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Initial Imbalance and Long Run Inequality: Numerical Evaluation of the Lucas Model

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Abstract

In a model of balanced growth applied to a sample of economies convergence does not mean long run equality. In the paper we show how to shed light upon the relation between initial imbalance and long run inequality. We define three different state value classifications which refer to initial, transitional and long run performance of economies. The formal concepts are evaluated with the numerical method of relaxation.

*I gratefully acknowledge the comments, suggestions and technical assistence of Benjamin Born, Karsten Hees and Timo Trimborn.

1 Introduction

The main goal of the paper is to provide a formal, analytical tool which inables us to analyze the relation between initial and long run economic variables of a given group of countries. On the basis of the respective growth model chosen we want to make theoretical predictions for long run distributions of ressources, income, consumption or utility given respective initial distributions. Wherever closed analytical solutions of the differential equations involved are not available we want to support the analysis with an appropriate numerical method.

Our formal framework will be that of differential equations as they arise from models of endogenous growth. We allow for optimal decisions to be taken based on initial states and intertemporal objective functions. In the long run we assume balanced growth to characterize the dynamics whereas the economies undergo transitional changes in the medium run. The dynamic properties should be captured by a differential equation in a space of state and control variables with a multidiemsional stable manifold of a unique balanced growth path *BGP* at it's core.

The state of the art of dealing with such a problem is limited by one crucial consideration: a balanced growth path is a one dimensional manifold of simple form, yet unbounded. Two very different strategies are followed in the literature to work around the mathematical difficulties related to non-compact domains. One is to take ratios of variables in order to reduce the system dimension by one and map the *BGP* to a single point. However, this identifies different positions along the *BGP* with the implication that long run distributions cannot be studied. The remaining object under study then is the pattern of convergence towards a steady state to the extent that these patterns can be stated in the ratios considered. This approach is acceptable in traditional neoclassical growth models with or without exogenous technical progress (e.g. Solow and Ramsey-CassKoopmans, to name only the two most influential examples). In models of endogenous growth this approach may be technically feasable, but it conceals almost all the distributional aspects we are interested in (e.g. Benhabib and Perli (1994)). The second strategy is to rescale the dynamics by reducing growth rates by appropriate constants equal to the respective balanced growth rates in order to freeze motion on the *BGP* (e.g. Lucas (1988), Caballe and Santos (1993)). Scale adjustment transforms the *BGP* into a center manifold of stationary equilibria *CSE* (c.f. Tu (1994)). In opposite to the first approach this is a one to one mapping which preserves all information about distributions.

The key idea of our paper is to identify long run positions in a distribution of the group of countries under consideration on the scale adjusted *BGP*. Initial conditions will be related to long run prospects along a path of development by classification methods. The classes can be identified as fibres of the dynamical system (c.f. Wiggins (1994)).

2 The Lucas (1988) model

The model under consideration in this paper is the one discussed by Caballe and Santos (1993) , Mulligan and Sala-i-Martin (1993), Benhabib and Perli (1994) and numerous other authors. It is based on the seminal contribution of Lucas (1988). Given that the model is well known, a very brief description should be sufficient at this place.

Assume final output is produced from physical and human capital, k and h. The stock of human capital can be split into a share u used for final output production and $1-u$ employed to increase human capital. Due to human capital spill over effects there are increasing returns to scale in the production sector. We consider the competitive version of the model where individual influence on the externality is not taken into account by the households. Intertemporal utility of consumption c with constant elasticity of intertemporal substitution σ^{-1} and discount rate ρ is to be maximized. There is neither population growth no depreciation.

First order conditions for optimal solutions can be computed in the usual way. In terms of growth rates (denoted by a hat) the system is

$$\hat{k} = APK - c/k \tag{1}$$

$$\hat{h} = \delta(1-u) \tag{2}$$

$$\hat{c} = \sigma^{-1}(\alpha APK - \rho) \tag{3}$$

$$\hat{u} = \frac{(\gamma-\alpha)\delta}{\alpha}(1-u) + \frac{\delta}{\alpha} - \frac{c}{k} \tag{4}$$

where $APK := A\,k^{\alpha-1}h^{1-\alpha+\gamma}u^{1-\alpha}$ denotes the average productivity of capital.

The transversality condition of the model implies that optimal solutions have to be balanced in the long run. Balanced growth requires that u, c/k as well as APK are constant. The latter requirement in turn demands for $(1-\alpha)\hat{k} = (1-\alpha+\gamma)\hat{h}$.

