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CO_2 mitigation in road transport: Gasoline taxation and/or fuel-efficiency regulation?

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Abstract: Although gasoline taxes are widely used (nearly) efficient CO_2 emission controls, *additional* fuel-efficiency regulation is applied e.g. in the USA and in Europe. In a simple *analytical* model, we specify the welfare implications of (i) gasoline taxes, (ii) of 'gas-guzzler taxes' (iii) of fuel-efficiency standards, and of combinations of the above. Both forms (ii) and (iii) of fuel-efficiency regulation turn out to produce the same suboptimally low emission rates. Combining (i) and (ii) is also distortionary, while efficiency *can* be secured by combining (i) and (iii). However, in the optimal mix of the latter two instruments the fuel-efficiency standard is redundant.

JEL Classifications: D61, H21, H22, Q52, Q53, Q58

Keywords: CO_2 emissions; road transport; fuel efficiency regulation; gasoline tax

1 The problem

Although the social cost of vehicular CO_2 emissions is found to be small relative to other external costs in the transportation sector,[1] rising pressures toward curbing greenhouse gas emissions and improving energy security warrant a closer look at instruments for controlling that externality. Road transport related CO_2 emissions make up a significant share of total CO_2 emissions amounting to about 24 percent of total CO_2 emissions (COM 2005, 261). The fossil energy share of vehicle transport still tends to increase - even in industrialized countries and even despite tightened gasoline taxation in some countries.

Since a one-to-one relation between gasoline consumption and vehicle CO_2 emissions is a reasonable approximation, "… the gasoline tax (or more generally, carbon content taxes) are nearly the perfect Pigouvian instrument for mitigating global warming." (Harrington and McConnell 2003, p. 254; see also Jansen and Denis 1999).[2] In other words, the gasoline tax is (nearly) a *CO_2 emissions tax* which is known as a cost effective policy tool to control CO_2 emissions.[3] As a corollary and in contrast, any other piece of regulation intended to curb CO_2 emissions must be suspected to cause allocative distortions, since it does not target these emissions precisely and exclusively but may be based, instead, on some vehicle characteristics that are only weakly associated with emissions. Such instruments must be expected to create distortions not only, when they are the only CO_2 emission controls but also when used jointly with other instruments, notably with CO_2 emissions taxes.

A relevant case in point is fuel-economy regulation defined here as regulation of vehicular *CO_2 emission-rates*[4] *either* by mandating average emission-rate standards *or* by taxing emission rates. The latter strategy means to levy an annual circulation tax that is precisely based on the CO_2 emission rate of the vehicle determined as an average value according to some

[1] For an extensive survey, see Harrington and McConnell (2003).
[2] For all other vehicle-related pollution externalities, such as emissions of carbon monoxide, nitrogen oxides, volatile organic compounds and particulate matter, no such convenient linear relation between fuel consumption and emissions exists. Since suitable measurement technologies are not available, taxing those emissions directly is not feasible. Fullerton and West (2002) and Fullerton and Gan (2005) focus on such pollutants and investigate the cost effectiveness of policies that are feasible.
[3] Eriksson (1993) argues that consumers use an exceptionally high implicit discount rate for fuel-efficiency investments, which would create a welfare bias against fuel taxes and in favor of emission-rate regulation. Kleit (1990) views new-vehicle markets as efficient, however, because buyers are reasonably well informed about fuel economy. Here we will neglect the market failure that might be caused by consumer myopia.
[4] The *emission rate* of a vehicle is its average emission per kilometer, which we will here take to be equal to its (average) *gasoline rate* or *fuel rate*. The latter indicator is inverse to the indicator *fuel economy* (= kilometers per liter of fuel or 'miles per galleon'): Increasing fuel economy means decreasing the emission rate and vice versa. The CO_2 emission rate is not entirely a technical attribute of vehicles because it also depends on how the car is driven (aggressive driving, cold start-ups, short-distance driving in towns or long-distance driving on highways). Yet there are standardized accepted procedures to calculate average emission rates for any type of vehicle, and this is what we focus on in the present paper.

acknowledged rule of measurement. Annual circulation taxes (or car ownership taxes) are levied in many countries with various different tax base characteristics. To the extent that their base is engine size or stroke volume, they are already weakly related to CO_2 emission rates. The EU Commission advocates a more specific proposal. In its proposal for a Council Directive on passenger car related taxes (COM 2005, 261) it has suggested the introduction of a CO_2-dependent element in the tax base of both annual circulation and registration taxes (or vehicle purchase taxes). The public discussion on that proposal has intensified recently after the EU committed to ambitious emission reduction targets for the years to come. The second form of emission-rate regulation, the standard-setting approach, is applied in the USA since 1978, known as the Corporate Automobile Fuel Efficiency (CAFE) standard. More recently, car manufacturers in the European Union have voluntarily agreed to reduce the average emission rate of new cars to 140 grams of CO_2 per kilometer by the year 2008, and the European Commission suggests bringing down further the emission rates to 130 grams CO_2 until 2012. These observations suggest that vehicle regulation targeting emission rates is empirically relevant and will likely be extended in the near future. It is therefore important to know how cost-effective the emission-rate regulation is, especially, when combined with taxes on emissions.

There is a rich theoretical and empirical literature on policies for curbing vehicle CO_2 emissions (and other emissions) surveyed by Harrington and McConnell (2003). That literature has placed much attention on the CAFE policy in the USA (Kleit (1990), NRC 2002, Portney et al. 2003). Tax policies have also been extensively explored and compared in many studies, notably in Jansen and Denis (1999), Fullerton and Gan (2005), Austin and Dinan (2005), and De Borger and Mayeres (2007).[5] These contributions focus on numerical analyses for policy advice. Accordingly, they are based on complex models capturing many institutional and other empirical details such as different types and engine sizes of vehicles, the age structure of vehicles, pollution control equipment or types of fuel. They deliver rich numerical results but, due to their complexity, they provide limited *analytical* insights only.

In a very complex analytical general equilibrium model, Fullerton and West (2002) study various forms of taxation – but no emission-rate standards – with respect to their ability to internalize emissions that (may) depend on miles driven, pollution-control equipment and per-gallon cleanliness. In their setting an ideal Pigouvian emissions tax is very complicated and impractical and therefore their main effort is on searching for alternative (combination of)

[5] In some of this literature, the focus is not explicitly on CO_2 emissions. See footnote 2.

taxes capable to achieve first best. In the present paper, we consider the fuel tax as a reasonable approximation of the Pigouvian tax, instead, and we aim at characterizing *analytically* the efficiency of emission-reduction strategies combining emission taxation and emission-rate regulation[6]. General information on the merits of the combined and overlapping use of emissions taxes and emission-rate regulation is desirable since in the USA, the CAFE regulation is combined with emissions taxation and since in Europe, with its tradition of quite high emission tax rates, the public discussion on EU-wide emission-rate standards and taxes on emissions rates has recently intensified. We will develop a simple but manageable analytical framework to clarify rigorously the implications for welfare and cost (in)effectiveness of vehicle emission regulation by means of (i) an emissions tax, (ii) a tax on the emission rate, (iii) a mandated emission-rate standard or (iv) a combination of the above.

Our approach differs from that of Fullerton and West (2002) in important aspects. We simplify greatly with regard to the determinants of emissions and model the market for passenger road transport as a market for road transport services. In contrast to Fullerton and West (2002) we derive the consumer demand for gasoline (and emissions) from a discrete choice model and we employ a more elaborate production technology and a monopolistic price-setting supplier of transport services while Fullerton and West (2002) assume fixed producer prices (per unit of car size and per unit of pollution-control equipment). Consequently, we will deal with two market imperfections in addition to the environmental externality: with a distortion due to the supplier's market power and with the emission rate of vehicles being a public consumption good. Due to the complexity of their model Fullerton and West (2002) can characterize efficiency only with the help of the pertaining marginal conditions whereas we are able to determine and compare explicitly the allocations attained with the various policies under consideration.

