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Aging in a Neoclassical Theory of Labor Demand

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Abstract. This paper introduces aging of workers into the neoclassical theory of labor demand. Among other things, it is shown that under reasonable hypotheses employment, even of younger workers, increases in the span of working life. Using the standard model without aging, the analysis of such issues has not been possible up to now. The main results derived for constant returns to scale carry over to the qualitatively equivalent steady state of a model with decreasing returns to scale. Thus, constant returns to scale loose much of their special flavor. The implied age-structure of optimum employment turns out to accord well with empirical evidence.

Keywords: Labor Demand – Aging – Adjustment Costs

JEL-Classification: D92; J23

1 Introduction

Adjustment costs were introduced into the neoclassical theory of investment by Eisner and Strotz (1963), who were followed by Lucas (1967) and others. Subsequently, adjustment costs have also been used in the dynamic theory of labor demand (cf. Nickell, 1986). Recently, the neoclassical theory of investment has been augmented by allowing for aging of capital goods (vintage capital) in a complete dynamic optimization framework by Feichtinger et al. (2001) and Barucci and Gozzi (2001), who also provide an extensive overview on various approaches to vintage capital in the literature. The purpose of the present paper is to introduce aging of workers into the theory of labor demand. As in the case of capital goods, this problem would be trivial in the absence of adjustment costs. For the firm's dynamic problem would be equivalent to an infinite sequence of static problems in which optimum demand for labor (of possibly different ages) could be determined at every instant without any consequences for the future.[1] Thus, aging of workers will be considered in the framework of a neoclassical adjustment costs model.

Although there are close similarities between the theory of labor demand and the theory of capital demand, there are also some important differences. First, while a firm usually accumulates the capital stock that it buys on the capital goods market, it just rents labor services at the current wage rate. Thus,

[1]E.g., demand for human capital in the general equilibrium model of vintage human capital by Chari and Hopenhayn (1991) has this simple structure. Their model is otherwise much more sophisticated than the present partial equilibrium approach, however.

The present paper is not concerned with the non-trivial empirical problem of complementarity or substitutability between workers of different age-groups, on which cf. e.g. Levine and Mitchell (1988).

the optimization problem with respect to labor demand is not equivalent to the optimization problem with respect to capital demand. Second, the span of working life may be influenced by political decisions and is therefore a reasonable candidate for a sensitivity analysis of labor demand. Third, a large part of labor adjustment costs does not arise for technical reasons but because of legal regulations, making it interesting to consider the impact of different adjustment costs on the employment decisions of a firm.

There are two principle approaches for handling aging in the framework of continuous time dynamic optimization models. While Feichtinger et al. (2001) use the maximum principle for systems with distributed parameters embedded in the McKendrick partial differential equation (PDE), Barucci and Gozzi (2001) transform the PDE into an ordinary differential equation with values in an infinite dimensional vector space. Both papers are concerned with explicit solutions of linear models arising from production functions with constant returns to scale.

The present paper follows the approach of Feichtinger et al. (2001), but vintage capital goods are replaced by aging workers. In addition to the already mentioned differences of the respective optimization problems and the focus of the analysis, the basic set-up will be extended by considering age dependent adjustment costs and the case of decreasing returns to scale. Although explicit solutions are merely numerically feasible in this case, analytical results can be obtained using phase diagram analysis and variational equations.

Among other things, it will be shown that under constant returns to scale a firm's employment increases in the span of working life provided that at the time of retirement of workers their marginal productivity exceeds their wage rate. The impact of adjustment costs on employment is negative. Moreover, some unrealistic implications of the case without aging under constant returns to scale disappear once aging is taken into account. Even without voluntary quitting, there exists a steady state with respect to calendar time in which employment is time invariant. With neither aging nor voluntary quitting, on the other hand, the firm either exits from the market or employment is ever increasing in calendar time, implying that equilibrium under perfect competition does not exist. Considering the steady state, the case of decreasing returns to scale turns out to be qualitatively equivalent to the case of constant returns to scale under some simplifying hypotheses.

The presented model is too simple for an immediate application to reality and data. Nevertheless, it is possible to put forward some plain hypotheses that allow for a macroeconomic interpretation of the results. It is remarkable that the implied age-structure of optimum employment turns out to accord well with recently observed patterns in G7-countries.

A basic model with constant returns to scale will be presented in Section 2. This model has the advantage of allowing for an explicit solution (which is derived in an appendix) with clear-cut economic implications. The case of decreasing returns to scale is analyzed in Section 3, which also includes a brief presentation of empirically observed patterns of the age-structure of employment. The final section concludes.

2 Constant Returns to Scale

2.1 Solution of the Optimization Problem

Consider a firm producing its output $Y(t)$ according to the constant returns to scale production function

$$Y(t) = \int_0^T a(\tau)L(t,\tau)\,d\tau$$

under perfect competition. Labor input of age τ, $\tau \in [0,T]$, at time t, $t \in [0,\infty)$, is denoted by $L(t,\tau)$ and $a(\tau)$ is the productivity of labor of age τ. Workers of age T leave the firm for free. Although there are no other inputs besides labor, it is reasonable to use the notion constant returns to *scale* as labor input is not homogenous but has possibly different productivities at different ages.