The common balanced growth rate μ of k and c can be computed by solving the system under balanced growth assumptions:

$$\mu = \frac{1-\alpha+\gamma}{(1-\alpha+\gamma)\sigma-\gamma}(\delta-\rho)$$

Growth is balanced if the four variables of the system satisfy three equations:

$$\begin{aligned}
1-u &= \frac{1-\alpha}{(1-\alpha+\gamma)\sigma-\gamma}(1-\rho/\delta) \\
c/k &= ((\gamma-\alpha)\psi\mu+\delta)/\alpha \\
k^{\alpha-1}h^{1-\alpha+\gamma} &= \frac{\sigma\mu+\rho}{\alpha A}(u^*)^{\alpha-1}
\end{aligned}$$

where $\psi := (1-\alpha)/(1-\alpha+\gamma)$.

The balanced growth conditions define a particular invariant manifold of the differential equation, the balanced growth path (*BGP*). The question arises whether other solutions initially suffering from initial imbalance converge to balanced growth in the long run. One method to check whether convergence occurs is scale adjustment. Without using the term Lucas (1988) and Caballe and Santos (1993) discuss and use this method. Scale adjustment slows down the motion of variables according to their respective balanced growth rates. The transformed variables are

$$ke^{-\mu t}, \quad he^{-\psi\mu t}, \quad ce^{-\mu t} \quad \text{and} \quad u$$

To avoid extra notation we continue to use the old designations of variables. The new, adjusted growth rates are reduced by the constants of adjustment, μ and $\psi\mu$, respectively. The growth rate of u remains unchanged.

$$\hat{k} = APK - c/k - \mu \tag{5}$$

$$\hat{h} = \delta(1-u) - \psi\mu \tag{6}$$

$$\hat{c} = \frac{1}{\sigma}(\alpha APK - \rho) - \mu \tag{7}$$

$$\hat{u} = (\gamma-\alpha)\frac{\delta}{\alpha}(1-u) + \frac{\delta}{\alpha} - \frac{c}{k} \tag{8}$$

Due to scale adjustment, the *BGP* of the original system turns into a curve of stationary equilibria with the same shape, which is labeled *CSE*. This curve represents a (saddle-point stable) center manifold of the new system.[1] In the economic literature there is no formal proposition stating the conditions for scale adjusted system to be autonomous. Nor is there a proof, that if two scale adjusted solutions converge to the same point on *CSE* the non-adjusted solutions converge! However, for the Lucas model these properties are given.

An optimal solution with unbalanced initial state conditions (k_0, h_0) now approaches a particular point on the curve *CSE*. Yet, there is no way to compute this limit point analytically. Numerical computation requires the solution of a differential equation system with two initial conditions and two final conditions. The initial conditions are given by the inital values of state variables $k(0) = k_0$, $h(0) = h_0$. Appropriate final conditions which determine the path are stationarity conditions for the state variables, implicitly defined by $\dot{k}(\infty) = 0$ and $\dot{h}(\infty) = 0$.

Balanced growth in this model is easy to describe whereas the characterization of the transition towards balanced growth is difficult (e.g. barro:94). In the literature one can find three approaches: One is to consider special cases of parameters which allow for analytical solution of the differential equation (see Boucekkine and Tamarit (2004) as a recent reference.) The second approach is the time elimination method proposed by Mulligan and Sala-i-Martin (1993). The last one is the reduction of the system by taking ratios of variables as shown in Barro and Sala-i-Martin (2004).

3 Numerical Computations

In this paper we elaborate on particular distributional aspects related to the Lucas model. We assume it to be the appropriate growth model for samples of economies. Under this strong assumption we analyse the relative performance of these economies at the beginning of their optimal growth path, along the growth path and in the long run. We see the questions we are asking under two different perspectives.

(a) A random shock affecting the physical or human capital stock may put a country out of balance. What will be the effect of the shock over the years?

[1] The scale adjusted system has one zero eigenvalue, which gives rise to a continuum of stationary equilibria (i.e. a center manifold). For details on the basic concept of center manifolds see, for instance, Tu (1994).

(b) Optimal growth will change the distribution of state variables, policy variables and resulting welfare. What shape will these distributions take over the years? How will inequity or inequality change?

This program as a whole of course is too much for a single paper. As a first step we will confine our investigation to the development of basic conceps and preliminary numerical examples.

