] Nonetheless, country B is shown (see the Appendix C) to gain unambiguously in terms of utility and country A loses from its own unilateral climate policy $(e^o_{A1}, e^o_{A2}, \tau_1, \tau_2 = 0)$.

other words, to shift some of country A's consumption (back) from the first to the second period. In that way an equilibrium on the first- and second-period (world) market for good X is secured. The ceiling policy of Proposition 5 differs markedly from the cost-effective cooperative policy regarding the fossil fuel price: $p_e > p_e^o$ results from the former policy but $p_e < p_e^o$ from the latter. Note also that keeping the emissions e_{A1} and e_{A2} constant at their laissez-faire levels does not imply that these emissions are unpriced. On the contrary, it is necessary to subsidize first-period emissions ($\pi_1 < 0$) and to tax second-period emissions ($\pi_2 > 0$). Our conjecture is that the ceiling policy of Proposition 5 is not cost effective.[18]

3.2 Cost-effective unilateral ceiling policies

3.2.1 The analytical approach

The previous section offered insights into the variety of ceiling policies country A has at its disposal. The multiplicity of ceiling policies in the case that government A disposes of emission taxes only (characterized in Eichner und Pethig 2011b) has been shown to expand when consumption taxes are added to the government's policy instruments. As compared to laissez faire, country A's unilateral action causes allocative distortions which, in turn, change the welfare of both countries and typically result in a welfare loss for country A.[19] That welfare loss represents country A's cost of its unilateral ceiling policy and depends on the particular policy chosen. Hence it is in country A's interest to select the least-cost policy among all feasible ceiling policies. The key question to be answered is whether country A is able to reduce its costs of implementing the ceiling - compared to the cost-effective emission-tax-only policy in Eichner and Pethig (2011b) - by an appropriate mix of consumption and emission taxes.

To characterize the unilateral ceiling policy that is cost effective for country A when government A has at its disposal taxes on both emissions and consumption we envisage a regulator who maximizes the utility of the consumer in country A through an appropriate choice of her policy instruments. For analytical convenience we will treat the quantities e_{A1} and e_{A2} as the regulator's decision variables while the tax rates π_1 and π_2 adjust endogenously in order to sustain e_{A1} and e_{A2}. The consumption tax rate τ_1 is no decision variable either. It is rather implied by the solution of the regulator's optimization problem as will be made precise below. In her optimization procedure the regulator takes into account

[18] All numerical examples of unilateral cost-effective ceiling policies in Section 3.2.2 below differ qualitatively from the ceiling policy of Proposition 5.

[19] That welfare loss is a partial welfare effect because the benefits from reduced climate damage are not accounted for.

- the equilibrium conditions (3) for the commodity markets,
- the fuel/emission constraints

$$e_{A1} + e_{B1} = \bar{e}_1 \quad \text{and} \quad e_{A2} + e_{B2} = \bar{e}_2 := \bar{e} - \bar{e}_1 \tag{15}$$

(where the equality signs in (15) are important[20]),

- the input demand and output supply functions of all producers
- and the consumption demand functions of the consumer in country B.

We proceed by demonstrating in several steps that under the constraints listed here the utility of the consumer in country A is, in fact, completely determined by the policy parameters e_{A1} and e_{A2}.

The supply side partial equilibrium. Consider (15) and the first-order conditions of profit maximization

$$X^A_{e_{A1}}(e_{A1}) = p_e + \pi_1, \tag{16}$$

$$X^B_{e_{B1}}(\bar{e}_1 - e_{A1}) = p_e, \tag{17}$$

$$p_x X^A_{e_{A2}}(e_{A2}) = p_e + \pi_2, \tag{18}$$

$$p_x X^B_{e_{B2}}(\bar{e}_2 - e_{A2}) = p_e. \tag{19}$$

These equations determine the supply-side system of the competitive economy for all (e_{A1}, e_{A2}) $\in [0, \bar{e}_1] \times [0, \bar{e}_2]$. They ensure an equilibrium of the fossil fuel markets in both periods and determine p_e, p_x, π_1, π_2 and x^s_{it} for $i = A, B$ and $t = 1, 2$ for every $(e_{A1}, e_{A2}) \in [0, \bar{e}_1] \times [0, \bar{e}_2]$.

The consumer in country B. Turn to the demand side. The consumer in country B maximizes her utility subject to the budget constraint

$$x_{B1} + p_x x_{B2} = X(e_{B1}) + p_x X(e_{B2}) + p_e(\alpha_B \bar{e} - e_{B1} - e_{B2}) =: y_B. \tag{20}$$

Resorting to the CES utility functions (14) the resulting consumption is

$$x_{B1} = \frac{\bar{\gamma}_B p_x^{\sigma_B} y_B}{p_x + \bar{\gamma}_B p_x^{\sigma_B}} =: B^1(e_{A1}, e_{A2}) \quad \text{and} \quad x_{B2} = \frac{y_B}{p_x + \bar{\gamma}_B p_x^{\sigma_B}} =: B^2(e_{A1}, e_{A2}), \tag{21}$$

where $\bar{\gamma}_i := \left(\frac{\gamma_1}{\gamma_2}\right)^{\sigma_i}$ for $i = A, B$ and where it follows from (15), (17), (19) and (20) that the consumption (x_{B1}, x_{B2}) in (21) is uniquely determined by (e_{A1}, e_{A2}).

[20] The consequence of the equality signs in (15) is that the tax rates π_1, π_2 may take on negative values.

Cost-effective regulation. The regulator of country A realizes that under consideration of (1), (15) and (21) the consumption x_{At} from (3) turns into

$$x_{At} = A^t(e_{A1}, e_{A2}) := X(e_{At}) + X(\bar{e}_t - e_{At}) - B^t(e_{A1}, e_{A2}) \quad \text{for } t = 1, 2. \tag{22}$$

She then maximizes $U^A\left[A^1(e_{A1}, e_{A2}), A^2(e_{A1}, e_{A2})\right]$ over $[0, \bar{e}_1] \times [0, \bar{e}_2]$ which yields the first-order conditions

$$\frac{U^A_{x_{A2}}}{U^A_{x_{A1}}} = -\frac{A^1_{e_{A1}}}{A^2_{e_{A1}}} = -\frac{A^1_{e_{A2}}}{A^2_{e_{A2}}}. \tag{23}$$

The two equations in (23) determine the cost-effective quantities e^*_{A1} and e^*_{A2} and the consumption demands $x^*_{At} := A^t(e^*_{A1}, e^*_{A2})$ follow from (22).[21]

Proposition 6 . *Suppose country A has at its disposal emission and consumption taxes in both periods and the functional forms (1) and (14) are given. Then the allocation of the cost-effective ceiling-policy equilibrium is invariant with regard to all combinations (τ_1, τ_2) satisfying $\tau_1 = G(\tau_2)$, where the properties of the function $G : \mathbb{R} \to \mathbb{R}$ are qualitatively the same as those of the function F from Proposition 3.*

The message of Proposition 6 is that if $(\pi_1, \pi_2, \tau_1, \tau_2)$ is a cost-effective ceiling policy for country A, then the consumption tax rates satisfy $\tau_1 = G(\tau_2)$. Moreover, different tax rate tuples $[G(\tau_2), \tau_2]$, $[G(\tau_2'), \tau_2']$ leave the allocation of the ceiling-policy equilibrium unchanged. Although that feature as well as the properties of the function G are qualitatively the same as those of the functions F from Proposition 3, it is obvious that, in general $F(\tau_2) \neq G(\tau_2)$ for all τ_2 in the relevant domain.[22]

Without loss of generality we set $\tau_2 = 0$ in the sequel. When the consumer of country A maximizes her utility (taking prices and income as given) the corresponding first-order condition reads $\frac{U^A_{x_{A2}}}{U^A_{x_{A1}}} = \frac{p_x}{1+\tau_1}$ which gives rise to the equivalence

$$\frac{U^A_{x_{A2}}}{U^A_{x_{A1}}} \gtreqless p_x \quad \Longleftrightarrow \quad \tau_1 \lesseqgtr 0. \tag{24}$$

Unfortunately, the equations (20) through (24) provide limited information only on the *analytical* characteristics of unilateral cost-effective ceiling-policies.[23] Therefore, we further

[21] The proof of Proposition 6 is similar to that of Proposition 3 and therefore omitted.

[22] Note also that Proposition 6 does not rule out the possibility that the function G satisfies $G(0) = 0$ in which case cost-effectiveness could be achieved without taxing consumption. In Section 3.2.2 we will show, however, that $G(0) > 0$ in our numerical analysis.