Labor costs are the wage rate $w(\tau)$ and adjustment costs $c(\tau)u(t,\tau)^2/2$, $c(\tau) > 0$, where $u(t,\tau)$ is the rate of hiring or firing of workers of age τ at time t. Both the wage rate and the adjustment costs generally depend on age τ. Adjustment costs are symmetric, strictly convex, and represented by a quadratic cost function. Hamermesh and Pfann (1996) reject this standard assumption as being not consistent with empirical evidence. It is adopted here for the sake of simplicity, however, as the analysis focuses on the effects of aging on labor demand. For an overview and assessment of other specifications of adjustment costs in the literature, cf. Hamermesh and Pfann (1996).

The firm's objective is to maximize the present value of all its cash flows from the present to the indefinite future. As the output price will be assumed to be constant, it may as well be normalized to $p \equiv 1$. Denoting partial derivatives by subscripts, the optimization problem can thus be written as

$$\max_{u(t,\tau)} \int_0^\infty \int_0^T \left[a(\tau)L(t,\tau) - w(\tau)L(t,\tau) - \frac{c(\tau)}{2}u(t,\tau)^2\right] e^{-rt}\,d\tau\,dt \tag{1a}$$

$$\text{subject to} \quad L_t(t,\tau) + L_\tau(t,\tau) = u(t,\tau) - \delta(\tau)L(t,\tau), \tag{1b}$$

$$L(0,\tau) = L_0(\tau), \quad L(t,0) = 0, \tag{1c}$$

where r is the constant interest rate (a non-constant interest rate could be introduced at the cost of complicating notation without adding much insight). Voluntary quitting of workers takes place at the percentage rate $\delta(\tau)$ and induces no direct costs. The functions a, w, c, and δ are assumed to be positive and twice continuously differentiable. For simplicity, the state constraint $L(t,\tau) \geqq 0$ is not considered explicitly, which should be kept in mind for the remainder of this paper.

The partial differential equation (1b) captures the fact that workers are getting older. E.g., if $u(t,\tau) = 0$ and $\delta(\tau) = 0$, then $L_t(t,\tau) = -L_\tau(t,\tau)$, that is, the rate of change of the number of workers of age τ with respect to calendar time would exactly balance the rate of change with respect to age. The boundary conditions (1c) indicate that at time 0 there is an initial distribution $L_0(\tau)$ of workers of ages $\tau \in [0,T]$ and that at each time $t \in [0,\infty)$ there are no workers of age zero. The model could be augmented to include a positive number of workers of age zero at each time t by introducing a boundary control $u_0(t)$ and setting $L(t,0) = u_0(t)$, cf. Feichtinger et al. (2001) for the case of investment demand. In order to simplify notation and as it would not add too much insight, this generalization will be neglected at the moment. A boundary control will be considered in Section 3, however, as it turns out to be important with respect to an explanation of empirically observed patterns of the age-structure of employment.

Setting up the current-value Hamiltonian,

$$H = [a(\tau) - w(\tau)]L(t,\tau) - \frac{c(\tau)}{2}u(t,\tau)^2 + \lambda(t,\tau)[u(t,\tau) - \delta(\tau)L(t,\tau)],$$

the following necessary conditions for optimality are derived:[2]

$$H_u = 0 \qquad \Longleftrightarrow \qquad u(t,\tau) = \lambda(t,\tau)/c(\tau) \tag{2a}$$

$$\lambda_t(t,\tau) + \lambda_\tau(t,\tau) = r\lambda(t,\tau) - H_L = [r + \delta(\tau)]\lambda(t,\tau) - a(\tau) + w(\tau), \tag{2b}$$

$$\lambda(t,T) = 0. \tag{2c}$$

These conditions must hold almost everywhere on $t \in [0,\infty)$ and $\tau \in [0,T]$.

The solution of the partial differential equation (2b), taking the boundary condition $\lambda(t,T) = 0$ into account, is[3]

$$\lambda(t,\tau) = \int_\tau^T [a(\sigma) - w(\sigma)]e^{-\int_\tau^\sigma [r+\delta(\rho)]\, d\rho}\, d\sigma.$$

Combining this result with the optimum condition (2a) immediately yields the optimum demand for new workers of age τ at time t (which may be negative if firing instead of hiring is optimal):

$$u(t,\tau;T) = \frac{1}{c(\tau)} \int_\tau^T [a(\sigma) - w(\sigma)]e^{-\int_\tau^\sigma [r+\delta(\rho)]\, d\rho}\, d\sigma. \tag{3}$$

The dependence of the optimum solution on the span of working life is emphasized by the inclusion of T as a variable. Notice that $u(t,\tau;T)$ is independent of calendar time t. Therefore, it is reasonable to define

$$\tilde{u}(\tau;T) := u(t,\tau;T). \tag{4}$$

As an immediate implication of equation (3), observe that if $a(T) < w(T)$, $\tilde{u}(\tau;T)$ may become negative at $\tau_1 < T$ even if $a(\tau_1) > w(\tau_1)$. Hence, if a firm expects that the marginal productivity of workers will fall short of their wage

[2]An introduction to the maximum principle for systems with distributed parameters can be found in Feichtinger and Hartl (1986, App. 5). More general optimum conditions have been derived by Feichtinger et al. (2000).