We use the relaxation algorithm proposed by Trimborn, Koch, and Steger (2005) as a powerful tool. We are able to compute optimal (scale adjusted) trajectories for any point of the sample or even for a fairly large mesh of initial state vectors.[2]

We use the following set of parameters for our simulations: $\alpha = 0.3$, $\gamma = 0.7$, $\delta = 0.1$, $\rho = 0.05$ and $\sigma = 1.5$. These parameters yield well defined optimal solutions, i.e. no indeterminacy (c.f Benhabib and Perli (1994)).

3.1 An arbitrary distribution of initial states

Consider a random sample of state values (k, h) centered at a point on the

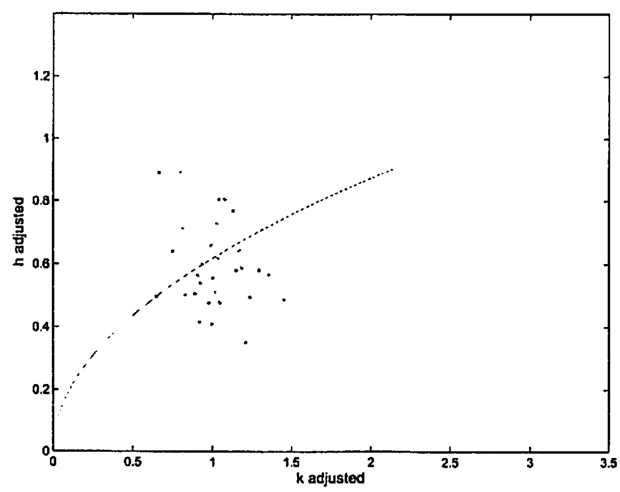

Figure 1: Random sample of state values

[2]On a standard PC it takes less than a minute of computing time for a sample of size 30 and 80 mesh points per trajectory. The computations for a mesh of 50 by 50 points took about half an hour.

curve of stationary values of the scale adjusted model (CSE). The deviation from the curve (CSE) represents initial imbalance in terms of state variables (see figure 1).

Roughly we can distinguish between two types of imbalance: (i) a lack of physical capital or an excess of human capital, or on the other hand (ii) an excess of physical capital or a lack of human capital. We will use the term "too capital intensive" for the latter case well knowing that due to the external effect the capital intensity is not constant along the balanced growth path. In our standard coordinates states below the curve (BGP) (or (CSE)) are too capital intensive.

Among other data, computations of optimal solutions for arbitrary initial states yield initial optimal consumption values. We may call the resulting relation $c(k, h)$ the consumption policy function. Figure 2 shows a represen-

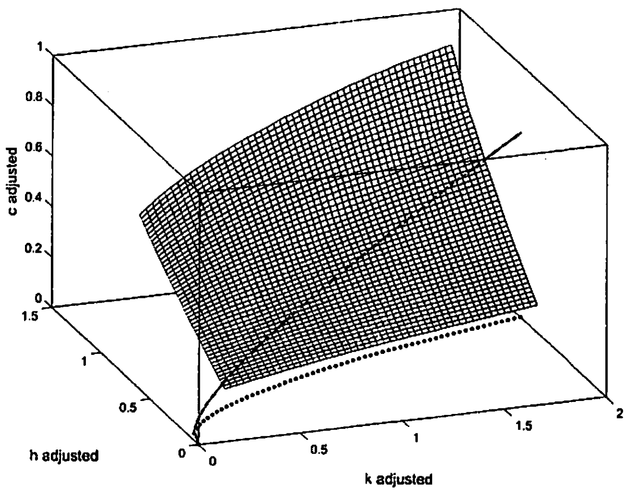

Figure 2: Initial optimal consumption for a mesh of states

tative part of the graph of $c(k, h)$ evaluated over a mesh of 50 by 50 state vectors. We see initial consumption rising with initial endowments in physical and human capital. Closer inspection shows a soft fold in the surface to the left of the curve of stationary equilibria (the solid line). It is due to the fact that for far too large endowments of human capital relative to physical capital the upper bound one on the choice of the share u is binding. In such a situation there will be no human capital allocated to the educational sector and at least part of the extra output will be consumed and not invested. Apart from the fold the surface appears quite flat, almost a plane. In other words, the substitution rates between physical and human capital under the assumption of constant initial optimal consumption shows very little variation.

The picture of the surface of optimal inital consumption may be helpful and instructive, but it reveals no dynamic effects. To see, how different the future development of economies can be we better draw on a sample of initial states again. Figure 3 depicts the evolution of the sample considered earlier.