[23] Scrutinizing these equations reveals that the sign and level of tax rates are related to the signs of imbalances in the trade of fossil fuel and consumption goods. For example, we found that

$$\begin{aligned} \Delta e_A := \alpha_A \bar{e} - e_{A1} - e_{A2} > 0 \quad &\Longrightarrow \quad \pi_t < 0 \quad \text{for} \quad t = 1, 2 \quad \text{or} \quad \pi_1 \cdot \pi_2 \leq 0, \\ \Delta e_A < 0 \quad &\Longrightarrow \quad \pi_t > 0 \quad \text{for} \quad t = 1, 2 \quad \text{or} \quad \pi_1 \cdot \pi_2 \leq 0. \end{aligned}$$

simplify the model by introducing the parametric functions

$$U\left(x_{i1}, x_{i2}\right) = x_{i1}^{\gamma} x_{i2}^{1-\gamma} \quad \text{and} \quad X^{i}\left(e_{it}\right) = a e_{it} - \frac{b}{2} e_{it}^{2} \quad \text{for } i = A, B;\ t = 1, 2. \tag{25}$$

With these simplifications we show in the Appendix D

Proposition 7. *Suppose country A has at its disposal emission taxes in both periods and a first-period consumption tax. Moreover, let the functional forms (25) be given and consider a symmetric competitive equilibrium in the absence of regulation* ($\pi_1 = \pi_2 = \tau_1 = \tau_2 = 0$, $\alpha_A = 0.5$).

(i) If country A implements the emission ceiling $\bar{e}_1 = \bar{e}_1^o + \mathrm{d}\bar{e}_1 < \bar{e}_1^o$, *the cost-effective tax rates have the signs* $\pi_1 > 0$, $\pi_2 < 0$ *and* $\tau_1 > 0$ *and the corresponding equilibrium prices satisfy* $p_e < p_e^o$ *and* $p_x < p_x^o$.

(ii) If government A has no consumption taxes at its disposal ($\tau_1 = \tau_2 \equiv 0$), *as in Eichner und Pethig (2011b), the cost-effective emission tax rates also have the signs* $\pi_1 > 0$ *and* $\pi_2 < 0$.

Although Proposition 7(i) relates to a fairly special case (symmetry and a ceiling very close to laissez-faire emissions) it is important in that it provides definite proof for the capacity of the consumption tax to reduce country A's costs of unilateral ceiling policy when it is levied in addition to emission taxes. Proposition 7(ii) is added for reference purposes. It demonstrates that with and without the option of taxing consumption the emission tax rates have the same sign and therefore their roles in securing cost effectiveness are similar in qualitative terms. More importantly, since setting $\tau_1 = \tau_2 = 0$ is a feasible choice of government A in the model of the present paper, where the consumption tax is at government A's disposal, the costs of country A's unilateral ceiling policy in Proposition 7(ii) are necessarily higher than those in Proposition 7(i). The subsequent numerical calculations confirm that conclusion and provide additional insights.

3.2.2 Numerical examples

In the numerical examples we are going to elaborate we will use the utility functions and production functions specified in (25). For the first example, denoted Example 1, we choose the parameter values $a = 1$, $b = 0.5$, $\bar{e} = 2$, $\bar{e}_1 = 1.20$ and $\gamma = 0.7$ and $\tau_2 \equiv 0$ and fix country A's share of the world stock of fossil fuel, $\bar{e}$, at $\alpha_A = 0.5$. Table 1 summarizes the results.

Yet we refrain from elaborating on such conditional characteristics of cost effectiveness because they fall short of definitive proof that non-zero commodity tax rates are a constituent part of cost-effective ceiling policies.

	Laissez faire	**R**egulation	**R** minus **L**
e_{A1}	0.672	0.542	−0.130
π_1	0	0.058	+0.058
e_{A2}	0.328	0.481	+0.153
π_2	0	−0.065	−0.065
e_{B1}	0.672	0.658	−0.014
e_{B2}	0.328	0.319	−0.009
$\Delta e_A{}^{a)}$	0	−0.023	−0.023
p_x	0.794	0.798	+0.004
p_e	0.664	0.671	+0.007
τ_1	0	0.705	+0.705
$\Delta x_{A1}{}^{b)}$	0	0.01	+0.01
Δx_{A2}	0	0.007	+0.007
x_{A1}/x_{A2}	1.854	1.102	−0.752
x_{B1}/x_{B2}	1.854	1.863	+0.009
u_A	0.46436	0.445	−0.01936
u_B	0.46436	0.46440	+0.00004

$^{a)}\Delta e_A = \alpha_A \bar{e} - e_{A1} - e_{A2} = -\Delta e_B$, $^{b)}\Delta x_{At} = -\Delta x_{Bt} = x^s_{At} - x_{At}$

Table 1: Example 1: Cost-effective ceiling policy and associated equilibrium allocation

It is worthwhile highlighting the main features of Example 1 compared to the no-regulation case, but also similarities and differences between Example 1 and the cost-effective cooperative ceiling policy characterized in Proposition 1.

To begin with, observe that the laissez-faire equilibrium of Example 1 is symmetric. From Table 1 we calculate $e^o_{A1} + e^o_{B1} =: e^o_1 = 1.34$, while the ceiling $\bar{e}_1$ is chosen to be $\bar{e}_1 = 1.20$. That amounts to a cut of total laissez-faire first-period emissions of about 9%. The results are surprising. In period 1 country A reduces its own emissions drastically, induced by a positive price for emissions ($\pi_1 = 0.058$), but it expands its own second-period emissions even more drastically by means of a subsidy on its second-period emissions ($\pi_2 = -0.065$). That contrasts strongly with the cooperative cost-effective regulation (Proposition 1(ia)) that leaves period 2 unregulated.

If one would set $\tau_1 = 0$ in the otherwise unchanged policy of Example 1, an excess demand [supply] of good X in the first [second] period would result. To avoid that disequilibrium the tax rate τ_1 is raised to the value $\tau_1 = 0.705$. Comparing the low (absolute) values of π_1 and π_2 with the relatively high value $\tau_1 = 0.705$ suggests that a major share of the regulatory burden rests with the tax on consumption. That observation is reinforced by the slight increase in the prices p_e and p_x that would have had to decline if the consumption tax were absent. The increase in p_e and p_x is also at variance with the conventional view that emission reductions in some parts of the world would tend to 'expropriate' fossil fuel owners. The increase in p_x is delicate because its substitution effect tends to increase consumption in the first period which counteracts country A's effort to stimulate second-period consumption (and production). While country A must tolerate the (small) increase in the ratio x_{B1}/x_{B2}, it effectively discourages the expansion of first-period consumption at home by raising τ_1. Another remarkable feature of regulation in Example 1 is that country B reduces its emissions in both periods as compared to its emissions in the laissez-faire equilibrium. That constitutes negative leakage. Thus somewhat ironically, country A implements the ceiling by inducing country B to reduce its emissions while country A's own overall emissions expand slightly (from $e^o_A = e^o_{A1} + e^o_{A2} = 1$ to $e_{A1} + e_{A2} = 1.02$). Nonetheless, as the last two rows of Table 1 show, country A suffers a loss while country B gains slightly. If country A's gain from the reduced climate damage (not considered in the formal model) is greater than 0.01936, country A's unilateral action passes the cost-benefit tests. Since country B free rides on the damage reduction, its net benefits are most likely greater than those of country A.[24]

As shown in Eichner und Pethig (2011b), there is a feasible unilateral ceiling policy for country A with tax rates $\pi_1 > 0$ and $\pi_2 = \tau_1 = \tau_2 = 0$ which is exactly the pattern of the cost-effective cooperative policy. Example 1 demonstrates that the ceiling policy relying on $\pi_1 > 0$ and $\pi_2 = \tau_1 = \tau_2 = 0$ is unnecessarily costly for country A. Country A minimizes its burden by multiple sticks and carrots which[25] appears to be a more general feature as will be shown in the discussion below.

Recall from Proposition 6 that every cost-effective unilateral ceiling-policy equilibrium can be sustained by multiple combinations of τ_1 and τ_2 and that we have set $\tau_2 = 0$ in our analysis following Proposition 6. In the Example 1 $\tau_1 = 0.705$ holds conditional on $\tau_2 = 0$ which corresponds to the point P in Figure 1. That Figure also depicts the graph of the function G defined in Proposition 6 which is a straight line with positive slope in case of Cobb-Douglas utility. Choosing any point on that graph other than $(\tau_1 = 0.705, \tau_2 = 0)$ would leave the second column of Table 1 unchanged.

[24] Modify Example 1 by choosing the parameters values $b = 0.1, \bar{e}_1 = 1.0, \alpha_A = 1.0$ (instead of $b = 0.5, \bar{e}_1 = 1.2, \alpha_A = 0.5$) and keep all other parameter values of Example 1 unchanged. In that modified example we calculate the changes in utility from their laissez-faire levels as $\Delta u_A = -0.007$ and $\Delta u_B = -0.011$. After having accounted for the welfare gains due to reduced climate change, country A may even turn out to be better off than country B.

[25] Interestingly, the stick $\tau_1 > 0$ can be replaced by a carrot $\tau_2 < 0$, as established in Proposition 6. In contrast, the emission-tax sticks and carrots are no substitutes.