[3]The solution of equation (1b) by the method of characteristics is explained in the appendix. Equation (2b) can be solved by the same method. In order to verify this and other solutions derived in this paper the reader should recall Leibniz's rule of differentiation of a definite integral.

rate in the future, it is rational to release workers at an age when their marginal productivity still exceeds their wage rate.

It is shown in the appendix that the solution of (1b) using the boundary conditions (1c) is

$$L(t,\tau;T) = L_0(\tau - t)e^{-\int_{\tau-t}^{\tau}\delta(\rho)\,d\rho} + \int_{\tau-t}^{\tau}\tilde{u}(s;T)e^{-\int_s^{\tau}\delta(\rho)\,d\rho}\,ds \quad \text{if} \quad t < \tau, \tag{5}$$

$$\tilde{L}(\tau;T) := L(t,\tau;T) = \int_0^{\tau}\tilde{u}(s;T)e^{-\int_s^{\tau}\delta(\rho)\,d\rho}\,ds \quad \text{if} \quad t \geqq \tau. \tag{6}$$

Observe that $L(t,\tau;T)$ is independent of t for $t \geqq \tau$, which accounts for the definition $\tilde{L}(\tau;T) := L(t,\tau;T)$. Since $\tau \leqq T$, this implies together with (4) that after no more than T years everything becomes time-invariant, yielding

Proposition 1 *The steady state of the solution to problem (1) with respect to calendar time is reached after no more than T years. Employment of workers of any age τ is constant in the steady state.*

An analogous result has been derived by Feichtinger et al. (2001) for the case of capital goods.

2.2 A Note on Perfect Competition

While the constant returns to scale technology is usually considered an important standard case due to the well-known *replication argument*, introducing adjustment costs into the standard model without aging leads to a contradiction if there is no voluntary quitting of workers. For the sake of simplicity, consider a production function with just one input, non-aging labor, and a firm under perfect competition striving to solve

$$\max_{u(t)} \int_0^{\infty} \left[(a-w)L(t) - \frac{c}{2}u(t)^2\right] e^{-rt}\,dt, \qquad \frac{dL(t)}{dt} = u(t) - \delta L(t), \quad L(0) = 0.$$

A straightforward application of Pontryagin's maximum principle shows that the solution to this problem is given by

$$u(t) = \frac{a-w}{cr}, \quad L(t) = \frac{a-w}{cr}t, \quad \text{if} \quad a \geqq w \quad \text{and} \quad \delta = 0,$$

$$u(t) = \frac{a-w}{c(r+\delta)}, \quad L(t) = \frac{a-w}{c\delta(r+\delta)} - \frac{a-w}{c\delta(r+\delta)}e^{-\delta t}, \quad \text{if} \quad a \geqq w \quad \text{and} \quad \delta > 0.$$

If $a < w$, then $u(t) = L(t) = 0$. Thus, without voluntary quitting ($\delta = 0$), if $a \leqq w$, the firm will exit the market, while if $a > w$, it will grow without bound, contradicting the assumption of a small firm under perfect competition. Therefore, in the presence of adjustment costs and if $\delta = 0$ in the standard model, an equilibrium under perfect competition does not exist under constant returns to scale. This assertion does not depend on labor being the only input. If δ is getting positive, the dynamic behavior of the system changes radically, that is, $\delta = 0$ is a bifurcation point at which the equilibrium $L = (a-w)/[c\delta(r+\delta)]$, which exists if $\delta > 0$, disappears.

Taking aging of workers into account, Proposition 1 shows that there is neither a contradiction if $\delta = 0$ nor is $\delta = 0$ a bifurcation point. In fact, if $a(\tau) = a$, $w(\tau) = w$, $c(\tau) = c$, and $\delta(\tau) = \delta$ are constants and $a > w$ in the model with aging, $\tilde{u}(\tau;T)$ and $\tilde{L}(\tau;T)$ can be explicitly calculated in order to directly compare the results. If $\delta = 0$, the solutions for $t \geqq \tau$ are

$$\tilde{u}(\tau;T) = \frac{a-w}{rc}\left(1 - e^{r(\tau-T)}\right), \tag{7}$$

$$\tilde{L}(\tau;T) = \frac{a-w}{rc}\left[\tau - \frac{1}{r}\left(e^{r(\tau-T)} - e^{-rT}\right)\right]. \tag{8}$$

As in the case without aging, the firm will exit if $a \leqq w$, but it will not grow without bound if $a > w$. Also, the bifurcation of behavior at $\delta = 0$ disappears. While an explicit solution is even possible if $\delta > 0$, the resulting expressions are rather tedious. The qualitative properties of the steady state solution in case of $\delta > 0$ will be derived in the next section, however.

2.3 Steady State Implications

According to Proposition 1, it is reasonable to concentrate on the steady state properties of the hiring or firing decisions and employment, respectively. The following propositions pertain to the steady state with respect to calendar time of the solution to problem (1).

Proposition 2 (Comparative Dynamics of Hiring and Firing)

(a) If $a(T) > w(T)$ ($a(T) < w(T)$, resp.), the rate of hiring of workers of all ages τ increases (decreases, resp.) in the span of working life T.

(b) The higher the adjustment costs $c(\tau)$ for workers of age τ, the lower is the hiring or firing rate of such workers.