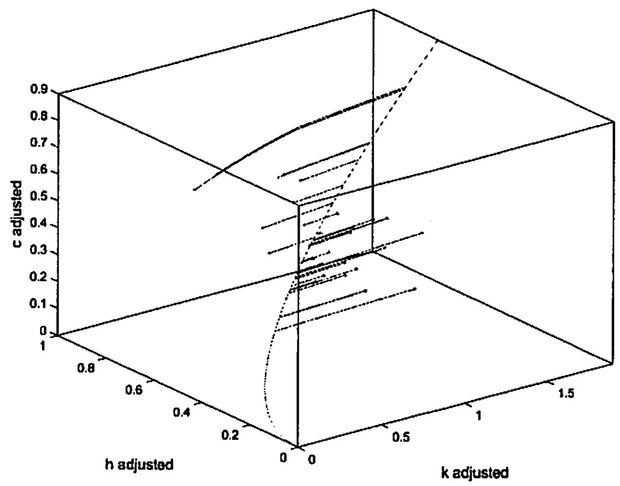

Figure 3: Optimal trajectories in the scale adjusted model

It shows scale adjusted solutions in k, h and c. We see that the respective trajectories converge to the curve of stationary equilibria (CSE). Moreover, they typically converge to different points. That means, in the long run their growth will be balanced, but on different levels. The further to the right the limit point is on the curve (CSE) the greater the long run lead of the corresponding economy.

Recalling figure 1 we can state that an inspection of the cloud in state coordinates hardly allows a prediction which economy will be ahead of others in the long run. But with the help of the computation underlying figure 3 we can classify points in the state space (k, h) with respect to their long run performance. We put this idea into a more formal definition.

Definition 1 (Convergence Classes) *Vectors of state variables belong to the same* Convergence Class *or growth fibre of a particular model of economic growth, if their respective growth trajectories converge in the long run.*

We can state a result which is quite obvious for the Lucas model:

Proposition 1 *The convergence classes of the Lucas model are the trajectories of the scale adjusted version of the model.*

The proof is easy and left to the reader. It only requires to check that convergence in the scale adjusted version is equivalent to pathwise convergence in the non-adjusted model.

Definition 2 (Balanced Base Point of a Convergence Class) *The common limit point of a convergence class on the curve* CES *is called the Balanced Base Point of the Convergence Class.*

In order to give a complete impression of the evolution of the particular sample considered above we add a figure in non-adjusted variables. Scale adjustment neither affects the cloud of initial states nor the shape of curve the stationary equilibria or balanced growth solutions, respectively. Only the shape of trajectories differs (figure 4).

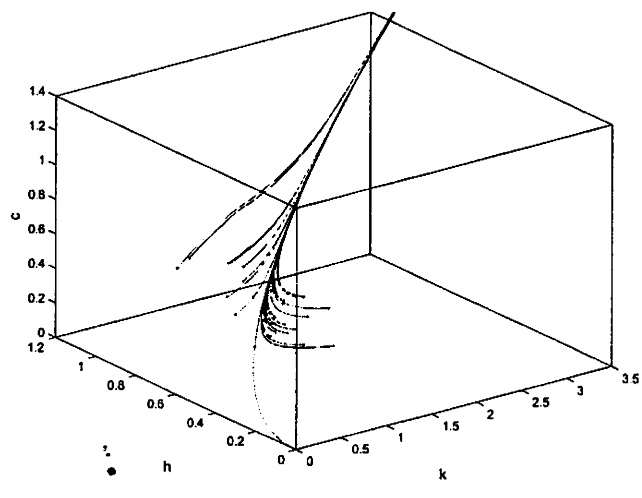

Figure 4: Optimal trajectories in the non-adjusted model

Inequality can be measured in various ways, with respect to all kinds of variables and with different measures. We confine our exposition to a single approach, the entropy measure due to Theil applied to consumption expenditures c. Figure 5 presents the time paths of this inequality measure

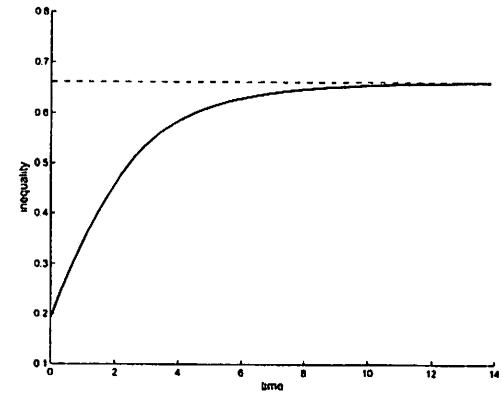

Figure 5: Consumption expenditure inequality

for the economies of the sample. We notice that in the sample considered inequality is monotonically increasing regardless of the fact that all economies converge to balanced growth! To our understanding this observation is an interseting point in itself beyond saddle stability of balanced growth.