Specifically, we will map the highly complex real-world road transport sector[7] in a very simple partial equilibrium model in which the demand for vehicle use and vehicle kilometers traveled is derived from the consumers' discrete choice between traveling by car or in a mass transportation system. Consumer preferences are heterogeneous with respect to vehicle size,

[6] Austin and Dinan (2005) present an *empirical* study for the USA of combined changes in CAFE standards and gasoline taxes. Emission-rate taxation combined with emissions taxes does not yet seem to have been addressed systematically. Janson and Denis (1999) consider supplementary differential fees on vehicles of different sizes to correct for myopia on the part of drivers about the value of better fuel economy. De Borger and Mayeres (2007) include in their applied study ownership taxation. In their concluding remarks, Fullerton and Gan (2005) suggest as an item for further research to study a tax collected annually on any vehicle at a rate that is proportional to its emission rate.

[7] We restrict the analysis to passenger transport excluding freight transport.

which in turn relates linearly to engine size, traveling comfort, and emission rate. On the supply side, a single price-setting producer rents her cars to consumers at a uniform and 'all-inclusive' price per kilometer traveled. Her cost of producing and operating vehicles increases with vehicle size. The market equilibrium simultaneously determines the type of vehicle supplied as well as the amount of vehicle kilometers traveled. The total fuel consumption resulting from the emission rate of the cars provided and from total kilometers traveled jointly fixes total fuel consumption, which determines, in turn, total CO_2 emissions and the associated environmental damage.

We are able to characterize analytically the relevant welfare and equilibrium implications. Our framework is therefore suited to explore the allocative effects of the regulation outlined above. It will be confirmed that emissions taxes, applied in isolation, achieve efficiency, while the isolated use of either form of emission-rate regulation results in excessive reduction (!) of total emissions. The combined use of pairs of instruments turns out to be always inefficient with the exception of the strategy that combines the use of an emissions tax with an emission-rate standard. In that case, we find the standard to be redundant.

Section 2 of the paper develops the model of individual road transport services. In Section 3, we characterize and compare the welfare maximizing provision of these services and the market equilibrium. After that, we explore in Section 4 the welfare implications of different regulatory schemes: direct emission-rate standards, taxes on emission rates, emission taxes and combined strategies. Section 5 concludes.

2 The model

Consumers have the choice of either traveling by car or by some mass transportation system, i.e. they consume either individual transport services or mass transport services. We refer to the former as *i-services*, for short, and to the latter as *m-services*. Although our focus is on i-services, deriving the demand for these services requires considering the interdependencies between both types of services.

The demand for individual transport services

For simplicity, only one type of vehicle is available for i-services in our model, and only one mode of mass transport is available to which consumers may switch, if they prefer m-services over i-services. Denote by p_i the uniform market price per kilometer of i-services and by p_m the price of m-services. Both prices are 'all inclusive', meaning that in the case of i-services p_i encompasses the rental rate of the vehicle per kilometer including the cost of the vehicle's fuel consumption per kilometer. All consumers are endowed with the same fixed budget for transport services of any kind, $y > 0$. Therefore, consumer $h \in [0,1]$ will either purchase y / p_i kilometers of i-services or y / p_m kilometers of m-services. For $j = i, m$, consumer h's valuation in monetary terms of one unit of j-services is denoted z_{hj}. We assume

$$z_{hm} = 1 \quad \text{for all } h \in [0,1] \qquad \text{and} \qquad z_{hi} = b^2 v_i - |h - v_i| \quad \text{for } h \in [0,1]. \tag{1}$$

In (1), b^2 is a positive parameter[8], and $v_i \geq 0$ characterizes the vehicle that is at the i-consumers' disposal for traveling. Abstracting from real-world complexities, we assume that v_i reflects the following vehicle attributes simultaneously:

(i) v_i stands for the size of vehicles with its related traveling convenience; with increasing v_i one moves from subcompacts, to compacts and up to luxury limousines;

(ii) v_i also measures the fuel consumption per kilometer of the vehicle, called the *fuel rate*, which is an indicator of fuel economy (footnote 4);

(iii) v_i represents the vehicle *emissions rate* as well, due to the fact that CO_2 emissions are (nearly) proportional to fuel consumption.

As v_i is a decision variable of the supplier of i-services, the consumers of these services take the prevailing v_i as given. Technically speaking, the emission rate is a public consumption good.

According to (1), consumer *preferences for m-services are uniform and flat* to keep the analysis as simple as possible. However, consumers have *heterogeneous preferences with respect to i-services*. Traveling comfort tends to increase all consumers' willingness-to-pay, as expressed by the term $b^2 v_i$ in (1). But preferences are not monotone increasing in v_i. To see this, fix $h \in \,]0,1[$ in (1) and increase v_i, starting at $v_i = 0$. The consumer's utility will then be linearly rising for all $v_i < h$. $v_i = h$ is the fuel rate of consumer *h's* favorite vehicle. With v_i increasing further, consumer *h's* utility declines linearly implying that she begins to dislike

[8] The only reason for introducing the unusual convention of denoting variables by squared letters is to ease notation in the analysis below.

what she perceives to be an 'excessive' vehicle size. In other words, for each h there is a threshold value of v beyond which she values fuel economy over vehicle size, which we can also interpret as an indication that her environmental awareness increasingly overcompensates her utility from comfortable but emission-intensive traveling. Each i-consumer then balances her preference for comfortable vehicles against her environmental concerns about high emissions rates of those comfortable vehicles. Since all consumers have different favorite cars ($v_i = h$), consumers differ with respect to their sensibility for environmental damage.

Summing up, for given p_i, v_i and p_m consumer h's discrete choice between both transport modes depends on which one she values more. We formalize her decision by[9]

$$\max \left\{ \frac{y}{p_m}, \ \frac{y}{p_i}\left(b^2 v_i - \left|h - v_i\right|\right) \right\}.$$

Following that rule of choosing the mode of transport, h spends her entire budget, y, on i-services for given p_i, p_m and v_i, if h satisfies

$$\frac{y}{p_i}\left(b^2 v_i - \left|h - v_i\right|\right) \geq \frac{y}{p_m} \qquad \text{or} \qquad b^2 v_i - \frac{p_i}{p_m} - \left|h - v_i\right| \geq 0. \tag{2}$$

In case of $h \leq v_i$, (2) yields $b^2 v_i - (p_i / p_m) + h - v_i \geq 0$ or $h \geq (1 - b^2) v_i + (p_i / p_m) = = v_i - b^2 v_i + (p_i / p_m) =: h_\ell$ and for $h \geq v_i$ (2) implies $b^2 v_i - (p_i / p_m) - h + v_i \geq 0$ or $h \leq (1 + b^2) v_i - (p_i / p_m) = v_i + b^2 v_i - (p_i / p_m) =: h_u$. Obviously, h_ℓ is the lower marginal user of i-services and h_u is the upper marginal user of i-services, if h_ℓ, $h_u \in [0,1]$. Therefore, the number of consumers of i-services is

$$N^i = \begin{cases} \min\,(h_u, 1) - \max\,(0, h_\ell), \ \textit{if} \ h_\ell < h_u, \\ 0, \ \textit{otherwise}. \end{cases} \tag{3}$$

Rather than embarking on a tedious study of all possible cases, we exclude the polar cases $N^i = 1$ and $N^i = 0$ as being of minor interest. To keep the analysis as conclusive and transparent as possible, we also disregard those cases, in which $N^i > 0$ but either $h_\ell < 0$ or $h_u > 0$.