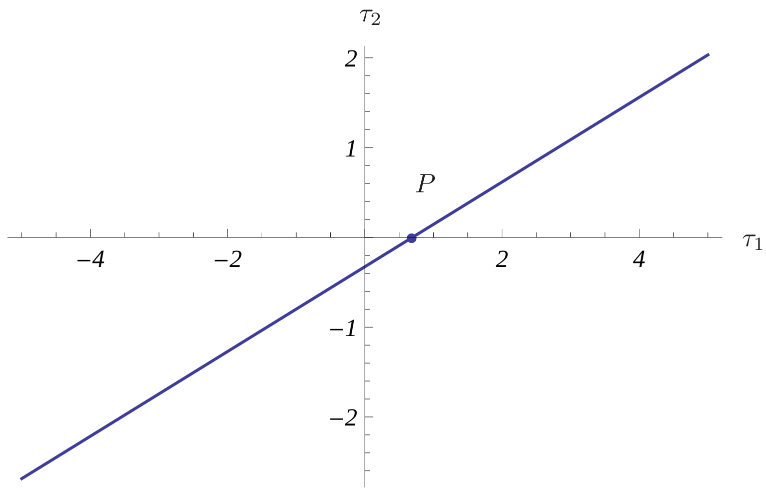

Figure 1: Consumption tax rates in Example 1

Figure 2 illustrates the supply-side displacement effects of Example 1 in the transition from laissez faire to regulation. D_{it} denotes the demand curve for fossil fuel with $i = A, B$ and $t = 1, 2$. Consider first the initial laissez-faire scenario indicated in Figure 2 by the letters with superscript "o". The two panels on the left [right] show the countries' first-period [second-period] fuel demand curves. First-period demands depend only on p_e while the demands in the second period depend on p_e and p_x. Hence the curves D_{A2} and D_{B2} on the right are drawn for specific fixed values of p_x. The middle panel of Figure 2 plots the aggregate fuel demand curves D_1^o and D_2^o and illustrates the equilibrium E^o in the fuel markets of both periods. The equilibrium quantities are $e_1^o = e_{A1}^o + e_{B1}^o$ and $\bar{e} - e_1^o = e_{A2}^o + e_{B2}^o$ and the equilibrium price in both periods is p_e^o.[26]

Suppose now country A implements the ceiling $\bar{e}_1 < e_1^o$ in unilateral action. To this end it levies the tax π_1^r [subsidy π_2^r] on domestic first-period [second-period] emissions (=fuel). The regulation scenario is indicated in Figure 2 by all letters with superscript "r". The new equilibrium point is E^r in the middle panel of Figure 2 and the equilibrium prices rise from p_e^o and p_x^o to p_e^r and p_x^r. The increase in p_x gives rise to an upward shift of the demand curves D_{A2}^o and D_{B2}^o in the right panels of Figure 2. Summarizing, Figure 2 replicates the signs of

[26] Recall that equilibrium on the fuel markets requires fixing simultaneously appropriate values p_e^o *and* p_x^o, because the price p_x determines the position of the second-period demand curves D_{A2} and D_{B2} in the right panels of Figure 2.

the differences $\pi_i^r - \pi_i^o, p_e^r - p_e^o, e_{At}^r - e_{At}^o$ for $t = 1, 2$ and $e_{B1}^r - e_{B1}^o$ listed in Table 1.[27]

The consumption tax τ_1 has no direct impact on those fuel market equilibria and is therefore absent from Figure 2. The role of τ_1 is rather to adjust consumption demand such that the commodity markets of both periods equilibrate. To understand the equilibrium mechanism observe first that according to Table 1 $(x_{B1}^r/x_{B2}^r) > (x_{B1}^o/x_{B2}^o)$. For $\tau_1 = 0$ that inequality would also hold for country A. Given identical Cobb-Douglas utility functions (that are homothetic functions), $\tau_1 = 0$ would clearly result in an excess demand [supply] in the first-period [second-period] market of good X because $(x_{A1}^r + x_{B1}^r)/(x_{A2}^r + x_{B2}^r) < (x_{A1}^o + x_{B1}^o)/(x_{A2}^o + x_{B2}^o)$. To avoid such commodity-market disequilibria country A needs to levy the first-period consumption tax in order to shift part of its first-period demand for good X to the second period.[28]

The columns 3, 4 and 5 of Table 2 provide insights into the effects of changes in the parameters $\bar{e}_1, a, b, \gamma, \alpha_A$ and σ on the magnitude (and signs) of the cost-effective tax rates τ_1, π_1 and π_2. All new examples listed in Table 2 differ from Example 1 in one of these parameters only, and for each parameter two examples are chosen, one with a lower and the other with a higher value than that in Example 1. To ease the comparison of the incidence of successive parameter changes, Table 2 also takes over from Table 1 the cost-effective tax rates of Example 1 printed in italic letters. For the sample of parameter values in Table 2 it is true that τ_1 is always positive, the tax rate π_2 is always negative. π_1 is positive except in the case where country A owns a large share of the world's stock of fossil fuel. We have been unable to find parameters constellations where emissions are taxed at positive rates in both periods which is shown in Eichner und Pethig (2011b) to be a possibility in the absence of taxing consumption. Table 2 is not only informative regarding the signs of tax rates, but also indicates directions of change in tax rates. Some tax rates are increasing in parameters (τ_1 in a, γ and σ; π_1 in a, b, γ and σ; π_2 in $\bar{e}_1$), some are decreasing (τ_1 in α_A, b and $\bar{e}_1$; π_1 in α_A, b and $\bar{e}_1$; π_2 in α_A a, b, γ and σ). Particularly interesting is that π_1 is decreasing in α_A and turns from positive to negative. That observation conforms with the general finding of Eichner and Pethig (2011b) for the case of $\tau_1 = \tau_2 = 0$ that with increasing α_A the unilateral cost-effective ceiling policy tends to consists of emission subsidies in both periods. With increasing α_A (and decreasing α_B) country A expands its exports of fossil fuel and therefore seeks to secure high export revenues by pushing up the fuel price. It is also worth noting that lowering the ceiling for a more stringent climate policy leads to increasing the consumption tax and reducing the second-period emission tax. On the whole, both the level and variability of τ_1 appear to be higher than those of the other tax rates, in particular

[27] $e_{B2}^r > e_{B2}^o$ in Figure 2 is the only qualitative deviation from Table 1, and we made it deliberately for expository reasons.

[28] See also the proof of Proposition 4 in the Appendix B.

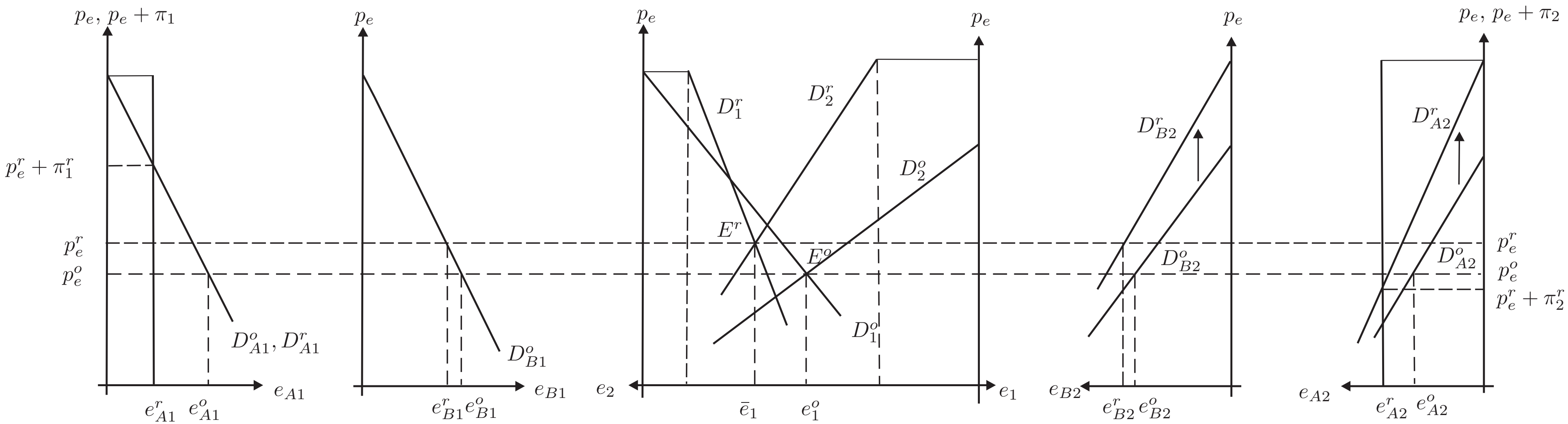

Figure 2: Unilateral cost-effective regulation of country A (Example 1)

with respect to changes in α_A,γ and $\bar{e}_1$.