The long run inequality of the smple can be derived immediately from the numerical solution of the scale adjusted model, because our inequality measure is scale invariant.

Proposition 2 *For scale invariant measures of inequality the long run inequality of a sample of economies is equal to the inequality between the balanced base points of the sample.*

Beyond inequality we compute the skewness of the distribution of consumption expenditures (figure 6). It turns out, that there is a peak in skewness at the beginning of the adjustment process and finally converges to a level higher than the initial level. There seems to be no obvious explanation for this observation and below we analyse a different sample, where the pattern is different.

A further distributional issue is mobility. The time path of consumption can be convex or concave and hence intersections of the consumption paths

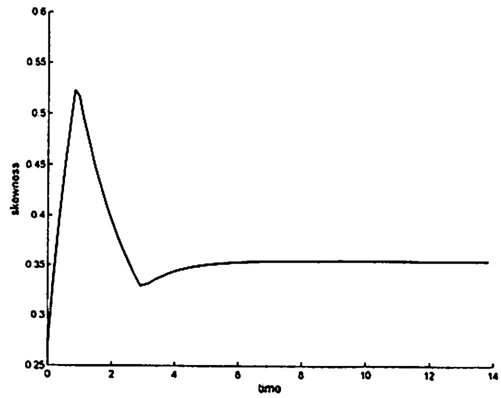

Figure 6: Skewness of the consumption expenditure distribution

of different economies can occur. Changes of the ranking of economies or mobility in the sample is the consequence. In general mobility occurs as figure 7 shows.

For the rest of the paper we specify the sample of initial states with respect to optimal policy. The definition of convergence classes already was one way to do that. There are two obvious and interesting alternatives. Initial optimal consumtion or utility may coincide within a sample or discounted utility.

3.2 Initial states with identical initial utility

In this section we consider a sample with identical optimal current utility under the assumption of intertemporal optimization. We can think of several reasons to deal with initial utility and not only with the seemingly more natural objective function of intertemporal optimization, the discounted utility function. First of all, a policy is easier to implement the more it favors those who are involved in the implementation. Second, current utility is one aspect of intergenerational justice. Last not least is it worthwile make an attempt to understand the difference between initial imbalance and initial inequality.

A formal definition specifies the appropriate class concept.

Definition 3 (Current Utility Class) *Vectors of state variables belong to the same* Current Utility Class *or utility fibre of a particular model of economic growth, if their optimal current utilities coincide.*

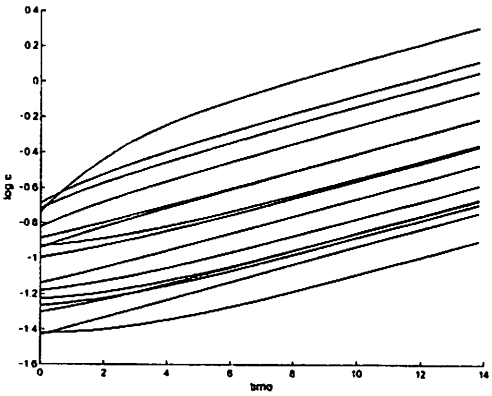

Figure 7: Mobility in the random sample

In state space coordinates we give a typical example in figure 8 (see next page). Our picture portrays the initial states of the sample and the resulting trajectories. The initial states form the current utility class. At the same time each trajectory is a section of a convergence class connecting an initial state of the sample with the balanced base point of the corresponding convergence class. The resulting balanced base points are spread over in a certain way along the curve *CSE*. This spread represents the long run inequality of the sample. On the other hand, the spread of initial points within the current utility class represents the initial imbalance. Figure 8 reveals the difference between convergence classes and current utility classes. The former ones are flatter impying a clear relation between imbalance and long run performance. Although the observation is robust against parameter changes we don't have a formal general proof, and so we present it in a less formal way.

Observation 1 *Take the parameter set underlying our simulation as given. Then we find: Within a current utility class a higher level of human capital induces a better long run performance.*

Scale adjustment conceals an intersting aspect of the dynamics of the system or at least obscures it. By contrast, the non-adjustred version (see figure 9) reveals it clearly: Convergence to balanced growth is much faster for capital intensive economies. The plausible explanation again is that extra output due to excess human capital can be consumed, which alleviates the pressure to remedy imbalance.