Proof: (a) The derivation of (3) with respect to T is

$$\tilde{u}_T(\tau;T) = \frac{1}{c(\tau)}[a(T) - w(T)]e^{-\int_\tau^T [r+\delta(\rho)]\, d\rho} \gtreqless 0 \quad \text{if} \quad a(T) \gtreqless w(T).$$

(b) Obvious from (3). □

Proposition 2 (a) shows that the chances of getting a job for younger workers depend on their expected span of working life. E.g., it pays to hire additional workers if their duration in the firm is extended and their marginal productivity at the moment when they leave the firm for free exceeds their wage rate at that time. Proposition 2 (b) has an obvious interpretation which coincides with the case without aging.

The impact of age τ on the hiring or firing decision is not unambiguous. Using Leibniz's rule, the derivative of (3) with respect to τ is

$$\tilde{u}_\tau(\tau;T) = \left[r + \delta(\tau) - \frac{c_\tau(\tau)}{c(\tau)}\right]\tilde{u}(\tau;T) - \frac{a(\tau) - w(\tau)}{c(\tau)}. \tag{9}$$

This equation reveals four effects of age τ on the hiring decision of the firm. Suppose that $\tilde{u}(\tau;T) > 0$. Then, first, $r\tilde{u}(\tau;T) > 0$ captures the positive *discounting effect*, as the interest costs of hiring new workers of age τ do not accrue if workers of age $\tau + \epsilon$, $\epsilon \to 0^+$, are hired. Second and similarly, $\delta(\tau)\tilde{u}(\tau;T) > 0$ is the positive *quitting effect*. Third, the *adjustment costs effect* $[-c_\tau(\tau)/c(\tau)]\tilde{u}(\tau;T)$ is positive (negative) if adjustment costs of older workers are lower (higher) than for younger workers. Fourth, $-[a(\tau) - w(\tau)]/c(\tau)$ is negative if $a(\tau) > w(\tau)$, since a worker hired at a marginally older age cannot contribute to the revenue of the firm at the time when he is of age τ (the *age effect*).[4] Thus, a general conclusion with respect to the sign of $\tilde{u}_\tau(\tau;T)$ is not possible. Proposition 4 below shows, however, that if $a(\tau)$, $w(\tau)$, $c(\tau)$, and $\delta(\tau)$ are constant and $a > w$, the firm will always hire more younger than older workers.

While Proposition 2 deals with the firm's hiring and firing decisions, the next proposition pertains to the level of employment.

Proposition 3 (Comparative Dynamics of Employment)

(a) If $a(T) > w(T)$ ($a(T) < w(T)$, resp.), employment of workers of all ages τ increases (decreases, resp.) in the span of working life T. Hence, even total employment (aggregated over τ) increases in T if $a(T) > w(T)$.

(b) Let $\tilde{L}(\tau;T) > 0$. If adjustment costs $c(\tau)$ for labor of all ages increase proportionally, employment decreases proportionally.

Proof: (a) The derivative of (6) with respect to T, taking the proof of Proposition 2 (a) into account, is

$$\tilde{L}_T(\tau;T) = \int_0^\tau \tilde{u}_T(s;T)e^{\int_\tau^s \delta(\rho)\, d\rho}\, ds \gtreqless 0 \quad \text{if} \quad a(T) \gtreqless w(T).$$

(b) Substituting (3) into (6) and replacing $c(\tau)$ by $\varepsilon c(\tau)$ yields

$$\begin{aligned} \tilde{L}(\tau;T,\varepsilon) &:= \int_0^\tau \frac{1}{\varepsilon c(s)} \int_s^T [a(\sigma) - w(\sigma)]e^{-\int_\tau^\sigma [r+\delta(\rho)]\, d\rho}\, d\sigma e^{\int_\tau^s \delta(\rho)\, d\rho}\, ds \\ &= \frac{1}{\varepsilon}\tilde{L}(\tau;T). \end{aligned}$$

Thus, $d\tilde{L}(\tau;T,\varepsilon)/d\varepsilon = -\tilde{L}(\tau;T) < 0$ evaluated at $\varepsilon = 1$, or $d\tilde{L}/\tilde{L} = -d\varepsilon/\varepsilon$ at $\varepsilon = 1$. □

Proposition 3 (a) has a similar economic explanation as Proposition 2 (a). In particular, it shows that under the reasonable hypothesis that the marginal productivity of workers exceeds their wage rate at the usual age of retirement, employment increases in the span of working life. In contrast, Proposition 3 (b) cannot readily be explained in terms of Proposition 2 (b), which states that the hiring and the firing rate both decrease in adjustment costs, leaving the overall

[4]Feichtinger et al. (2001) derive similar effects for the case of investment in machines. As they do not consider age dependent adjustment costs, however, there is no related effect in their model, which instead involves an effect of acquisition costs of new machines which are not considered here.

effect on employment open at first sight. Nevertheless, Proposition 3 (b) shows that the level of employment decreases in adjustment costs. As Bertola (1992) has shown using an asymmetric adjustment costs model without aging, however, it is possible that the average level of employment increases in firing costs. A more thorough analysis of the impact of firing costs on average employment in the present model would require asymmetric adjustment costs not considered here. As in Bertola (1992), it would also be reasonable to include cyclical behavior of revenues, e.g. by modelling the price of output as a trigonometric function of time. Such issues will be left for future research at the moment.

Definite implications with respect to the age-structure of employment can be derived under the additional assumptions of the following proposition. Notice that in case of constant productivities and wages an interesting problem requires that $a > w$.