[9] While the all-or-none feature is not entirely realistic, it is essential for keeping the analysis tractable.

Hence we restrict our analysis to a subset of parameters for which N^i satisfies the condition $0 < h_\ell < h_u < 1$. This turns (3) into[10,11]

$$N^i = h_\ell - h_u = 2\left(b^2 v_i - \frac{p_i}{p_m}\right) > 0. \tag{4}$$

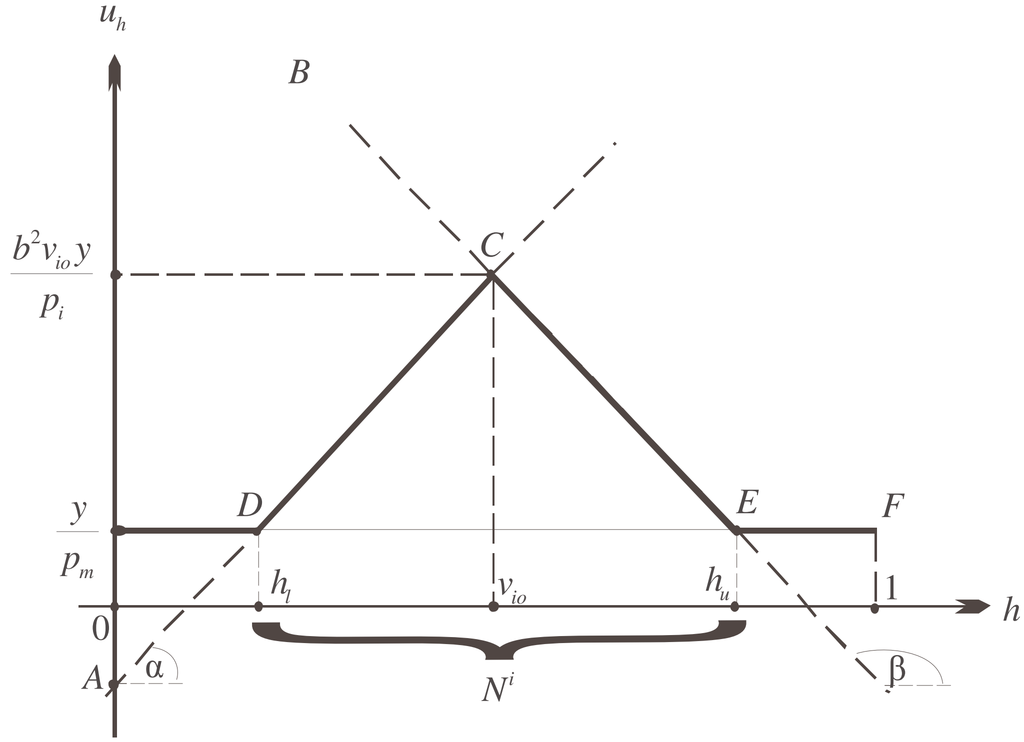

Figure 1: Utility of all consumers $h \in [0,1]$ for given p_i, p_m and v_{io}

Figure 1 illustrates the utility of all consumers $h \in [0,1]$ for given p_i, p_m and $v_{io} < \left[1 + (p_i / p_m)\right] / (1 + b^2)$. The slopes of the cone DCE are $tan\ \alpha = y / (p_i / p_m)$ and $tan\ \beta = -y / (p_i / p_m)$. Individuals $h \in [0, h_\ell]$ with low preferences for a vehicle of type v_{io} consume m-services. So do also all individuals $h \in [h_u, 1]$ but for a different reason: They would want to switch to i-services if, at the same relative price (p_i / p_m), a more comfortable

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[10] Note that $b^2 v_i > p_i / p_m$ is equivalent to the condition $h_\ell < h_u$

[11] In view of (3) and the definitions of h_ℓ and h_u, $N^i = h_\ell - h_u$ is the correct specification of N^i if and only if the constraints $p_m v_i (1 - b^2) + p_i \geq 0$ and $p_i + p_m \left[1 - v_i (1 + b^2)\right] \geq 0$ are satisfied. One can show that there is a set of parameter values for which these conditions hold in the relevant range of the model solutions.

type of vehicle would be available.[12] With $h_u < 1$, Figure 1 depicts a situation in which N^i is given by (4).

We have shown that consumer h goes for i-services if and only if $h \in [h_\ell, h_u]$. The aggregate demand for i-services therefore is $(y/p_i)N^i$.

The supply of individual transport services

There is a single firm that is capable to produce any size of vehicle, $v_i \in [0,1]$. However, actually the firm provides one and only one size. Its production costs consist of the following components:

kv_i^2 = fixed cost of producing automobiles of size v_i ($k > 0$ and constant)[13];

av_i = marginal cost of producing, operating and maintaining i-services of type v_i including fuel cost per kilometer of i-services ($a > 0$ and constant);

Specifying the fixed cost and the unit cost of i-services as increasing in v_i reflects our assumption that more powerful and therefore less fuel-efficient engines go along with larger and more comfortable vehicles. Due to this simplifying assumption, increasing v_i also means to increase the size of the vehicle, which in turn raises the costs of production and operation.

Putting together these cost components, it follows that with aggregate demand for i-services equal to $(y/p_i)N^i$ the firm's *total costs* are $K^i := kv_i + \frac{av_i y}{p_i}N^i$. With this specification of total costs and the revenue $R^i := p_i \frac{y}{p_i} N^i = yN^i$, the firm's profit is

$$\Pi^i := R^i - K^i = \frac{y}{p_i} N^i (p - av_i) - kv_i^2. \tag{5}$$

[12] We obviously want to restrict the parameter space to such values for which i-services are better than m-services for at least some v_i and some h which requires $b^2 v_i y > p_i$ to hold for some prices exceeding marginal cost.

[13] In a very long-run perspective, one may also add to the fixed costs the cost of designing the types of vehicles. However, even if the blueprints of all types are already freely available, substantial upfront investment is needed before the first vehicle can leave the assembly line.

Interrelatedness of individual and mass transport services

In the model developed so far, the demand for i-services, $\left(y/p_i\right)N^i$, not only depends on the price of i-services, p_i, but also on the price of m-services, p_m. Since $h\in[0,1]$ by assumption, the number of consumers of m-services is obviously $1-N^i$, and the corresponding aggregate demand for m-services is $\left(y/p_m\right)\left(1-N^i\right)$. Obviously, the demand for m-services depends on p_i and p_m as well implying that both segments of the transportation sector are interrelated through the demand for their services. That interdependence also extends to CO_2 emissions because the services of both subsectors generate emissions.

No doubt, an encompassing approach coping with that interrelatedness would clearly be desirable. However, to keep the analysis transparent and manageable and to avoid indeterminate results in a very complex model, we will restrict our focus on i-services in the remaining part of this paper. We do so by keeping the price for m-services constant, fixing it at $p_m \equiv 1$, and neglect the welfare associated to m-services. Having thus effectively cut off all links of our model to the subsector of mass transportation, we also drop the index *i* on all variables for convenience of notation.

3 Welfare and the road transport market

The welfare maximum as a benchmark

The welfare related to individual transport services[14] is defined as $W := \Pi + S - D$. Π are profits from (5), S is consumer surplus, and D is environmental damage caused by CO_2 emissions from consuming the fuel, $F = vyN/p$, in the process of providing i-services. For simplicity, environmental damage is taken to be proportional to total emissions, i.e. $D := \delta F$, where $\delta \geq 0$ and constant. As for the consumer surplus, observe that the consumers' aggregate willingness-to-pay is[15] $yN + \left(\frac{b^2vy}{p} - y\right)\frac{N}{2}$. With total expenditure on individual transport

[14] Alternatively, the regulator – as well as the firm – could implement the welfare-maximizing allocation by charging personalized (Lindahl) prices. To that end, one would have to replace the inequalities (2) by equalities and substitute the uniform price *p* by the personalized prices p_h (which are defined by those equalities, in fact). As a result, total consumer surplus is transformed into revenue. We will not pursue this approach here, however, because in practice the informational requirements for charging Lindahl prices appear to be insurmountable.