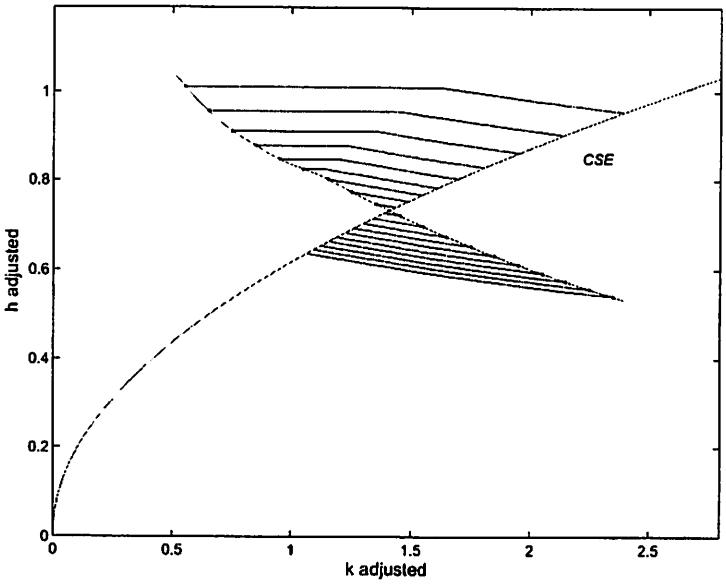

Figure 8: Optimal trajectories of a current utility class

Regarding inequality the picture for utility classes is not much different from that of the the general sample. Hence we take the liberty to skip it. The time pattern of skewness is slightly different. The peak ist sharper and convergence after the peak is monotonic now. We include the picture although we have no idea to explane the shape. Whereas we observed mobility with repsect to consumption expenditure (or utility) in the random sample, the opposite is the case in utility classes. Without an explanation we present the result. Starting off with identical consumption levels the less capital intensive countries turn out to reach higher consumption levels for all future points in time.