Proposition 4 (Age-Structure of Hiring and Employment) *Suppose that* $a(\tau) = a$, $w(\tau) = w$, $c(\tau) = c$, *and* $\delta(\tau) = \delta$, $0 \leqq \delta < 1$ *are constants and that* $a > w$. *Then:*

(a) The firm always hires more younger than older workers.

(b) The age-structure of employment is single peaked, that is, employment first increases and eventually decreases in τ. *If* $\delta = 0$, *the peak occurs at* $\tau = T$.

Proof: (a) Under the above hypotheses, (9) reduces to

$$\tilde{u}_\tau(\tau;T) = \frac{a-w}{c}\left(1 - e^{(r+\delta)(\tau-T)}\right) - \frac{a-w}{c} = -\frac{a-w}{c}e^{(r+\delta)(\tau-T)} < 0.$$

(b) Let $\delta > 0$. As $L_t = 0$ in a steady state, (1b) and (4) imply that

$$\tilde{L}_\tau(\tau;T) = \tilde{u}(\tau;T) - \delta\tilde{L}(\tau;T).$$

As $\tilde{u}_\tau(\tau;T) < 0$ according to (a) and $\tilde{u}(T;T) = 0$ from (3), it follows that $\tilde{u}(0;T) > 0$. Since $\tilde{L}(0;T) = L(t,0;T) = 0$ according to (1c) and $\tilde{L}(T;T) > 0$ from (6) (taking (3) and $a > w$ into account), it follows that

$$\tilde{L}_\tau(0;T) = \tilde{u}(0;T) - \delta\tilde{L}(0;T) > 0,$$
$$\tilde{L}_\tau(T;T) = \tilde{u}(T;T) - \delta\tilde{L}(T;T) < 0.$$

Thus, there exists $\tau_1 \in (0,T)$ such that $\tilde{L}_\tau(\tau_1;T) = 0$. Now, observe that

$$\tilde{L}_{\tau\tau}(\tau_1;T) = \tilde{u}_\tau(\tau_1;T) - \delta\tilde{L}_\tau(\tau_1;T) = \tilde{u}_\tau(\tau_1;T) < 0,$$

implying that there is an $\epsilon > 0$ such that $\tilde{L}_\tau(\tau;T) < 0$ for $\tau_1 < \tau < \tau_1 + \epsilon$, which by substitution into $\tilde{L}_{\tau\tau}(\tau;T)$ yields $\tilde{L}_{\tau\tau}(\tau;T) < 0$ and $\tilde{L}_\tau(\tau;T) < 0$ for all $\tau > \tau_1$. Thus, $\tilde{L}$ assumes a unique and global maximum at τ_1, and $\tilde{L}$ increases (decreases) in τ if $\tau < \tau_1$ ($\tau > \tau_1$), proving (b) if $\delta > 0$. If $\delta = 0$, the proposition follows immediately from $\tilde{L}_\tau(\tau;T) = \tilde{u}(\tau;T) > 0$. □

Although the constancy hypotheses of Proposition 4 are rather restrictive, its implications are convincing. It will be shown in the next section that the age-structure of employment according to Proposition 4 is also valid in case of decreasing returns to scale under similar hypotheses and that it accords well with recently observed patterns in G7-countries.

3 Decreasing Returns to Scale

3.1 The Age-Structure of Employment in the Steady State

Although the maximum principle for systems with distributed parameters applies to more general cases, using it similar to the case of constant returns to scale requires that the production function is additively separable in labor of different ages. A suitable formulation is given by

$$Y(t) = \int_0^T a(\tau)L(t,\tau)^\alpha \, d\tau, \quad 0 < \alpha < 1. \tag{10}$$

One of the objectives of this section is to show that the model of labor demand with aging is consistent with empirical observed age-structures of employment in industrialized countries. It turns out that it is reasonable to include a boundary control $u_0(t)$ to this end, which indicates the number of workers of age 0 hired at time t. (Notice the difference between hiring $u_0(t)$ and the *rate* of hiring or firing, $u(t,\tau)$.) Thus, the relevant optimization problem now reads

$$\max_{u(t,\tau),u_0(t)} \int_0^\infty \int_0^T \left[a(\tau)L(t,\tau)^\alpha - w(\tau)L(t,\tau) - \frac{c(\tau)}{2}u(t,\tau)^2 \right] e^{-rt} \, d\tau \, dt$$

$$- \int_0^\infty \left[\frac{c(0)}{2} u_0(t)^2 \right] e^{-rt} \, dt \tag{11a}$$

$$\text{subject to} \quad L_t(t,\tau) + L_\tau(t,\tau) = u(t,\tau) - \delta(\tau)L(t,\tau), \tag{11b}$$

$$L(0,\tau) = L_0(\tau), \quad L(t,0) = u_0(t). \tag{11c}$$

In addition to the usual optimum conditions, it is now necessary that $u_0(t)$ maximizes the boundary Hamiltonian

$$\mathcal{H} = -\frac{c(0)}{2}u_0(t)^2 + \lambda(t,0)u_0(t).$$

Thus, the following first-order conditions for optimality must hold:

$$u(t,\tau) = \lambda(t,\tau)/c(\tau) \tag{12a}$$

$$u_0(t) = \lambda(t,0)/c(0) \tag{12b}$$

$$\lambda_t(t,\tau) + \lambda_\tau(t,\tau) = [r + \delta(\tau)]\lambda(t,\tau) - a(\tau)\alpha L(t,\tau)^{\alpha-1} + w(\tau), \tag{12c}$$

$$\lambda(t,T) = 0. \tag{12d}$$

Although there is no general theorem available asserting that the optimum solution must asymptotically approach a steady state with respect to calendar time, it will be shown that the steady state solution satisfies the sufficient optimum conditions for a suitable initial distribution $L_0(\tau)$. Thus, consider the steady state solution where

$$\lambda_t(t,\tau) \equiv L_t(t,\tau) \equiv 0.$$

Defining $\tilde{\lambda}(\tau) := \lambda(t,\tau)$ and $\tilde{L}(\tau) := L(t,\tau)$, (12c) reduces to

$$\tilde{\lambda}_\tau(\tau) = [r + \delta(\tau)]\tilde{\lambda}(\tau) - \underbrace{a(\tau)\alpha\tilde{L}(\tau)^{\alpha-1}}_{=:\tilde{a}(\tau)} + w(\tau) = [r + \delta(\tau)]\tilde{\lambda}(\tau) - \tilde{a}(\tau) + w(\tau)$$

in the steady state. Comparing this equation with (2b) and noting that the optimum condition for u has been left unchanged, it is obvious that formally the steady state solution in case of decreasing returns can be obtained completely analogously to the solution under constant returns to scale. In order to compare the solutions immediately, disregard the boundary control $u_0(t)$ at the moment. The optimum steady state solution for u would then be given by (3) with $a(\sigma)$ replaced by $\tilde{a}(\sigma)$. Using this solution in equation (6) would yield optimum employment in the steady state. As $\tilde{a}(\tau)$ depends on $\tilde{L}(\tau)$, however, equations (3) and (6) would now be interdependent rather than recursive. Substituting (3) into (6) would therefore yield a non-linear integral equation in $\tilde{L}(\tau)$ that could be solved numerically but not analytically.

Being interested in analytical rather than numerical results, an alternative, graphical approach under the additional assumption that $a(\tau) = a$, $w(\tau) = w$, $c(\tau) = c$, and $\delta(\tau) = \delta$, $0 \leqq \delta < 1$, are constant is chosen. For this matter, the optimum conditions pertaining to the steady state with respect to time are rewritten as

$$\tilde{L}_\tau(\tau) = \tilde{u}(\tau) - \delta\tilde{L}(\tau), \quad \tilde{L}(0) = \tilde{u}(0), \tag{13a}$$

$$\tilde{u}_\tau(\tau) = (r+\delta)\tilde{u}(\tau) - (a/c)\alpha\tilde{L}(\tau)^{\alpha-1} + w/c, \quad \tilde{u}(T) = 0, \tag{13b}$$

where, as $\lambda_t = L_t = 0$, (13a) follows from (11b), while (13b) follows from (12c) and $\tilde{u}(\tau) := u(t,\tau) = \tilde{\lambda}(\tau)/c$ according to (12a). Equations (12a) and (12b) imply that $\tilde{u}(0) := u(t,0) = u_0(t) = \text{const.}$, as $\lambda_t \equiv 0$. Together with (11c), this implies the boundary condition $\tilde{L}(0) = \tilde{u}(0)$. The boundary condition $\tilde{u}(T) = 0$ follows immediately from (12a) and (12d).

Equations (13) can be analyzed using the phase diagram in Figure 1. Inspection of the isoclines $\tilde{L}_\tau = 0$ and $\tilde{u}_\tau = 0$ reveals that the optimum trajectory must qualitatively look like the one starting at $\tilde{u}(0)$ (where $\tilde{L}(0) = \tilde{u}(0)$ along the 45°-line) below the stable arm of the saddle-point and terminating at $\tilde{L}(T)$ (where $\tilde{u}(T) = 0$). All other trajectories would violate the boundary conditions in (13).

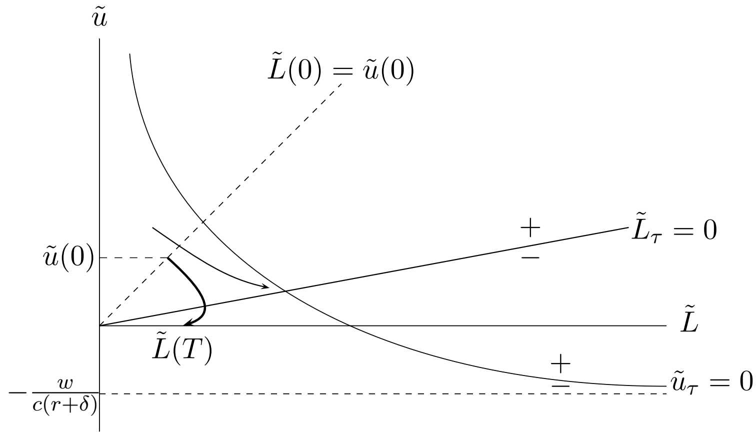

Figure 1. Solution of System (13)

The optimum trajectory in Figure 1 reveals that $\tilde{u}$ is decreasing in τ and that $\tilde{L}$ is first increasing and eventually decreasing in τ, confirming the results obtained for the case of constant returns to scale. Thus, the firm always hires more younger than older workers and the distribution of employment with respect to age is single-peaked. It is also immediate from Figure 1 that the employment peak would occur at $\tau = T$ if $\delta = 0$, that is if there was no voluntary quitting. Although it has been assumed that a and w are constant, there is no need to make an assumption about the relative magnitude of these parameters to derive these results. The reason is that the marginal productivity of labor of any age τ increases without bound when $L(\tau) \to 0$.