[15] The graphical equivalent to this willingness-to-pay term is the area $h_\ell DCEh_u$ in Figure 1.

services equal to yN, consumer surplus is easily calculated as $S := \frac{1}{2}\left(\frac{b^2 v}{p} - 1\right) yN$. Using this information, we invoke (4) to rewrite welfare as

$$W(p,v) = \Pi(p,v) + S(p,v) - \delta F(p,v), \tag{6}$$

where $\Pi(p,v) = \frac{2y\left(b^2 v - p\right)[p - av]}{p} - kv^2$, $F(p,v) = v\frac{y}{p}N = \frac{2vy\left(b^2 v - p\right)}{p}$ and $S(p,v) = \frac{y\left(b^2 v - p\right)^2}{p}$.

Suppose now, the regulator is capable to fix[16] p and v directly and aims to maximize welfare. The result is

Proposition 1: [17] *(Welfare optimum)*

Suppose a welfare-maximizing regulator's policy variables are p and v.

(i) The regulator then chooses

$$v_w := \frac{y(b-q)^2}{2k} \qquad \textit{and} \qquad p_w := bqv_w, \qquad \textit{where}^{18}\ q^2 := 2a - b^2 + 2\delta \tag{7}$$

(and where the subscript w indicates welfare-maximizing values of v and p).

(ii) For sufficiently small environmental damage $(\delta = 0$ or small$)$ the price per kilometer of i-services, p_w, is below marginal costs, and the welfare-maximizing emission rate, v_w, is greater than the emission rate resulting from marginal-cost pricing.

(iii) The welfare-maximizing emission rate and total emissions are strictly decreasing in δ.

The welfare-maximizing pricing rule of Proposition 1 is unexpected. Note first that marginal-cost pricing would not allow covering all costs. Therefore, setting the price below mar-

[16] An alternative interpretation is to think of a welfare-maximizing public enterprise allowed to run a deficit.
[17] The proofs of propositions are delegated to the Appendix.
[18] Note that (6) presupposes parameter values securing $q^2 > 0$. In scenarios with $\delta = 0$ the condition $2a > b^2$ is required for $q^2 > 0$.

ginal costs creates a deficit, a fortiori. Yet the intriguing question is, as to why it is optimal to charge a price below marginal cost for sufficiently small δ. The reason is the link between i-services, which are ordinary private consumption goods priced in the market, and the emission rate *v*, which is an unpriced public good, in technical terms. The optimizing regulator needs to balance the consumers' increasing aggregate willingness-to-pay for larger vehicles against the exclusion effect of pricing i-services whose marginal costs increase in the emission rate and hence in the size of cars. Since marginal cost is greater than p_w for sufficiently small δ, price exclusion is more stringent with marginal-cost pricing than with p_w. As a result, the public good 'car size' is undersupplied in case of marginal-cost pricing.

According to Proposition 1(iii), growing environmental damage calls for enhancing the fuel economy of vehicles, which conforms our intuition. The price for transport services is subject to opposing effects. It tends to shrink with increasing δ because of decreasing v_w and it tends to rise because q becomes greater with increasing δ. $\partial p_w / \partial \delta < 0$ would raise total vehicle kilometers traveled. However, this positive partial effect on total fuel consumption is overcompensated by the change in total fuel consumption induced by $\partial v_w / \partial \delta < 0$ because according to Proposition 1(iii) the sign of $\partial F\left(p_w, v_w\right) / \partial \delta$ is unambiguously negative.

In the real world, there is no omniscient regulator who implements the optimal allocation by fixing p and v directly. There rather is in operation a market for transport services, and real-world regulators do account for this market. The next step is, therefore, to determine the market allocation.

The market for individual transport services

Throughout the remaining part of the paper, the firm supplying transport services is assumed to determine its sales price *p*. It also chooses the fuel rate *v,* unless the regulator directly fixes that rate. Alternatively or in addition, the firm may have to pay the following linear taxes:

- a *gasoline tax* (= vehicle emissions tax) with rate $t \geq 0$ and tax base $F := \left(vy / p\right)N$, the total (absolute) fuel consumption of i-services;
- a *gas guzzler tax* (= tax on the emission rate of vehicles) with rate $\tau \geq 0$ and tax base v.

Gasoline taxes are applied in most countries although their rates are often differentiated e.g. with respect to types of fuel and types of vehicles. Pure gas-guzzler taxes are not yet applied in practice to our knowledge but annual car circulation taxes levied by many countries resemble gas-guzzler taxes to some extent. Car circulation taxes are paid by car owners. Since in our model consumers of transport services do not buy the vehicles they use but rather *rent* them at prices strictly based on kilometers traveled, the only way to introduce a gas-guzzler tax in the present model is to let the producer pay that tax independent of the amount of transport services provided.

We now consider both taxes in equation (5) to obtain the firm's after-tax profit

$$\Pi = \frac{2y\left(b^2 v - p\right)\left[p - c^2 v\right]}{p} - (kv + \tau)v, \quad \text{with} \quad c^2 := a + t. \tag{8}$$

The straightforward implications of profit maximization are presented in

Proposition 2: (Profit maximum)

(i) *The firm's profit-maximizing production is*

$$v_\pi := \frac{2y(b-c)^2 - \tau}{2k} \quad \textit{and} \quad p_\pi = bcv_\pi. \tag{9}$$

(ii) *Denote by v_π^o the emission rate chosen by the firm when no taxes are levied and by $v_w(\delta)$ the welfare-maximizing emission rate for $\delta \geq 0$. There is $\tilde{\delta} := \left(b^2 - a\right)/2$ such that*

$$v_w(\delta) \gtreqless v_\pi^o \quad \Leftrightarrow \quad \delta \lesseqgtr \tilde{\delta} > 0. \tag{10}$$

Proposition 2(ii) implies that in case of absent or sufficiently low environmental damage (δ = 0 or small) the regulator would have to encourage the firm to provide less fuel-efficient cars (!). Clearly, we presume that in the real world δ is high enough to warrant the exclusive focus on scenarios where regulation aims at inducing more fuel-efficient cars. The reason why, for small δ, cars are optimal which are less fuel-efficient than those provided by the monopolistic firm is essentially the same as in Proposition 1(ii) because in the absence of taxation the price charged by the firm, $p_\pi = ba_t v_\pi$, is above marginal costs, av_π, owing to $b > a_t = \sqrt{a}$. Setting p_π above marginal costs is necessary to make a profit which accrues only when revenues exceed both the marginal (equal to average) costs of i-services, av_π, and the fixed costs, kv_π. Recall that the undersupply result for δ = 0 or small is not confined to monopolistic pricing. According to Proposition 1 it also holds for marginal-cost pricing and therefore also for Ramsey pricing.

The firm's production (p_π, v_π) from (9) clearly depends on the tax rates τ and t. The emission rate v_π from (9) is decreasing in both tax rates, τ and t. Moreover, the firm reduces p_π when τ is raised but it may increase or lower its price when t is increased.

4 Regulation

Fuel-efficiency standards (Policy I)

The first policy option we explore now is regulation by means of setting a mandatory fuel-efficiency standard. The regulator will take as given the market power of the firm, that is, when fixing her standard she will anticipate how the firm reacts to her policy. She does so by accounting for the firm's profit-maximizing pricing rule $p = bcv$ from (7). In formal terms, the regulator maximizes welfare $W(p,v)$ from (6) with respect to v subject to $p = bcv$ and $t = \tau = 0$.