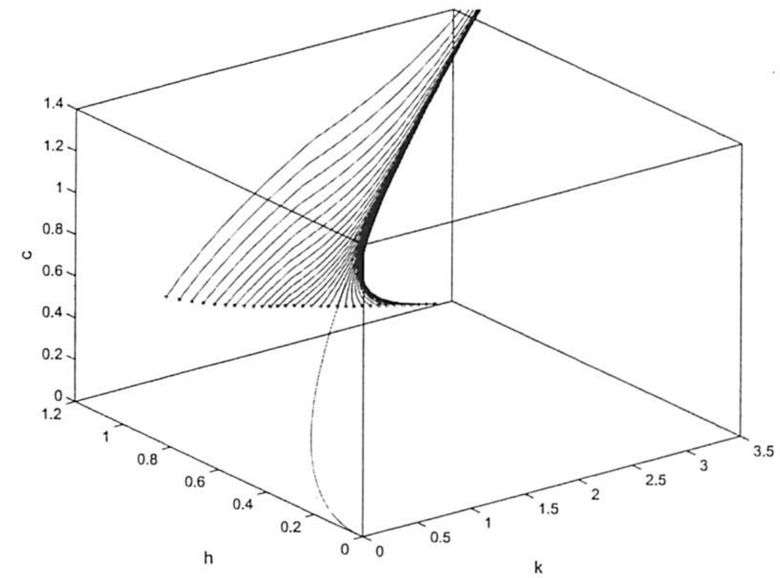

Figure 9: Optimal non-adjusted trajectories of a current utility class

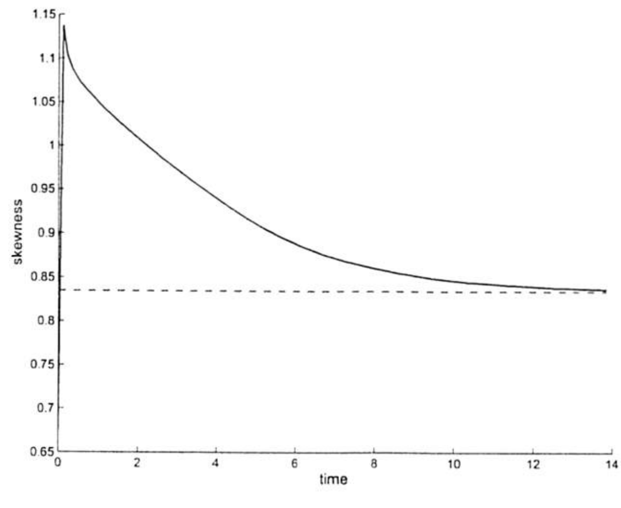

Figure 10: The time pattern of skewness of a utility class

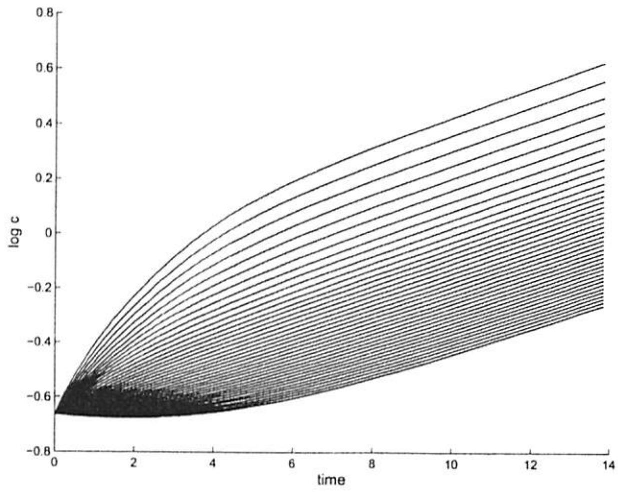

Figure 11: Immobility in current utility classes

3.3 Initial states with identical discounted utility

Discounted utility, the objective function of the representative agent of a single economy, is a natural candidate to define state space classes. Starting off from different positions economies may well achive the same value of discounted utility over the infinite time horizon. None of the classification criteria considered before is equivalent to this last one, but it turns out the long run similarity as expressed in terms of convergence classes is closer related to the discounted utility approach than current utility. As far as numerical accuracy is concerned the computation of discounted utility is difficult to assess. However, our calculations are accurate enough to give the correct qualitative picture.

As in the preceding sections we first give a definition of the concept of classification.

Definition 4 (Discounted Utility Class) *Vectors of state variables belong to the same* Discounted Utility Class *or discounted utility fibre of a particular model of economic growth, if their optimal discounted current utilities coincide.*

Figure 12 shows a typical discounted utility class. The discounted utility classes turn out to be almost identical to convergence classes because of fast convergence to balanced growth. The non-adjusted version are added for the sake of completeness of the illustration of discounted utility classes (figure 13). Inequality within a discounted utility class must develop in a different way than before. By the definition of convergence classes there is no long run inequality. As the shape of discounted utility classes is close to that of convergence classes, the long run inequality must be small, but positive. Initial imbalance gives rise to initial inequality which is balanced by small inequality in the long run (see figure 14). The contrast between short run and long run in a discounted utility class implies a particular mobility pattern. Initially capital intensive economies start with a lower level of consumption - as mentione dbefore. Identical discounted utility there requires a reversed pattern in the long run: Those with lower initial utility must balance total discounted utility through slightly higher long run consumption (c.f. figure 15).