So far we have discussed the columns 3, 4 and 5 of Table 2. For the purpose of comparison, the column 6 [columns 7 and 8] list(s) the cost-effective tax rate(s) under the additional constraint that government A has at its disposal only consumption taxes [emission taxes]. The general features of these policies are:

(i) If only consumption taxes are available, there exists a cost-effective ceiling policy for all parameter constellations; with $\tau_2 \equiv 0$ the tax rates τ_1 are all positive and the direction of change of tax rates in parameter values in column 6 is exactly as in column 3.

(ii) If only emission taxes are available, there are parameter constellations for which no (cost-effective) ceiling policy exists. In Table 2 these examples are marked by empty boxes in the columns 7 and 8. As in column 5 all π_2 in column 8 are negative and in the columns 4 and 7 all π_2 are positive. The only exception being the parameter constellation $\alpha_A = 0.75$ (3rd row) where $\pi_1 < 0$ in column 4 but $\pi_1 > 0$ in column 7. The direction of change of tax rates in parameter values is not always the same in the columns 4 and 7 and 5 and 8, respectively.

A straightforward conclusion from Table 2 is that if both kinds of tax instruments are at government A's disposal, by making use of both it can reduce the costs of its ceiling policy below the costs accruing in scenarios in which only one kind of tax instrument is available. For an explanation of that result it is useful to compare the level of tax rates in the three types of policies in Table 2 (columns 3 - 5, column 6, column 7 and 8). All tax rates τ_1 in column 3 are smaller than those in column 6, all tax rates π_1 in column 4 are smaller than those in column 7 and all tax rates in column 5 are smaller than those in column 8. In other words, if taxes on emissions and consumption are available, the cost-effective tax rates are lower than in the case where government A can apply only consumption taxes or only emission taxes. The reason is, as indicated in the introduction (Section 1), that taxes on both emissions and consumption are distortionary and with the use of one kind of taxes the distortions to reach the ceiling are more pronounced and require higher tax rates than in policies applying both kinds of taxes. Since the distortions tend to increase progressively in tax rates, an appropriately chosen policy mix minimizes country A's total costs of implementing the ceiling.

The non-existence of some unilateral ceiling policies under conditions of the columns 7 and 8 of Table 2 reminds us of the preconditions for feasibility that we have implicitly taken to be satisfied in our analytical studies above. In fact, we have also found that in economies differing from Example 1 only with respect to the value of $\bar{e}_1$ there exists no ceiling policy equilibrium anymore, if $\bar{e}_1$ is lower than 1.1. That is suggestive for the conjecture that the set of feasible ceiling policies shrinks and eventually turns into an empty set when the ceiling is successively lowered. Another conjecture whose study is beyond the scope of the present paper is that country A's ceiling policies become infeasible if country A is too small relative to country B where the relative size depends on wealth, i.e. on the ownership of fossil fuel (α_A) and the abundance of domestic factors of production.

		$\tau_2 \equiv 0$ and			$\pi_1 = \pi_2 \equiv 0$ $\tau_2 \equiv 0$ and	$\tau_1 = \tau_2 \equiv 0$ and	
		τ_1	π_1	π_2	τ_1	π_2	π_2
1	2	3	4	5	6	7	8
	0.25	0.959	0.0185	−0.04	2.278	0.354	−0.247
α_A	*0.5*	*0.705*	*0.05*	*−0.06*	*1.017*	*0.24*	*−0.116*
	0.75	0.569	−0.065	−0.102	0.654	0.14	−0.217
	0.75	0.492	0.05	−0.05	0.855	0.148	−0.09
$a =$	*1*	*0.705*	*0.05*	*−0.06*	*1.017*	*0.24*	*−0.116*
	2	1.006	0.066	−0.09	1.201	0.282	−0.499
	0.1	1.186	0.017	−0.025	1.285	–	–
$b =$	*0.5*	*0.705*	*0.05*	*−0.06*	*1.017*	*0.24*	*−0.116*
	0.7	0.455	0.067	−0.060	0.817	0.178	−0.123
	0.65	0.219	0.025	−0.024	0.310	0.091	−0.083
$\gamma =$	*0.7*	*0.705*	*0.05*	*−0.06*	*1.017*	*0.24*	*−0.116*
	0.9	9.353	0.122	−0.214	16.757	–	–
	1.0	2.375	0.11	−0.181	4.44	–	–
$\bar{e}_1 =$	*1.2*	*0.705*	*0.05*	−0.06	*1.017*	*0.24*	−0.116
	1.3	0.182	0.02	*−0.017*	0.249	0.052	−0.138
	0.5	0.029	0.002	−0.002	0.992	0.002	−0.117
$\sigma =^{*)}$	*1*	*0.705*	*0.05*	*−0.06*	*1.017*	*0.24*	*−0.116*
	10	0.804	0.144	−0.143	1.039	–	–

$^{*)}$CES production function with $\gamma_1 = 0.7$ and $\gamma_2 = 0.3$

Table 2: A gallery of examples centering around Example 1

The information provided in Table 2 and discussed above is based on a small sample of parameters, to small to assess the robustness of our results. We therefore do not claim presenting general results not least because our model is very simple. Nonetheless, Table 2 clearly demonstrates that consumption taxes do play a significant role in reducing the burden of the country that carries out a unilateral ceiling policy. Table 2 also raises various intriguing questions the answer to which is beyond the scope of the present paper. Is $\tau_1 > 0$

(in case of $\tau_2 = 0$) a general feature of unilateral cost-effective ceiling policies? Are there conditions under which such policies show the signs ($\pi_1 > 0, \pi_2 > 0, \tau_1 > 0$)? What are the reasons for cost-effective tax rates being monotone or non-monotone in model parameters?

4 Concluding remarks

As in Eichner und Pethig (2011b) we have studied here a two-period model in which some countries cooperate in a sub-global climate coalition (country A) to implement a binding ceiling on the world's medium-term emissions while the rest of the world (country B) refrains from taking action. Eichner und Pethig (2011b) showed that if the government of country A has at its disposal sign-unconstrained emission taxes its *unilateral* cost-effective ceiling policy requires, in general, taxing emissions in both periods at rates that may be positive, negative or mixed in sign. That contrasts markedly with the cost-effective ceiling policy of the *global* climate coalition (consisting of the countries A and B) which would require a uniform tax on all first-period emissions leaving second-period emissions unregulated. The present paper extends Eichner und Pethig (2011b) by providing government A with the option of taxing consumption in addition to taxing emissions.

The somewhat unexpected general conclusion is that with both kinds of stand-alone tax instruments the ceiling can be implemented, in cooperative as well as in unilateral action. However, while for the global climate coalition both instruments are perfectly equivalent for meeting the ceiling cost-effectively, and hence perfect substitutes, they turn out to be imperfect substitutes in country A's unilateral cost-effective ceiling policy. The striking result is that no tax instrument dominates the other. Rather the policy mix turns out to be better than policies making use of consumption taxes only or emission taxes only. The reason is that in case of unilateral action both kinds of taxes create different types of distortions. In an economy where the ceiling constraint is imposed and required to be satisfied in unilateral action reaching the target via two moderate distortions is better for country A than enforcing the ceiling by creating a massive distortion of one kind.

Moreover, we have shown that if the ceiling is not too low and country A is not too small relative to country B there exists a large set of unilateral ceiling policies which maps into a large set of utility tuples (u_A, u_B). Under standard assumptions, as applied in our model, the corresponding utility possibility frontier is negatively sloped. With its ceiling policy government A chooses that point on the frontier which maximizes the utility of country A at the expense of country B's utility. In that sense country A shifts as much burden as possible of its unilateral ceiling policy on country B.

Although our model is extremely simple and omits many important features of the real world, calculations turned out to be very involved allowing for limited *analytical* insight only into the characteristics of unilateral cost-effective ceiling policies. For example it is still difficult to fully understand the driving forces and determinants of the signs and magnitudes of tax rates that constitute such policies or the observation that a given cost-effective ceiling-policy equilibrium can be sustained by various combinations of first- and second-period consumption taxes. We were able to make considerable headway by resorting to numerical examples all of which confirmed the substantive role of consumption taxes in cost-effective unilateral ceiling policies. Nonetheless, more work is desirable to further improve our understanding of such policies. As it is clear that accounting for more real-world complexities would soon render intractable non-numerical analyses, CGE models would seem to promise further advances.