Consideration of $p = bcv$ in (6) yields,[19] after some rearrangement of terms

$$W^I(v) := -kv^2 + \frac{y(b-a_t)(cb-q^2)}{c}v. \tag{11}$$

The straightforward implication is

Proposition 3: *(Optimal fuel-efficiency standard)*

Suppose the regulator maximizes welfare by fixing the emission rate directly.

(i) The regulator chooses $v_I := \dfrac{y(b-c)(bc-q^2)}{2ck}$.

[19] Note that in the following equation we have $a_t = \sqrt{a}$ because $t = \tau = 0$ is presupposed.

(ii) *Denote by* $v_I(0)$ *the emission rate that is optimal under Policy I, when emissions do not cause environmental damage* $(\delta = 0)$. *The emission rate* $v_I(0)$ *satisfies* $v_w(0) > v_I(0) > v_\pi^o$, *where* $v_w(0)$ *and* v_π^o *are defined as in Proposition 2(ii).*

(iii) *If emissions do cause environmental damage* $(\delta > 0)$,

(a) *there is a threshold value* $\delta_I := \dfrac{b(b-c)}{2} > 0$ *such that* $v_I(\delta) \gtreqless v_\pi^o \quad \Leftrightarrow \quad \delta \lesseqgtr \delta_I$;

(b) $v_w(\delta) > v_I(\delta)$ *and* $F(v_w, p_w) > F(v_I, p_I)$ *for all* $\delta > 0$.

The remarkable message of Proposition 3 is that the *optimal* regulation by means of a fuel-efficiency standard implies the production of vehicles whose fuel efficiency is excessive, i.e. suboptimally high (!). To put it differently, the public good 'emission rate' or 'size of car' is undersupplied for all $\delta \geq 0$ and consequently total fuel consumption and total emissions fall short of their welfare-maximizing level. The reason why the standard-setting policy fails to produce the efficient allocation is the difference in price formation. Suppose, for some given $\delta \geq 0$ the standard-setting regulator fixes the standard $v = v_w(\delta)$. In that case, the firm reacts with the price $p_\pi = v_w(\delta) b\sqrt{a}$, while the associated efficiency-securing price would be $p_w = v_w(\delta) b\sqrt{2a - b^2 + 2\delta}$.

Proposition 3 characterizes the second-best allocation when the regulator's only instrument is the fuel-efficiency standard. Since the welfare function *W* from (6) attains a unique maximum we clearly have $W(bcv_I, v_I) < W(p_w, v_w)$. Additional information can be gained by exploring how Policy I fares when the regulator fixes the fuel-efficiency standard to implement a predetermined emissions target (or emissions cap) in form of some upper bound, $\bar{f} > 0$, on total fuel consumption. In that scenario Policy I performs as stated in

Proposition 4: *(Implementing an emissions cap with Policy I)*

Suppose a regulator fixes the fuel-efficiency standard such that some predetermined emissions cap $\bar{f} > 0$ *is implemented. That policy yields*

$$v_I(\bar{f}) > v_w(\bar{f}) \qquad \text{and} \qquad p_I(\bar{f}) > p_w(\bar{f}),$$

where $v_I(\bar{f})$ *denotes that particular fuel-efficiency standard which implements the emissions cap* $\bar{f}$, *where* $p_I(\bar{f})$ *is the corresponding price of transport services, and where* $v_w(\bar{f})$ *and* $p_w(\bar{f})$ *denote the respective values attained when* $\bar{f}$ *is implemented in a cost-effective way.*

To see the reason for the inequality $v_I(\bar{f}) > v_w(\bar{f})$ observe that for any given v total fuel consumption is $f = \frac{2y(b-c)v}{c}$ in case of monopolistic pricing and $f = \frac{2y(b-q)v}{q}$ in case of efficient pricing. (See also the proof of Proposition 4 in the Appendix). Since f is decreasing in q and $c > q$ it follows that for any given v the efficient total fuel consumption is greater than under Policy I. To reach the same total fuel consumption under Policy I as in the cost-efficient case we need to raise v under Policy I. Hence $v_I(\bar{f}) > v_w(\bar{f})$.

Invoking $W(p,v)$ from (6) Proposition 4 allows quantifying the excess costs of Policy I as the difference

$$W\left[p_w(\bar{f}), v_w(\bar{f})\right] - W\left[p_I(\bar{f}), v_I(\bar{f})\right].$$

That difference can be shown to depend on the model parameters in a quite complex way. We refrain from further elaborating on the excess costs, however, because one would have to take great care in securing strictly binding emissions caps and values of emission rates, v, that satisfy the conditions under which N from (4) holds. (See footnote 11).

Another feature having received attention in the literature (Harrington and McConnell 2003) is the so-called rebound effect of tightening an existing fuel-efficiency standard. The rebound effect is the additional fuel consumption. With total fuel consumption equal to $f = (y/p) \cdot N \cdot v$ we obtain in view of $p = bcv$ and $N = 2(b^2v - p) = 2b(b-c)v$:

$$\frac{df}{dv} = \underbrace{\underbrace{Nv\frac{d(y/p)}{dv}}_{[1]} + \underbrace{\frac{vy}{p}\frac{dN}{dv}}_{[2]}}_{\textit{rebound effect}} + \frac{Ny}{p}\frac{dv}{dv} = \underbrace{\underbrace{-\frac{2y(b-c)}{c}}_{[1]} + \underbrace{\frac{2y(b-c)}{c}}_{[2]}}_{\textit{rebound effect}} + \frac{Ny}{p}.$$

To fix our ideas suppose $dv < 0$. Term [1] then denotes the extra fuel that each individual consumes if she is (or if she were) a user of individual transport services before and after the reduction in v. Term [2] gives us the reduction in fuel that occurs because the number of consumers of individual transport services declines. The rebound partial effects [1] and [2] exactly compensate and hence the total rebound effect is zero.[20] Although it is true that those consumers who remain users of individual transport services after the fuel-efficiency standard has been tightened increase their fuel consumption, the aggregate effect is zero because the number of those consumers shrinks.

Taxing gas guzzlers (Policy II)

As in case of policy I, the regulator takes into consideration the firm's profit-maximizing pricing rule $p = bcv$. Yet rather than fixing the emission rate directly, she now chooses the rate τ of a tax on emission rates as her (only) instrument. The firm will react to that tax by adjusting its emission rate according to (9). The regulator, in turn, considers that reaction in her decision-making. Hence, she solves the problem:

$$\underset{(\tau)}{Maximize} \; W(p,v) \text{ subject to } t = 0, \; v = \frac{2y(b-c)^2 - \tau}{2k} \text{ and } p = bcv.$$

Inserting $p = bcv$ in (6) turns this optimization problem into

$$\underset{(\tau)}{Maximize} \left[-kv^2 + \frac{y(b-c)(cb-q^2)}{c} \right] \; s.\,t. \;\; v = \frac{2y(b-c)^2 - \tau}{2k} \;\; and \;\; q^2 := 2a - b^2 + 2\delta .$$

We characterize the straightforward solution in

Proposition 6: *(Policy II: Optimal gas guzzler tax)*

Suppose a welfare-maximizing regulator's only policy variable is a tax on emission rates.

The regulator chooses $\tau_{II} := \frac{y(b-c)}{c}\left[2\delta - b(b-c)\right]$ *and the pertaining fuel rate is*

[20] According to Portney et al. (2003, p. 210) empirical estimates suggest that the rebound effect "… offsets 10-20 percent or more of the initial fuel reduction from tighter CAFE standards."

$$v_{II} := \frac{2y\left(b-c\right)^2 - \tau_{II}}{2k} = v_I \,.$$

Proposition 5 establishes the equivalence of the Policies I and II. The specification of τ_{II} clearly implies $\tau_{II} > 0$ if and only if $\delta > b\left(b-c\right)/2$. This is just another way of observing that the emission rate chosen by the firm in the absence of regulation and environmental damage is smaller than the welfare-maximizing rate.