It will now be shown that the trajectory $[\tilde{u}(\tau), \tilde{L}(\tau)]$ solving system (13) is an optimum solution for an initial distribution $L_0(\tau)$ which is consistent with the steady state. Sufficient conditions for control problems with distributed parameters have been derived by Haurie et al. (1984). Although the present model is no special case of the models in Haurie et al. (1984), a minor variation of the proof of their Proposition 4 (p. 146) shows that conditions (12) are sufficient if in addition the Hamiltonian H is concave in (L, u) and

$$\lim_{t\to\infty} \int_0^T e^{-rt} \lambda(t,\tau)[L(t,\tau) - \tilde{L}(\tau)]\, d\tau \geqq 0, \tag{14}$$

where $L(t,\tau)$ denotes any other admissible solution (cf. also Feichtinger and Hartl, 1986, p. 524). Concavity of H is assured as the integrand in (11a) is concave and the state equation is linear. To prove (14), note that while the non-negativity of $L(t,\tau)$ has not been explicitly imposed in problem (11), using it would not have affected the derivation of the single trajectory in Figure 1 meeting the necessary optimum conditions, which does not violate the non-negativity of $L(t,\tau)$ anyway. Thus, the solution in Figure 1 would also apply if the condition $L(t,\tau) \geqq 0$ had been imposed. Hence, it is possible to proceed under the (reasonable) assumption that all admissible solutions for $L(t,\tau)$ must be non-negative. As $\infty > \tilde{L}(\tau) > 0$ and $\infty > \lambda(t,\tau) \geqq 0$ for all $\tau \in [0, T]$, it follows that condition (14) is met. Thus, $[\tilde{u}(\tau), \tilde{L}(\tau)]$ is an optimum solution if the system starts in the steady state with respect to calendar time.

Although the present model refers to an individual firm, the following simple hypotheses can be put forward to give a macroeconomic interpretation of the model's results. First, the firm is a representative firm under perfect competition, second, the wage rate is fixed, e.g. due to union power, and third, labor supply of any age τ does not fall short of labor demand for age τ. It is straightforward from the shape of the optimum solution in Figure 1 that the aggregate age-structure of employment will qualitatively look as shown in the final panel of Figure 2. This theoretical distribution is obtained by entering discrete data points and joining them by straight lines, just as it is done with respect to the empirically observed age-structures of employment in G7-countries of 2001.[5]

[5] G7 instead of G8 has been used as data for Russia were neither available nor would it be too sensible to compare them with the *steady state*-distribution of a theoretical model of a market economy.

While the model is too simple for an immediate application to reality and data, it is remarkable that the implied age-structure of employment accords well with empirically observed patterns. As shown in Figure 2, with the exception of Japan, the empirical distributions in G7-countries 2001 match the theoretical distribution well. (Data for the United Kingdom were merely available in highly aggregated form.) Notice that without the boundary control $u_0(t)$ the theoretical distribution would start on the age-axis. Thus, including the boundary control appears to be important with respect to empirical applications of the model.