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Appendix

A. Proof of Proposition 3

Observe that (4), (5), (12) and the first-order conditions

$$X^A_{e_{A1}}(e_{A1}) = X^B_{e_{B1}}(\bar{e}_1 - e_{A1}) = p_x X^A_{e_{A2}}(e_{A2}) = p_x X^B_{e_{B2}}(\bar{e} - \bar{e}_1 - e_{A2}) \tag{A1}$$

determine e_{A1}, e_{A2}, p_e and p_x for given $\bar{e}_1$. Hence country B's income

$$y_B := X^B(\bar{e}_1 - e_{A1}) + p_x X^B(\bar{e} - \bar{e}_1 - e_{A2}) + p_e[\alpha_B \bar{e} - (\bar{e} - e_{A1} - e_{A2})]$$

is fixed and so are the levels of consumption, x_{B1} and x_{B2}. From the market clearing condition $x^s_{A1} + x^s_{B1} = x_{A1} + x_{B1}$ therefore follows that the equilibrium value of x_{A1}, say

$$x^*_{A1} := x^s_{A1} + x^s_B - x_{B1}, \tag{A2}$$

is uniquely determined by $\bar{e}_1$. Consider next country A's demand functions for the consumption good derived from CES utility (14)

$$x_{A1} = \frac{\bar{\gamma}_A (p_x + \tau_2)^{\sigma_A} y_A}{\bar{\gamma}_A (1+\tau_1)(p_x+\tau_2)^{\sigma_A} + (p_x+\tau_2)(1+\tau_1)^{\sigma_A}}, \tag{A3}$$

$$x_{A2} = \frac{y_A}{p_x} - \frac{x_{A1}}{p_x}, \tag{A4}$$

where

$$\begin{aligned} y_A &:= \underbrace{X^A(e_{A1}) + p_x X^A(e_{A2}) + p_e(\alpha_A \bar{e} - e_{A1} - e_{A2})}_{=y_{Ao}} + \tau_1 x_{A1} + \tau_2 x_{A2}, \\ &= y_{Ao} + \tau_1 x_{A1} + \tau_2 x_{A2}. \end{aligned}$$

In (A1) and (A2) we already have accounted for all conditions of market clearing other than that of the second-period market of the consumption good. The latter is taken care of by Walras Law which is why we disregard equation (A4). Closer inspection of (A3) reveals that any value of x_{A1} and therefore also the value $x_{A1} = x^*_{A1}$, can be attained through various combinations of τ_1 and τ_2. Formally speaking, there is a function $F : \mathbb{R} \to \mathbb{R}$ such that $\tau_1 = F(\tau_2)$, if and only if x_{A1} from (A3) is equal to x^*_{A1}. In order to specify the properties of that function F rearrange terms in (A3) to obtain

$$x_{A1}\left[1 + (p_x+\tau_2)^{1-\sigma_A}(1-\tau_1)^{\sigma_A}\bar{\gamma}_A^{-1}\right] = y_{Ao} + \tau_2 x_{A2}. \tag{A5}$$

Differentiate (A5) and take into account that $\mathrm{d}x_{A1} = \mathrm{d}x_{A2} = \mathrm{d}y_{Ao}$ to get

$$\begin{aligned} &x_{A1}\left[(1-\sigma_A)(p_x+\tau_2)^{-\sigma_A}(1+\tau_1)^{\sigma_A}\bar{\gamma}_A^{-1}\mathrm{d}\tau_2\right] \\ &+\sigma_A(p_x+\tau_2)^{1-\sigma_A}(1+\tau_1)^{\sigma_A-1}\bar{\gamma}_A^{-1}\mathrm{d}\tau_1 = x_{A2}\mathrm{d}\tau_2 \end{aligned} \tag{A6}$$

or equivalently

$$\frac{\mathrm{d}\tau_1}{\mathrm{d}\tau_2} = F_{\tau_2} = \frac{x_{A2} - (1-\sigma_A)x_{A1}(p_x+\tau_2)^{-\sigma_A}(1+\tau_1)^{\sigma_A}\bar{\gamma}_A^{-1}}{\sigma_A x_{A1}(p_x+\tau_2)^{1-\sigma_A}(1+\tau_1)^{\sigma_A-1}\bar{\gamma}_A^{-1}}. \tag{A7}$$

Account for $\frac{x_{A1}}{x_{A2}} = \bar{\gamma}_A \left(\frac{p_x+\tau_2}{1+\tau_1}\right)^{\sigma_A}$ to turn (A7) into

$$\frac{\mathrm{d}\tau_1}{\mathrm{d}\tau_2} = \frac{\sigma_A x_{A2}}{\sigma_A x_{A1}(p_x+\tau_2)^{1-\sigma_A}(1+\tau_1)^{\sigma_A-1}\bar{\gamma}_A^{-1}} > 0. \tag{A8}$$

Inspection of (A3) shows that the term on the right side of (A3) can attain any non-negative value either if $\tau_2 = 0$ and τ_1 is varied on $\mathbb{R}$ or if $\tau_1 = 0$ and τ_2 is varied on $\mathbb{R}$.

B. Proof of Proposition 4

Consider first the supply-side partial equilibrium established by the equations

$$\begin{aligned}
e_{A1} + e_{B1} &= \bar{e}_1, \qquad e_{A2} + e_{B2} = \bar{e}_2 := \bar{e} - \bar{e}_1, && \text{(B1)}\\
X^A_{e_{A1}}(e_{A1}) &= p_e + \pi_{A1}, && \text{(B2)}\\
X^B_{e_{B1}}(\bar{e}_1 - e_{A1}) &= p_e, && \text{(B3)}\\
p_x X^A_{e_{A2}}(e_{A2}) &= p_e + \pi_{A2}, && \text{(B4)}\\
p_x X^B_{e_{B2}}(\bar{e}_2 - e_{A2}) &= p_e. && \text{(B5)}
\end{aligned}$$

These equations determine p_e, p_x, π_1, π_2 and x^s_{it} for $i = A, B$ and $t = 1, 2$ for every $(e_{A1}, e_{A2}) \in [0, \bar{e}_1] \times [0, \bar{e}_2]$. That equilibrium is partial because the demand for good X still needs to be specified in order to establish equilibrium on the commodity markets,[29]

$$x_{A1} + x_{B1} = x^s_{A1} + x^s_{B1}. \tag{B6}$$

The CES utility functions yield the demands

$$\begin{aligned}
x_{A1} &= \frac{\left(\frac{\gamma_1 p_x}{(1+\tau_1)\gamma_2}\right)^{\sigma}(y_{Ao} + \tau_1 x_{A1})}{(1+\tau_1)\left(\frac{\gamma_1 p_x}{(1+\tau_1)\gamma_2}\right)^{\sigma} + p_x} = \frac{(\gamma_1 p_x)^{\sigma} y_{Ao}}{(\gamma_1 p_x)^{\sigma} + (1+\tau_1)^{\sigma}\gamma_2^{\sigma} p_x}, && \text{(B7)}\\
x_{B1} &= \frac{\left(\frac{\gamma_1 p_x}{\gamma_2}\right)^{\sigma} y_B}{p_x + \left(\frac{\gamma_1 p_x}{\gamma_2}\right)^{\sigma}}, && \text{(B8)}
\end{aligned}$$

where y_{Ao} and y_B are incomes (see (10)) with y_{Ao} representing income before the tax revenue $\tau_1 x_{A1}$ is recycled to the consumer. Note that if $\sigma > 0$ and $\tau_1 \in \mathbb{R}$, x_{A1} in (B7) and x_{B1} in

[29] If (B6) is satisfied, the second-period market for good X is also in equlibrium owing to Walras law.

(B8) are fully determined for every $(e_{A1}, e_{A2}) \in [0, \bar{e}_1] \times [0, \bar{e}_2]$.

We first investigate how x_{B1} varies with σ. Since utility maximization implies

$$\frac{x_{B1}}{x_{B2}} = \left(\frac{\gamma_1 p_x}{\gamma_2}\right)^{\sigma}, \tag{B9}$$

the straightforward conclusion from (B9) and constant y_B is

$$\frac{\mathrm{d}x_{B1}}{\mathrm{d}\sigma} \begin{cases} > 0, \text{ if } (\gamma_1 p_x/\gamma_2) > 1 & (\text{and } x_{B1} \to y_B, \text{ if } \sigma \to \infty), \\ = 0, \text{ if } (\gamma_1 p_x/\gamma_2) = 1, & \\ < 0, \text{ if } (\gamma_1 p_x/\gamma_2) < 1 & (\text{and } x_{B1} \to 0, \text{ if } \sigma \to \infty). \end{cases} \tag{B10}$$

Consequently, disregarding the knife-edge case $(\gamma_1 p_x/\gamma_2) = 1$ we find that for every (e_{A1}, e_{A2}) there is some set $S \subset \mathbb{R}$ with non-empty interior such that

$$x_{B1} \leq x^s_{A1} + x^s_{B1} \quad \forall \sigma \in S. \tag{B11}$$

Since $x_{A1} \geq 0$, (B11) is clearly a necessary condition for (B6). Given (B11), sufficient for (B6) is $x_{A1} = x^s_{A1} + x^s_{B1} - x_{B1}$. For $\tau_1 = 0$ this equality will not hold, in general. But x_{A1} varies with τ_1. Differentiation of (B7) with respect to τ_1 yields

$$\frac{\mathrm{d}x_{A1}}{\mathrm{d}\tau_1} = -\frac{(\gamma_1 p_x)^{\sigma} y_{Ao} \sigma (1+\tau_1)^{\sigma-1}}{\left[(\gamma_1 p_x)^{\sigma} + (1+\tau_1)^{\sigma} \gamma_2^{\sigma} p_x\right]^2} \quad \text{and} \quad \lim_{\sigma \to 0} \frac{\mathrm{d}x_{A1}}{\mathrm{d}\tau_1} = 0. \tag{B12}$$

It follows that if $\sigma \in S$ and $x_{A1} \neq x^s_{A1} + x^s_{B1} - x_{B1}$ for $\tau_1 = 0$, one can change the magnitude of x_{A1} by an appropriate choice of τ_1 such that (B6) is satisfied. This completes the proof of Proposition 4.