Gas taxes, fuel-efficiency regulation, and the inefficiency of mixed taxation

We now assume that the regulator has at her disposal a gasoline tax, which she may or may not combine either with a gas-guzzler tax or with an emission-rate standard. Suppose first, the regulator's instruments are a gas tax and a gas-guzzler tax. As in case of Policy II, she accounts for the firm's first-order conditions of profit maximization and thus solves the problem: Maximize with respect to *p* and *v* welfare $W\left(p,v\right)$ from (6) subject to $p = bcv$ and

$$v = \frac{2y\left(b-c\right)^2 - \tau}{2k}, \quad q^2 := 2a - b^2 + 2\delta \text{ and } c^2 = a + t \,. \tag{12}$$

This optimization problem is equivalent to

$$\underset{(t,\,\tau)}{Maximize} \left[\frac{y\left(b-c\right)\left(bc - q^2 + 2t\right)}{c} v - 2v^2 \right] \text{ subject to (12).} \tag{13}$$

Although the objective function of (13) is not concave in *t*, it is possible to determine the solution. The result is presented in

Proposition 6: *(Optimality of taxing gasoline and inefficiency of mixing tax policies)*

(i) *If the regulator's only policy instrument is a gasoline tax, the optimal solution* $\left(p_w, v_w\right)$ *characterized in Proposition 2 can be achieved by the tax rate* $t_{III} := a - b^2 + 2\delta$.

(ii) *If the regulator has at her disposal a gas tax as well as a gas guzzler tax, the unique optimal policy is* $t = t_{III}$ *and* $\tau = 0$.

Proposition 6(i) presents – and confirms - the well-known standard proposition in environmental economics that an environmental externality can be internalized by an ecological tax, if and only if the tax base is the emission of the pollutant that creates the environmental damage. In the present context, emissions are therefore the appropriate 'ideal' tax base. Recall from our comment on Proposition 2 that for small values of δ the regulator would need to encourage the firm to build larger vehicles. This conclusion is made precise here because the tax rate t_{III} is negative for small values of δ (because of $b^2 > a$). Obviously, if δ happens to be equal to $\left(b^2 - a\right)/2 > 0$, then $t_{III} = 0$, and in this particular case v_π is optimal.

Whenever a tax intended to curb emissions does not directly and precisely target emissions it tends to induce allocative distortions irrespective of whether the regulator applies it separately or combined with other policy instruments. The gas-guzzler tax is such a tax, and this explains the result of Proposition 6(ii) that combining the gasoline tax with a gas-guzzler tax is inefficient. As mentioned above, some countries levy annual circulation taxes roughly approximating a gas-guzzler tax, and there is an ongoing discussion in the European policy arena about transforming prevailing circulation and/or registration taxes into taxes on vehicle emission rates proper. In view of Proposition 6(ii), such an 'amendment' would result in an efficiency loss, however, because all pairs of tax rates (t,τ) fail to implement the welfare maximum unless $(t,\tau) = (t_{III}, 0)$.[21]

It remains to ask the question as to what the merits are of the strategy of combining a gasoline tax with a fuel efficiency standard.[22] The straightforward answer is given in

Proposition 7: *(Optimality of combining a gasoline tax with a fuel efficiency standard)*

If a welfare-maximizing regulator has at her disposal a gasoline tax and can also impose a fuel efficiency standard, she is able to implement the optimal allocation (p_w, v_w) by fixing the rate of the gas tax at $t_{III} := a - b^2 + 2\delta$ and by setting any fuel efficiency standard $v \in [v_w, 1]$.

[21] Owing to their assumption of consumer myopia, Jansen and Denis (1999) find that combining both taxes is superior to applying both instruments individually.

[22] For a brief non-technical discussion of this issue see Portney et al. (2003, p. 210).

Proposition 7 suggests that the optimal combination of both instruments is to fix the standard in such a way that it is redundant or weakly binding at most. To clarify further the message of Proposition 7, suppose the regulator has fixed the policy parameters $(\bar{t},\bar{v})$ and denote by $v(\bar{t})$ the emission rate that the firm would have chosen when subjected to a gasoline tax at rate $\bar{t}$ in the absence of a fuel efficiency standard. We wish to demonstrate that in a two-instrument strategy $(\bar{t},\bar{v})$ either instrument can be binding.

If $(\bar{t},\bar{v})$ satisfies $v(\bar{t})<\bar{v}$, the standard $\bar{v}$ is not binding. Therefore, any standard $\bar{\bar{v}}\in[v(\bar{t}),1]$ leaves the allocation unchanged and is hence redundant. To put it differently, for all $\bar{\bar{v}}\in[v(\bar{t}),1]$ the policies $(\bar{t},\bar{\bar{v}})$ are the same in allocative terms as the tax policy $\bar{t}$ in the absence of a fuel efficiency standard. The regulator does not choose an arbitrary $t=\bar{t}$, of course. Poised to maximize welfare she sets $\bar{t}=t_{III}$, yielding the welfare maximum determined in Proposition 7.

Next consider policies $(\bar{t},\bar{v})$ satisfying $\bar{t}>0$ and $v(\bar{t})>\bar{v}$. In that case, the firm would like to choose $v(\bar{t})$ but is forced to abide by the more stringent standard $\bar{v}$. Efficiency cannot be attained with such a strategy. To see that, suppose $\bar{v}=v_w$. Since we presupposed $v(\bar{t})>\bar{v}=v_w$, $\bar{t}<t_w$ follows. However, efficiency requires $\bar{t}=t_w$. The distortions caused by policy $(\bar{t},\bar{v})$ satisfying $\bar{t}>0$ and $v(\bar{t})>\bar{v}$ also differ from those induced by a policy that employs a fuel efficiency standard only. Since the latter is simply the special case of a mixed strategy $(\bar{t},\bar{v})$ with $\bar{t}=0$, the difference between both strategies is the payment of the gasoline tax. That has an effect on the price charged by the firm because if $\bar{t}>0$ the firm sets its price $p_\pi=b\bar{v}\sqrt{a+t}$ that is greater than the price $p_\pi=b\bar{v}\sqrt{a}$ in the absence of the gas tax. Consequently, the allocation differs from the case of a standard policy without gas tax.

We conclude that as long as the tax rate $\bar{t}$ is (weakly) binding, the fuel efficiency standard has no impact at all. It is then redundant, and the regulator could drop it altogether[23]. On the other hand, if the standard is strictly binding, positive tax rates $\bar{t}$ induce additional allocative distortions as compared to the case $\bar{t}=0$. It follows that "$\bar{t}=t_{III}$" is a necessary condition and "$\bar{t}=t_{III}$ combined with $v(t_{III})\leq\bar{v}$" are necessary and sufficient conditions for a cost-effective policy $(\bar{t},\bar{v})$.

[23] The standard should be dropped, in fact, as far as it causes administration costs.