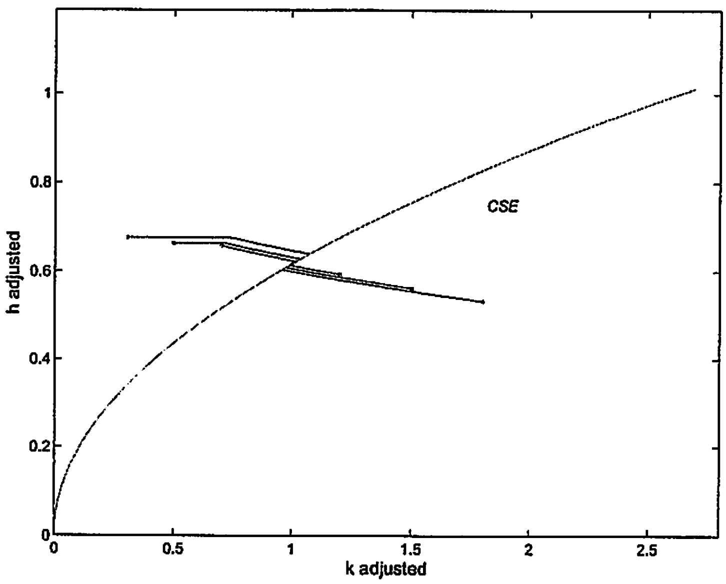

Figure 12: Optimal trajectories of a discounted utility class

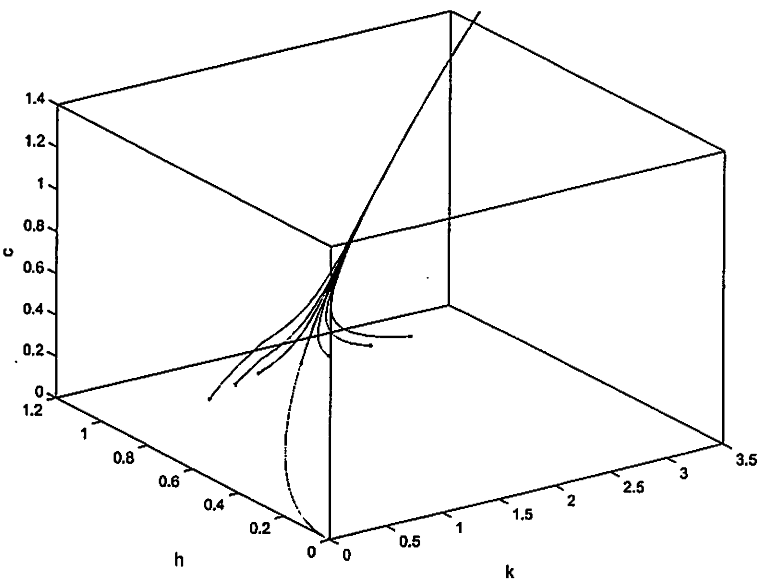

Figure 13: Optimal non-adjusted trajectories of a discounted utility class

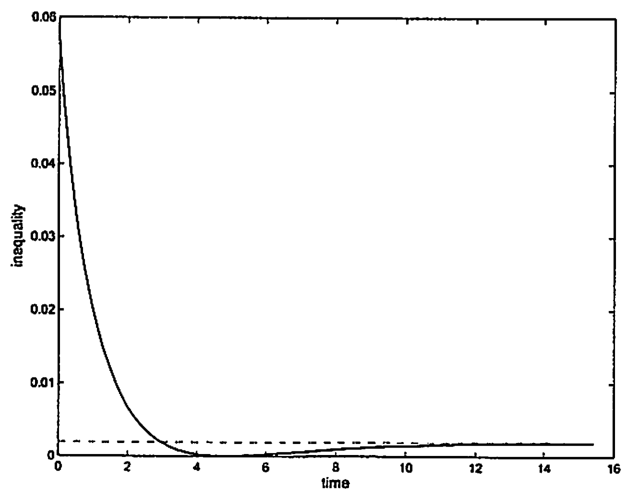

Figure 14: The time path of inequality in a discounted utility class

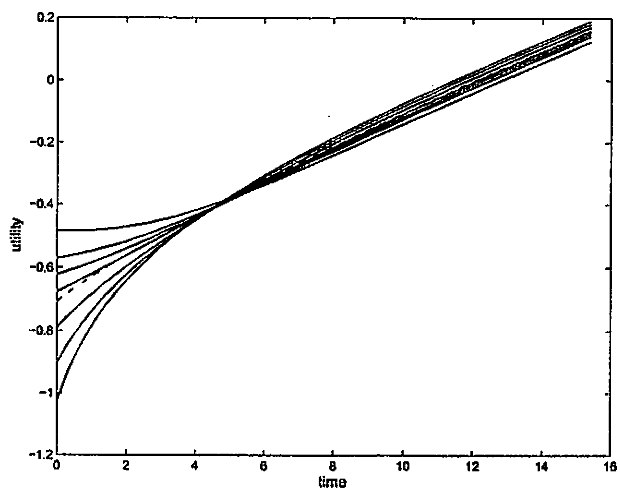

Figure 15: Mobility in a discounted utility class

4 Conclusion

The goal of the paper was to provide tools to investigate the relation between initial imbalance and long run inequality.

We established and characterized three different classifications of initial states for models of endogenous growth. Convergence classes are defined with respect to the long run performance, current utility classes with respect to optimal initial decisions, and discounted utility classes take the development as a whole into account.

For the Lucas model we evaluated the concepts and demonstrated their power. Initial imbalance can appear in different form and with different long run implications. Mobility occurs to relate initial imbalance to final distributions. The classes help to identifiy typical patterns.

The concepts are evaluated numerially. The numerical method of relaxation can be applied to higher dimensional models without problems. However, the analytical work lacks behind. More analysis on the shape of classes is possible and reasonable in order not to limit the method.

Future work may proof that these or similar classification methods improve the characterization of the relation between imbalance and ineuqality in models of economic growth.

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