C. Proof of Proposition 5

A unilateral ceiling-policy equilibrium with policy $(\pi_1, \pi_2, \tau_1, \tau_2 = 0)$ is characterized by the following equations

$$X^A_{e_{A1}} - p_e - \pi_1 = 0, \tag{C1}$$

$$X^B_{e_{B1}} - p_e = 0, \tag{C2}$$

$$p_x X^A_{e_2} - p_e - \pi_2 = 0, \tag{C3}$$

$$p_x X^B_{e_{B2}} - p_e = 0, \tag{C4}$$

$$e_{A1} - \bar{e}_{A1} = 0, \tag{C5}$$

$$e_{A2} - \bar{e}_{A2} = 0, \tag{C6}$$

$$e_t - e_{At} - e_{Bt} = 0, \quad t = 1, 2, \tag{C7}$$

$$\bar{e} - e_1 - e_2 = 0, \tag{C8}$$

$$X^A(e_{A1}) - x_{A1} + p_x\left[X^A(e_{A2}) - x_{A2}\right] + p_e\Delta e_A = 0, \tag{C9}$$
$$X^B(e_{B1}) - x_{B1} + p_x\left[X^B(e_{B2}) - x_{B2}\right] + p_e\Delta e_B = 0, \tag{C10}$$
$$X^A(e_{A2}) + X^B(e_{B2}) - x_{A2} - x_{B2} = 0, \tag{C11}$$
$$\frac{U_{x_{A2}}}{U_{x_{A1}}} - \frac{p_x}{1+\tau_1} = 0, \tag{C12}$$
$$\frac{U_{x_{B2}}}{U_{x_{B1}}} - p_x = 0, \tag{C13}$$

where $\Delta e_i := \alpha_i \bar{e} - e_{i1} - e_{i2}$ for $i = A, B$. The endogenous variables determined by the 14 equations (C1)-(C13) are e_{A1}, e_{A2}, e_{B1}, e_{B2}, e_1, e_2 x_{A1}, x_{A2}, x_{B1}, x_{B2}, p_e, p_x, π_1 and π_2. The emissions $\bar{e}_{A1}$, $\bar{e}_{A2}$ and the tax rate τ_1 are treated here as exogenous parameters. Total differentiation of (C1) - (C13) yields, after some rearrangement of terms,

$$\frac{1}{\eta_{A1}}\hat{e}_{A1} - \delta_1\hat{p}_e - (1-\delta_1)\hat{\pi}_1 = 0, \tag{C14}$$
$$\frac{1}{\eta_{B1}}\hat{e}_{B1} - \hat{p}_e = 0, \tag{C15}$$
$$\hat{p}_x + \frac{1}{\eta_{A2}}\hat{e}_{A2} - \delta_2\hat{p}_e - (1-\delta_2)\hat{\pi}_2 = 0, \tag{C16}$$
$$\hat{p}_x + \frac{1}{\eta_{B2}}\hat{e}_{B2} - \hat{p}_e = 0, \tag{C17}$$
$$\hat{e}_{A1} - \hat{\bar{e}}_{A1} = 0, \tag{C18}$$
$$\hat{e}_{A2} - \hat{\bar{e}}_{A2} = 0, \tag{C19}$$
$$e_1\hat{e}_1 - e_{A1}\hat{e}_{A1} - e_{B1}\hat{e}_{B1} = 0, \tag{C20}$$
$$e_2\hat{e}_2 - e_{A2}\hat{e}_{A2} - e_{B2}\hat{e}_{B2} = 0, \tag{C21}$$
$$e_1\hat{e}_1 + e_2\hat{e}_2 = 0, \tag{C22}$$
$$\pi_1 e_{A1}\hat{e}_{A1} + \pi_2 e_{A2}\hat{e}_{A2} - x_{A1}\hat{x}_{A1} - p_x x_{A2}\hat{x}_{A2} + \Delta e_A p_e\hat{p}_e + \Delta x_{A2}p_x\hat{p}_x = 0, \tag{C23}$$
$$-x_{B1}\hat{x}_{B1} - p_x x_{B2}\hat{x}_{B2} + \Delta e_B p_e\hat{p}_e + \Delta x_{B2}p_x\hat{p}_x = 0, \tag{C24}$$
$$(p_e+\pi_2)e_{A2}\hat{e}_{A2} + p_e e_{B2}\hat{e}_{B2} - p_x x_{A2}\hat{x}_{A2} - p_x x_{B2}\hat{x}_{B2} = 0, \tag{C25}$$
$$\hat{x}_{A2} - \hat{x}_{A1} + \hat{p}_x\sigma - \frac{\sigma\tau_1}{1+\tau_1}\hat{\tau}_1 = 0, \tag{C26}$$
$$\hat{x}_{B2} - \hat{x}_{B1} + \hat{p}_x\sigma = 0, \tag{C27}$$

where hat variables are defined as $\hat{y} := \mathrm{d}y/y$, and $\delta_t := \frac{p_e}{p_e+\pi_{At}}$, $\Delta x_{it} := x^s_{it} - x_{it}$ and $\eta_{it} := X^i_{e_{it}} / \left(e_{it}X^i_{e_{it}e_{it}}\right)$ for $t = 1, 2$ and $i = A, B$. Next, we derive the comparative static results of increases in τ_1 when the emissions $\bar{e}_{A1}$ and $\bar{e}_{A2}$ are constant. Formally, we set $\hat{\bar{e}}_{A1} = \hat{\bar{e}}_{A2} = 0$ and insert (C26) in (C23) and (C27) in (C24) to obtain

$$\hat{x}_{A2} = \frac{\Delta e_A p_e}{y_A}\hat{p}_e + \frac{p_x\Delta x_{A2} - \sigma x_{A1}}{y_A}\hat{p}_x + \frac{\sigma\tau x_{A1}}{(1+\tau_1)y_A}\hat{\tau}_1, \tag{C28}$$
$$\hat{x}_{B2} = \frac{\Delta e_B p_e}{y_B}\hat{p}_e + \frac{p_x\Delta x_{B2} - \sigma x_{B1}}{y_B}\hat{p}_x, \tag{C29}$$

where $\Delta e_i := \alpha_i \bar{e} - e_{i1} - e_{i2}$ for $i = A, B$. Using (C20) in (C15), and (C21), (C22) in (C17) to get

$$\hat{p}_e = \frac{e_1}{e_{B1}\eta_{B1}}\hat{e}_1, \tag{C30}$$

$$\hat{p}_x = \frac{e_{B1}\eta_{B1} + e_{B2}\eta_{B2}}{e_{B1}\eta_{B1}e_{B2}\eta_{B2}} e_1\hat{e}_1. \tag{C31}$$

Inserting $\hat{e}_{B2}e_{B2} = -\hat{e}_1 e_1$ in (C25) yields

$$-p_e e_1 \hat{e}_1 = p_x \sum_{i=A,B} x_{i2}\hat{x}_{i2}. \tag{C32}$$

We make use of (C28) and (C29) in (C32) and obtain

$$\begin{aligned}\frac{p_x x_{A2} x_{A1} \sigma \mathrm{d}\tau_1}{(1+\tau_1)y_A} &= -p_e e_1 \hat{e}_1 - \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B}\right)\Delta e_A p_e \hat{p}_e \\ &- \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B}\right) p_x \Delta x_{A2}\hat{p}_x + \sigma \left(\frac{p_x x_{A1} x_{A2}}{y_A} + \frac{p_x x_{B1} x_{B2}}{y_B}\right)\hat{p}_x \end{aligned} \tag{C33}$$

which with the help of (C30) and (C31) can be rearranged to

$$\begin{aligned}\frac{p_x x_{A1} x_{A2} \sigma \mathrm{d}\tau_1}{(1+\tau_1)y_A \mathrm{d}e_1} &= -p_e + \frac{1}{e_{B2}\eta_{B2}}\left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B}\right)\Delta e_A \\ &+ \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B}\right)\Delta x_{A1}\left(\frac{e_{B1}\eta_{B1} + e_{B2}\eta_{B2}}{e_{B1}\eta_{B1}e_{B2}\eta_{B2}}\right) \\ &+ \sigma\left(\frac{p_x x_{A1} x_{A2}}{y_A} + \frac{p_x x_{B1} x_{B2}}{y_B}\right)\left(\frac{e_{B1}\eta_{B1} + e_{B2}\eta_{B2}}{e_{B1}\eta_{B1}e_{B2}\eta_{B2}}\right). \end{aligned} \tag{C34}$$