	Gasoline tax	Fuel-efficiency regulation	
		Gas-guzzler tax	Fuel-efficiency standard
Gasoline tax	[1]: Emission rate & emissions efficient (Prop. 6)	---	---
Gas-guzzler tax	[4]: Efficiency reduced by gas guzzler tax (Prop. 6)	[2]: Emission rate & emissions too low (Prop. 5)	---
Fuel-efficiency standard	[5]: Fuel-efficiency standard redundant (Prop. 7)	[6]: same as [2] (Prop. 3 & Prop.5)	[3]: same as [2] (Prop. 3)

Table 1: Overview of regulation results

Table 1 provides a compact summary of our regulation results. As expected, the gas tax can achieve efficiency when applied separately (box [1] in Table 1). Strategies of separate emission-rate regulation lead to suboptimally low emission rates and total emissions ([2] and [3]). Both forms of emission-rate regulation are equivalent and hence equally distortionary. However, the form of emission-rate regulation matters greatly when combined with a gas tax. If the regulator applies a gas tax jointly with a gas-guzzler tax ([4]), she cannot attain efficiency anymore: The overlapping gas-guzzler tax reduces the efficiency of the gas tax instrument. In contrast, if the regulator applies a gas tax jointly with a fuel-efficiency standard ([5]), she is able to attain efficiency but that result is exclusively owed to the incentive properties of the gas tax. The optimal combination of both instruments is such that the standard is not - or only weakly - binding. Finally, for the performance of a policy that combines both taxes ([6]), it follows from the assessment of their separate use that the emission rate as well as total emissions will be inefficiently low.

5 Concluding remarks

The present paper offers some general, and partly unexpected, *conceptional* insights in the efficiency characteristics of a set of CO_2 emission control strategies in the road transportation sector. In a stylized analytical and parametric model, we assume the size of vehicle to be inversely related to its fuel efficiency. Technically, we model fuel efficiency as a public consumption good, and therefore price exclusion and market power of price-setting suppliers play a role for the efficiency of the market mechanism and of CO_2 mitigating regulation. As far as such regulation - in form of separately applied or mixed policy instruments - does not target CO_2 emissions directly, allocative distortions are to be expected. While this is well-established knowledge in principle and also the result of various numerical (large-scale) models, the contribution of the present paper is to present specific analytical results on the efficiency performance of individual instruments and mixed policies.

It is also clear, however, that due to the high level of abstraction and the narrow focus of the analysis the results obtained do not qualify as prescriptions for *practical* CO_2 mitigating regulation. In the real world, consumers buy cars and then use them rather than renting vehicles that are priced strictly based on vehicle kilometers traveled. Many types and sizes of vehicles with different fuel-economy characteristics coexist and the age structure of the stock of vehicles matters. In our model, we ignore several relevant environmental and other vehicular externalities related to factors such as fuel consumption, driving habits, travel amount, emission-control equipment, vehicle maintenance, vehicle ownership etc. Therefore, policy recommendations are incomplete unless they are based on an *integrated* assessment of multiple externalities as done by some of the literature cited in Section 1.

Moreover, for the benefit of specific analytical information about the efficiency characteristics of strategies for reducing vehicle-related CO_2 emissions, we restricted our focus on individual vehicle transport although the model as developed in Section 2 is designed and capable to capture the links between individual and mass transport. A comprehensive welfare analysis clearly calls for including both modes of transport, not only because of demand interdependencies but also because both transportation subsectors release CO_2. Cutting off mass transport altogether means to overstate the effectiveness of CO_2 emission controls in the subsector of individual vehicle transport because discouraging individual travel translates into increased mass transport and increasing CO_2 emissions in that subsector.

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Appendix

Proof of Proposition 1

Proposition 1(i): In view of (6), welfare is

$$W(p,v) = \frac{2y\left(b^2v-p\right)[p-av]}{p} - kv^2 + \frac{y\left(b^2v-p\right)^2}{p} - \delta\frac{2vy\left(b^2v-p\right)}{p}.$$

The derivatives are $W_p = -\frac{y}{p^2}\left(p^2 - b^2q^2v^2\right)$ and $W_v = -2kv + \frac{y}{p}\left[\left(b^2+q^2\right)p - 2b^2q^2v\right]$, where $q^2 := 2a - b^2 + 2\delta$. W is strictly concave in p and v. The first-order conditions $W_v = W_p = 0$ readily yield $p = bqv$ and $\frac{y}{p}\left[\left(b^2+q^2\right)p - 2b^2q^2v\right] - 2kv = 0$. The solution to these two equations is (7).

Proposition 1(ii): The marginal cost of i-services is av and the optimal price is $p_w = bqv_w$. $p_w < av_w$ follows, if and only if $bq < a$. For $\delta = 0$ this inequality is equal to $b\sqrt{2a-b^2} < a$ or $(b-a)^2 > 0$. Hence $p_w < a^2 v_w$.

Turning to the second part of Proposition 1(ii) we first determine v_{MC}^o, the fuel rate corresponding to marginal-cost pricing. Under the condition $\delta = 0$ we calculate $W = -kv^2 + \left[y\left(b^2 v - p\right)^2 / p \right] v$. Inserting $p = av$ (= marginal-cost pricing) gives us $W = -kv^2 + \left[y\left(b^2 - a\right)^2 / a^2 \right] v$. Solving this function for *v* yields $v_{MC}^o := \frac{y\left(b^2 - a\right)^2}{2ak}$.

We want to show that $v_w^o > v_{MC}^o$. Closer inspection yields

$$v_w^o > v_{MC}^o \quad \Leftrightarrow \quad (b-q)\sqrt{2a} > b^2 - a \quad \Leftrightarrow \quad X(a,b) := b\sqrt{2a} - q\sqrt{2a} + a - b^2 > 0.$$

Note first that in the limiting case $a = b^2$ we have $X(a,b) = 0$ and $v_w^o = v_{MC}^o = 0$. Hence $v_w^o > v_{MC}^o$ is verified if and only if $X_b > 0$. The derivative is $X_b = \sqrt{2a} + \frac{b\sqrt{2a}}{q} - 2b = \frac{\sqrt{2}}{q}\left(q\sqrt{a} + b\sqrt{a} - bq\sqrt{2}\right)$. Therefore $X_b > 0$, if and only if $q\sqrt{a} + b\sqrt{a} > bq\sqrt{2}$. Since this inequality is equivalent to $\left(a - b^2\right)^2 > 0$, $X_b > 0$ follows implying $v_w^o > v_{MC}^o$.

Proposition 1(iii): Taking the derivative of v_w with respect to δ yields $\frac{dv_w}{d\delta} = -\frac{2y(b-q)}{kq}$. To show that $\left(dv_w / d\delta\right) > 0$, observe first that $S > 0$ is a necessary condition for i-services to be beneficial. By definition of *S,* one has $S > 0$ if and only if $b^2 v - p > 0$. Combined with the first-order condition $p = bqv$ this inequality is equivalent to $bv(b-q) > 0$. Hence $\left(dv_w / d\delta\right) > 0$, i.e. the optimal emission rate is strictly declining in δ. □

Proof of Proposition 2

Proposition 2(i): We rewrite (8) as $\Pi = \frac{2y}{p}\left[\left(b^2+c^2\right)pv - p^2 - b^2c^2v^2\right] - \left(kv+\tau\right)v$. The derivatives of that function are

$$\Pi_v = \frac{2y}{p}\left[\left(b^2+c^2\right)p - 2b^2c^2v\right] - 2kv - \tau,$$

$$\Pi_p = -\frac{2y}{p^2}\left[\left(b^2+c^2\right)pv - p^2 - b^2c^2v^2\right] + \frac{2y}{p}\left[\left(b^2+c^2\right)v - 2p\right] =$$

$$= -\frac{2y}{p^2}\left[\left(b^2+c^2\right)pv - p^2 - b^2c^2v^2 - \left(b^2+c^2\right)pv + 2p^2\right] = -\frac{2y}{p^2}\left(p^2 - b^2c^2v^2\right).$$

Since $\Pi_{vv} = -\frac{4b^2c^2y}{p} - 2k < 0$ and $\Pi_{pp} = -\frac{4b^2c^2v^2y}{p^3} < 0$, Π is strictly concave in p and v. The first-order conditions $\Pi_v = \Pi_p = 0$ readily yield

$$p = bcv \quad \text{and} \quad \frac{2y}{p}\left[\left(b^2+c^2\right)p - 2b^2c^2v\right] - 2kv - \tau = 0.$$

We solve these equations for v and p to obtain (9) after some rearrangement of terms.

Proposition 2(ii): First we show that $v_w(\delta) > v_\pi^o$ for $\delta = 0$. Making use of the respective definitions one obtains the inequalities

$$v_w(\delta=0) > v_\pi^o \iff \frac{y\left(b-\sqrt{2a-b^2}\right)^2}{2k} > \frac{y\left(b-\sqrt{a}\right)^2}{k} \iff \left(b-\sqrt{2a-b^2}\right)^2 > 2\left(b-\sqrt{a}\right)^2$$

$$\iff \sqrt{2a-b^2} < b - 2\sqrt{a} \iff \left(b-\sqrt{a}\right)^2 > 0.$$

Therefore $v_w(0) > v_\pi^o$. Recall form Prop. 1(iii) that $v_w(\delta)$ is strictly decreasing in δ. Combined with $v_w(0) > v_\pi^o$ that proves the existence of a positive threshold $\tilde{\delta}$ satisfying (10). The specific value $\tilde{\delta} = \left(b^2 - a\right)/2$ of that threshold will be derived in Proposition 5 below. □

Proof of Proposition 3

Proposition 3(i): v_1 in Prop. 3(i) is the straightforward maximum of the welfare function W from (11).

Proposition 3(ii): For $t=\delta=0$ we turn the emission rate v_I into

$$v_I^o := \frac{y(b-c)\left(bc-q^2\right)}{2ck} = \frac{y(b-c)^2}{k} \cdot \frac{bc-q^2}{2c(b-c)} = v_\pi \cdot \frac{bc-q^2}{2c(b-c)}.$$

$v_I^o > v_\pi$ follows, if and only if $\frac{bc-q^2}{2c(b-c)} > 1$. By definition of q this inequality is satisfied (for $\delta=0$) if and only if $b>c$ which holds by assumption. It remains to show that $v_I^o < v_w^o$, which is equivalent to showing that $\frac{bc-q^2}{2c(b-c)} < \frac{(b-q)^2}{2(b-c)^2}$ or $c(b-q)^2 > (b-c)\left(bc-q^2\right)$. We consider $q^2 = 2a-b^2$ and rearrange terms to show that this inequality is equivalent to $(c-q)^2 > 0$.

Proposition 3(iii): The fact that there is a threshold value $\delta_I > 0$ follows from the observation that $(dv_I / d\delta) > 0$. The specific value of δ_I is straightforward from Proposition 4 (below).

As defined in (6), total fuel consumption is $F(v,p) = 2y\left(\frac{b^2v^2}{p} - 1\right)$. It is obvious, therefore, that $F(v_w, p_w) > F(v_I, p_I) \Leftrightarrow \frac{v_w^2}{p_w} > \frac{v_\pi^2}{p_\pi}$. This inequality is satisfied because $p_w < p_I = p_\pi$ (Proposition 2i) and $v_w > v_I$. □

Proof of Proposition 4

Recall that total fuel consumption is $f = \frac{yvN}{p} = \frac{2y\left(b^2v-p\right)}{p}$, where $N = 2\left(b^2v-p\right)$ from (4). In case of Policy I we have $p = bcv$ and hence $f = \frac{2y(b-c)v}{c}$. For $f = \overline{f}$ this equation is satisfied by $v_I\left(\overline{f}\right) := \frac{c\overline{f}}{2y(b-c)}$. The benchmark for cost efficiency is determined analogously. Since $p = bqv$ is a necessary condition for welfare maximization the corresponding total fuel consumption is $f = \frac{2y(b-q)v}{q}$ for any given v. With $f = \overline{f}$ this equation is satisfied by $v_w(\bar{f}) := \frac{\bar{f}q}{2y(b-q)}$. Making use of the definitions $c = \sqrt{a}$ and $q = \sqrt{2a-b^2}$ for $t = \delta = 0$ the comparison of $v_I(\bar{f})$ and $v_w(\bar{f})$ yields

$$v_I(\bar{f}) \gtreqless v_w(\bar{f}) \quad \Leftrightarrow \quad \frac{c}{b-c} \gtreqless \frac{q}{b-q} \quad \Leftrightarrow \quad c \gtreqless q \quad \Leftrightarrow \quad b^2 \gtreqless a.$$

Since $b^2 > a$ by assumption, $v_I(\bar{f}) > v_w(\bar{f})$ follows. Moreover, $p_I(\bar{f}) := bcv_I(\bar{f})$ and $p_w(\bar{f}) := bqv_w(\bar{f})$ combined with $v_I(\bar{f}) > v_w(\bar{f})$ and $c > q$ yield $p_I(\bar{f}) > p_w(\bar{f})$. □

Proof of Proposition 5

The first-order condition of maximizing $W(v)$ from (11) over τ is

$\frac{dW}{d\tau} = W_v \frac{dv}{d\tau} = \left[-2kv + \frac{y(b-c)(bc-q^2+2t)}{c} \right] \frac{dv}{d\tau} = 0$. So $v_{II} = \frac{y(b-c)(bc-q^2)}{2ck} = v_I$ is obvious. Since it is also true by assumption that $v_{II} = \frac{2y(b-c)^2 - \tau_{II}}{2k}$, we compute τ_{II} by solving the equation $v_{II} = \frac{y(b-c)(bc-q^2)}{2ck} = \frac{2y(b-c)^2 - \tau_{II}}{2k}$. □

Proof of Proposition 6

Proposition 6(i): Consider the specific tax rate $t_{III} := a - b^2 + 2\delta$, to which the firm responds by choosing $v_{III} := \frac{y(b-c_{t,III})^2}{k}$ and $p_{III} = bc_{t,III}v_{III}$, where $c_{t,III} := \sqrt{a+t_{III}}$. We want to prove Proposition 6(i) by showing that $(p_{III}, v_{III}) = (p_w, v_w)$, where (p_w, v_w) characterizes the welfare maximum according to Proposition 1. The first step is to prove that $p_{III} = p_w$, if and only if $v_{III} = v_w$. Starting from the observation that $p = bc_{t,III}v_{III}$ and $p_w = bqv_w$, $p_{III} = p_w$ is obviously satisfied for $v_{III} = v_w$, if and only if $q = c_{t,III} = \sqrt{a+t}$ or, equivalently,

$t_{III} := a - b^2 + 2\delta$. It remains to show that $v_{III} = v_w$. To this end, we make use of the equality $q = c_{t,III}$ to transform v_{III} as follows:

$$v_{III} := \frac{y\left(b - c_{t,III}\right)^2}{k} = \frac{y\left(b^2 - 2bc_{t,III} + c_{t,III}^2\right)}{k} = \frac{y\left(b^2 - 2bq + q^2\right)}{k} = \frac{y\left(b - q\right)^2}{k} = v_w .$$

Proposition 6(ii): Suppose there is a pair of tax rates, (t,τ), such that $p = p(t,\tau) = p_w$ and $v = v(t,\tau) = v_w$. Since $p_w = bqv_w$, that tuple is the solution of the equations $v(t,\tau) = v_w$ and $bc(t)v(t,\tau) = bqv_w$, where $c(t)^2 = a + t$ and $q^2 := 2a - b^2 + 2\delta$. Obviously, the equations $v(t,\tau) = v_w$ and $bc(t)v(t,\tau) = bqv_w$ are satisfied if and only if $c(t) = q$, and this equation holds if and only if $t = t_{III}$. As shown in the proof of Proposition 6(i), the equations $t = t_{III}$ and $v(t,\tau) = v_w$ imply $\tau = 0$. □