For CES functions (14) holds $x_{A1} = x_{A2}\left(\frac{\gamma_1 p_x}{\gamma_2(1+\tau_1)}\right)^\sigma$, $x_{B1} = x_{B2}\left(\frac{\gamma_1 p_x}{\gamma_2}\right)^\sigma$ and hence

$$\frac{p_x x_{A2}}{x_{A1} + p_x x_{A2}} = \frac{p_x}{p_x + \left(\frac{\gamma_1 p_x}{\gamma_2(1+\tau_1)}\right)^\sigma} \geq \frac{p_x x_{B2}}{x_{B1} + p_x x_{B2}} = \frac{p_x}{p_x + \left(\frac{\gamma_1 p_x}{\gamma_2}\right)^\sigma}. \tag{C35}$$

From inserting (C30) and (C31) in (C14) and (C16), respectively, follows

$$\hat{\pi}_{A1} = -\frac{\delta_1}{(1-\delta_1)e_{B1}\eta_{B1}} e_1\hat{e}_1 \tag{C36}$$

$$\hat{\pi}_{A2} = \left[\frac{1}{e_{B1}\eta_{B1}} + \frac{1}{(1-\delta_2)e_{B2}\eta_{B2}}\right] e_1\hat{e}_1 \tag{C37}$$

We start from a symmetric (laissez-faire) equilibrium without any policy, i.e. with $\Delta e_A = \Delta x_{A1} = \Delta x_{A2} = \pi_1 = \pi_2 = \tau_1 = 0$ and $\bar{e}_{A1} = e^o_{A1}$, $\bar{e}_{A2} = e^o_{A2}$. The comparative static results are summarized in Table 3.

	$\mathrm{d}e_1$	$\mathrm{d}p_e$	$\mathrm{d}p_x$	$\mathrm{d}x_{B2}$	$\mathrm{d}x_{A2}$	$\mathrm{d}e_{A1} = \mathrm{d}e_{A2}$	$\mathrm{d}e_{B1} = -\mathrm{d}e_{B2}$
$\mathrm{d}\tau_1 > 0$	$-$	$+$	$+$	$-$	$+$	0	$-$
follows from eq.	(C34)	(C30)	(C31)	(C29)	(C32)		

	$\mathrm{d}\Delta x_{A2} = -\mathrm{d}\Delta x_{B2}$	$\mathrm{d}\Delta e_A = \mathrm{d}\Delta e_B$	$\mathrm{d}\Delta x_{A1} = \mathrm{d}\Delta x_{B1}$	$\mathrm{d}\pi_1$	$\mathrm{d}\pi_2$
$\mathrm{d}\tau_1 > 0$	$-$	0	$+$	$-$	$+$

Table 3: The comparative statics of increases in τ_1

Next, we further increase the tax rate ($\tau_1 > 0$ and $\mathrm{d}\tau_1 > 0$) and account for the information of Table 3, in especially $\Delta e_A = 0$ and $\Delta x_{A1} > 0$. This comparative static analysis yields again the signs listed in Table 3.

D. Proof of Proposition 7

Ad (i): For the quadratic production functions (25) the supply-side partial equilibrium (B1) - (B5) turns into

$$\begin{aligned} p_e &= a - b(\bar{e}_1 - e_{A1}), && \text{(D1)}\\ p_x &= \frac{a - b(\bar{e}_1 - e_{A1})}{a - b(\bar{e}_2 - e_{A2})}, && \text{(D2)}\\ \pi_{A1} &= b(\bar{e}_1 - 2e_{A1}), && \text{(D3)}\\ \pi_{A2} &= \frac{[a - b(\bar{e}_1 - e_{A1})]b(\bar{e}_2 - 2be_{A2})}{a - b(\bar{e}_2 - e_{A2})} && \text{(D4)} \end{aligned}$$

and

$$\frac{\mathrm{d}p_e}{\mathrm{d}e_{A1}} = b, \qquad \frac{\mathrm{d}p_e}{\mathrm{d}e_{A2}} = 0, \tag{D5}$$

$$\frac{\mathrm{d}p_x}{\mathrm{d}e_{A1}} = \frac{b}{a - b(\bar{e}_2 - e_{A2})} = \frac{bp_x}{p_e}, \qquad \frac{\mathrm{d}p_x}{\mathrm{d}e_{A2}} = -\frac{p_e b}{[a - b\bar{e}_2 - e_{A2}]^2} = -\frac{bp_x^2}{p_e}. \tag{D6}$$

Next, we determine x_{A1} and x_{A2} with the help of country B's demand

$$x_{B1} = \gamma y_B, \qquad x_{B2} = \frac{(1-\gamma)y_B}{p_x}. \tag{D7}$$

and with

$$X^A(e_{At}) + X^B(e_{Bt}) = x_{At} + x_{Bt} \qquad t = 1, 2. \tag{D8}$$

Observe that

$$x_t^s := X^A(e_{At}) + X^B(e_{Bt}) = a(e_{At} + e_{Bt}) - \frac{b}{2}\left(e_{At}^2 + e_{Bt}^2\right). \tag{D9}$$

From $e_{At}^2+e_{Bt}^2 = (e_{At}+e_{Bt})^2 - 2e_{At}e_{Bt}$ follows $x_t^s = x_t^{s*} - b\left(\frac{\bar{e}_t^2}{4} - e_{At}e_{Bt}\right)$ with $x_t^{s*} := a\bar{e}_t - \frac{b\bar{e}_t^2}{4}$. Combined with (D8) we get

$$x_{At} = x_t^s - x_{Bt} = x_t^{s*} - \frac{b\bar{e}_t^2}{4} + be_{At}e_{Bt} - x_{Bt}. \tag{D10}$$

Making use of (D7) and inserting (D10) into the Cobb-Douglas utility function yields

$$u_A = \left[\underbrace{\left(x_1^{s*} - \frac{b\bar{e}_1^2}{4}\right) + be_{A1}(\bar{e}_1 - e_{A1}) - \gamma y_B}_{x_{A1}}\right]^{\gamma}\left[\underbrace{\left(x_2^{s*} - \frac{b\bar{e}_2^2}{4}\right) + be_{A2}(\bar{e}_2 - e_{A2}) - \frac{(1-\gamma)y_B}{p_x}}_{x_{A2}}\right]^{1-\gamma}. \tag{D11}$$

We differentiate u_A in (D11) with respect to e_{A1} and e_{A2} and account for (D5), (D6) and

$$\frac{\mathrm{d}y_B}{\mathrm{d}e_{A1}} = \frac{bp_x x_{B2}^s}{p_e} - b\Delta e_A, \tag{D12}$$

$$\frac{\mathrm{d}y_B}{\mathrm{d}e_{A2}} = -\frac{b}{p_e}p_x^2 x_{B2}^s, \tag{D13}$$

to obtain after rearrangement of terms

$$\frac{\mathrm{d}u_A}{\mathrm{d}e_{A1}} = \frac{bu_A}{p_e}\left[\frac{\gamma}{x_{A1}}[p_e(e_{B1}-e_{A1}) - \gamma(y_B - x_{B1}^s)] + \frac{(1-\gamma)^2 x_{B1}^s}{p_x x_{A2}}\right], \tag{D14}$$

$$\frac{\mathrm{d}u_A}{\mathrm{d}e_{A2}} = \frac{bu_A}{p_e}\left[\frac{\gamma^2 p_x^2 x_{B2}^s}{x_{A1}} + \frac{1-\gamma}{x_{A2}}[p_e(e_{B2}-e_{A2}) - (1-\gamma)(y_B - p_x x_{B2}^s)]\right]. \tag{D15}$$

Next, we use (D14), (D15) and $\frac{x_{A1}(1-\gamma)}{x_{A2}\gamma} = \frac{p_x}{1+\tau_1}$ in $\frac{\mathrm{d}u_A}{\mathrm{d}e_{A1}} = 0$ and $\frac{\mathrm{d}u_A}{\mathrm{d}e_{A2}} = 0$, respectively, to get

$$\frac{\mathrm{d}u_A}{\mathrm{d}e_{A1}} = 0 \iff (1+\tau_1)\left[p_e(e_{B1}-e_{A1}) - \gamma(y_B - x_{B1}^s)\right] + (1-\gamma)x_{B1}^s = 0, \tag{D16}$$

$$\frac{\mathrm{d}u_A}{\mathrm{d}e_{A2}} = 0 \iff p_e(e_{B2}-e_{A2}) + (1+\tau_1)\gamma p_x x_{B2}^s - (1-\gamma)(y_B - p_x x_{B2}^s) = 0. \tag{D17}$$

Summing up (D16) and (D17) we obtain

$$(1+\tau_1)(e_{B1}-e_{A1}) + (e_{B2}-e_{A2}) = -(1+\gamma\tau_1)\Delta e_A. \tag{D18}$$

Then we make use of (D1) - (D4) and get

$$(1+\tau_1)\pi_1 + \frac{\pi_2}{p_x} = -(1+\gamma\tau_1)b\Delta e_A. \tag{D19}$$

Observe that (D8) for $t = 1$, (D14) and (D19) determine the cost-effective policy (e_{A1}, e_{A2}, τ_1). Total differentiation of (D19) yields

$$-(3+2\tau_1+\tau_1\gamma)\mathrm{d}e_{A1} - (3+\tau_1\gamma)\mathrm{d}e_{A2} + (\bar{e}_1 - 2e_{A1} + \gamma\Delta e_A)\mathrm{d}\tau_1 + \tau_1\mathrm{d}\bar{e}_1 = 0. \tag{D20}$$

At the laissez faire values $\tau_1 = 0$, $e_{A1}^0 = e_{B1}^0 = \bar{e}_1^0$, $\Delta e_A^0 = 0$, (D20) simplifies to

$$3\mathrm{d}e_{A1} + 3\mathrm{d}e_{A2} = 0. \tag{D21}$$

Next we account for $x_{B1} = \gamma y_B$ and $x_{A1} = \underbrace{\frac{1}{1+\tau_1(1-\gamma)}}_{=:\check{\tau}_1} \gamma \underbrace{[X(e_{A1}) + p_x X(e_{A2}) + p_e \Delta e_A]}_{=:y_{Ao}}$ and obtain after total differentiation of the equation

$$x_{A1} + x_{B1} - X(e_{A1}) - X(\bar{e}_1 - e_{A1}) = 0 \tag{D22}$$

$$\begin{aligned}
&\left[\check{\tau}_1\gamma(X_{e_{A1}} - p_e) + \frac{bp_x x^s_{A2}}{p_e} + b\Delta e_A + \gamma\left(\frac{bp_x x^s_{B2}}{p_e} - b\Delta e_A\right) - X_{e_{A1}} + X_{e_{A1}}\right] \mathrm{d}e_{A1} \\
+ \quad &\left[\check{\tau}_1\gamma\left(p_x X_{e_{A2}} - p_e - \frac{bp_x^2 x^s_{A2}}{p_e}\right) - \left(\frac{\gamma b p_x^2 x^s_{B2}}{p_e}\right)\right] \mathrm{d}e_{A2} - \frac{(1-\gamma)}{[1+\tau_1(1-\gamma)]^2}\gamma y_{Ao} \mathrm{d}\tau_1 \\
+ \quad &\left[\check{\tau}_1\gamma\left(-\frac{2bp_x x^s_{A2}}{p_e} - \frac{bp_x^2 x^s_{A2}}{p_e} - b\Delta e_A\right)\right. \\
&\qquad \left. + \gamma\left(-\frac{bp_x x^s_{B2}}{p_e} - \frac{bp_x^2 x^s_{B2}}{p_e} - b\Delta e_B\right) - X_{e_{B1}}\right] \mathrm{d}\bar{e}_1 = 0.
\end{aligned} \tag{D23}$$

Assessing (D23) at laissez faire, which is characterized by $\tau_1 = 0$, $X_{e_{A1}} = p_e$, $p_x X_{e_{A2}} = p_e$, $X_{e_{A1}} = X_{e_{B1}}$, $\Delta e_A = \Delta e_B = 0$, $\gamma y_A = X(e_{A1}) = x^s_{A1} = x^s_{B1}$, $x^s_{B2} = x^s_{A2}$, we get

$$\frac{2\gamma b p_x x^s_{A2}}{p_e}\mathrm{d}e_{A1} - \frac{2\gamma b p_x^2 x^s_{A2}}{p_e}\mathrm{d}e_{A2} - (1-\gamma)x^s_{B1}\mathrm{d}\tau_1 = \left[2\gamma\left(\frac{bp_x x^s_{A2}}{p_e} + \frac{bp_x^2 x^s_{A2}}{p_e}\right) + p_e\right]\mathrm{d}\bar{e}_1. \tag{D24}$$

Finally, we differentiate (D14) to get

$$\begin{aligned}
&\left[(\bar{e}_1 - 2e_{A1})\frac{\mathrm{d}p_e}{\mathrm{d}e_{A1}} - 2p_e - \gamma\left(\frac{\mathrm{d}y_B}{\mathrm{d}e_{A1}} + X_{e_{B1}}\right) - \frac{(1-\gamma)}{1+\tau_1}X_{e_{B1}}\right]\mathrm{d}e_{A1} \\
+ \quad &\left[(\bar{e}_1 - 2e_{A1})\frac{\mathrm{d}p_e}{\mathrm{d}e_{A2}} - \gamma\frac{\mathrm{d}y_B}{\mathrm{d}e_{A2}}\right]\mathrm{d}e_{A2} - \frac{(1-\gamma)x^s_{B1}}{(1+\tau_1)^2}\mathrm{d}\tau_1 \\
+ \quad &\left[(\bar{e}_1 - 2e_{A1})\frac{\mathrm{d}p_e}{\mathrm{d}\bar{e}_1} - \gamma\left(\frac{\mathrm{d}y_B}{\mathrm{d}\bar{e}_1} - X_{e_{B1}}\right) + \frac{(1-\gamma)}{1+\tau_1}X_{e_{B1}}\right]\mathrm{d}\bar{e}_1 = 0.
\end{aligned} \tag{D25}$$

Assessing (D25) at laissez-faire, (D25) turns into

$$\begin{aligned}
&\left[-3p_e - \frac{\gamma b p_x x^s_{B2}}{p_e}\right]\mathrm{d}e_{A1} + \frac{\gamma b p_x^2 x^s_{B2}}{p_e}\mathrm{d}e_{A2} - (1-\gamma)x^s_{B1}\mathrm{d}\tau_1 \\
&\qquad = -\left[\gamma\left(\frac{bp_x x^s_{B2}}{p_e} + \frac{p_x^2 x^s_{B2}}{p_e}\right) + p_e\right]\mathrm{d}\bar{e}_1.
\end{aligned} \tag{D26}$$

(D21), (D24) and (D26) jointly determine $\mathrm{d}e_{A1}$, $\mathrm{d}e_{A2}$ and $\mathrm{d}\tau_1$. In matrix notation, these equations read

$$\begin{bmatrix} 3 & 3 & 0 \\ 2a & -2ap_x & -(1-\gamma)x^s_{B1} \\ -3p_e - a & ap_x & -(1-\gamma)x^s_{B1} \end{bmatrix} \begin{bmatrix} \mathrm{d}e_{A1} \\ \mathrm{d}e_{A2} \\ \mathrm{d}\tau_1 \end{bmatrix} = \begin{bmatrix} 0 \\ [2(a + ap_x) + p_e]\,\mathrm{d}\bar{e}_1 \\ (-a - ap_x - p_e)\,\mathrm{d}\bar{e}_1 \end{bmatrix}, \tag{D27}$$

where $a := \frac{\gamma b p_x x^s_{B2}}{p_e}$. Solving the equation system (D27) by using Cramer's rule yields

$$\frac{\mathrm{d}e_{A1}}{\mathrm{d}\bar{e}_1} = \frac{3(a + ap_x) + 2p_e}{3(a + ap_x p_e)} > 0, \tag{D28}$$

$$\frac{\mathrm{d}e_{A2}}{\mathrm{d}\bar{e}_1} = -\frac{3(a + ap_x) + 2p_e}{3(a + ap_x p_e)} < 0, \tag{D29}$$

$$\frac{\mathrm{d}\tau_1}{\mathrm{d}\bar{e}_1} = -\frac{5p_e(a + ap_x) + 3p_e}{3x^s_{B1}(1 - \gamma)(a + ap_x p_e)} < 0. \tag{D30}$$

The associated changes of the prices p_e and p_x follow from making use of (D28) and (D29) in (D5) and (D6).

Ad (ii): In case of $\tau_1 = \mathrm{d}\tau_1 = 0$, (D24) and (D26) jointly determine $\mathrm{d}e_{A1}$ and $\mathrm{d}e_{A2}$. In matrix notation, these equations read

$$\begin{bmatrix} 2a & -2ap_x \\ -3p_e - a & ap_x \end{bmatrix} \begin{bmatrix} \mathrm{d}e_{A1} \\ \mathrm{d}e_{A2} \end{bmatrix} = \begin{bmatrix} [2(a + ap_x) + p_e]\,\mathrm{d}\bar{e}_1 \\ (-a - ap_x - p_e)\,\mathrm{d}\bar{e}_1 \end{bmatrix}. \tag{D31}$$

Solving the equation system (D31) we obtain

$$\frac{\mathrm{d}e_{A1}}{\mathrm{d}\bar{e}_1} = \frac{ap_x p_e}{6ap_x p_e} = \frac{1}{6} > 0, \tag{D32}$$

$$\frac{\mathrm{d}e_{A2}}{\mathrm{d}\bar{e}_1} = -\frac{5a + 6ap_x + 3p_e}{6ap_x} < 0. \tag{D33}$$