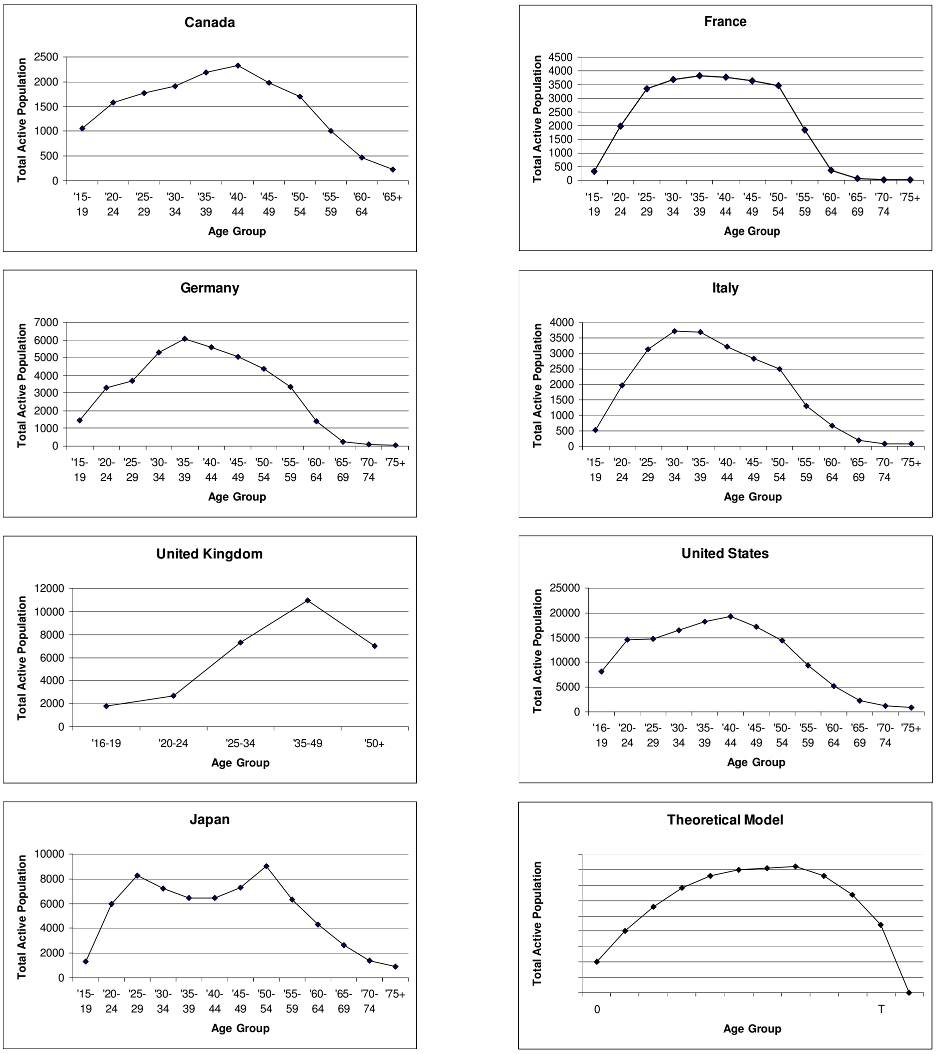

Figure 2. Age-Structure of Employment in G7-Countries 2001, Numbers of Active Population in Thousands, Data Source: International Labor Organization, ILO Bureau of Statistics.

3.2 Comparative Dynamics of the Steady State

The effects of varying the span of working life T can be determined using variational equations. To this end, turn system (13) hypothetically into an initial value-problem with initial values

$$\tilde{u}(0) = \mathfrak{u}, \quad \tilde{L}(0) = \mathfrak{u}. \tag{15}$$

The solutions of this initial value-problem are denoted $\tilde{u}(\tau;\mathfrak{u})$ and $\tilde{L}(\tau;\mathfrak{u})$. The variational equations are obtained by substituting these solutions into (13) with the boundary conditions replaced by (15) and differentiating both with respect to $\mathfrak{u}$:[6]

$$\frac{\partial \tilde{L}_\tau(\tau;\mathfrak{u})}{\partial \mathfrak{u}} = \frac{\partial}{\partial \tau}\frac{\partial \tilde{L}(\tau;\mathfrak{u})}{\partial \mathfrak{u}} = \frac{\partial \tilde{u}(\tau;\mathfrak{u})}{\partial \mathfrak{u}} - \delta\frac{\partial \tilde{L}(\tau;\mathfrak{u})}{\partial \mathfrak{u}}, \qquad \frac{\partial \tilde{L}(0;\mathfrak{u})}{\partial \mathfrak{u}} = 1,$$
$$\frac{\partial \tilde{u}_\tau(\tau;\mathfrak{u})}{\partial \mathfrak{u}} = \frac{\partial}{\partial \tau}\frac{\partial \tilde{u}(\tau;\mathfrak{u})}{\partial \mathfrak{u}} = (r+\delta)\frac{\partial \tilde{u}(\tau;\mathfrak{u})}{\partial \mathfrak{u}} + \alpha(1-\alpha)\frac{a}{c}\tilde{L}(\tau;\mathfrak{u})^{\alpha-2}\frac{\partial \tilde{L}(\tau;\mathfrak{u})}{\partial \mathfrak{u}},$$
$$\frac{\partial \tilde{u}(0;\mathfrak{u})}{\partial \mathfrak{u}} = 1.$$

Setting $z_1 := \partial\tilde{L}/\partial\mathfrak{u}$, $z_2 := \partial\tilde{u}/\partial\mathfrak{u}$, and $A(\tau) := \alpha(1-\alpha)a\tilde{L}(\tau;\mathfrak{u})^{\alpha-2}/c$, it follows that the partial derivatives of $\tilde{u}(\tau;\mathfrak{u})$ and $\tilde{L}(\tau;\mathfrak{u})$ with respect to $\mathfrak{u}$ are the solutions of the linear, non-autonomous, and homogeneous initial value problem

$$z_{1\tau} = -\delta z_1 + z_2, \quad z_1(0) = 1$$
$$z_{2\tau} = A(\tau)z_1 + (r+\delta)z_2, \quad z_2(0) = 1.$$

The signs of z_1 and z_2 are determined by means of Figure 3. It is straightforward that the trajectory starting at $(1,1)$ cannot cross the isocline $z_{1\tau} = 0$. Thus, z_1 and z_2 are positive for all $\tau \in [0,T]$, that is

$$\frac{\partial \tilde{L}(\tau;\mathfrak{u})}{\partial \mathfrak{u}} > 0, \quad \frac{\partial \tilde{u}(\tau;\mathfrak{u})}{\partial \mathfrak{u}} > 0 \quad \forall \tau \in [0,T]. \tag{16}$$

To complete the analysis, the relation between $\mathfrak{u}$ and T has to be established. Note that $\mathfrak{u}$ must be chosen optimally depending on the value of T. The boundary condition in (13b) implies $\tilde{u}(T,\mathfrak{u}) \equiv \tilde{u}(T) \equiv 0$. By the rule of implicit differentiation,

$$\frac{d\mathfrak{u}}{dT} = -\frac{\partial \tilde{u}(T,\mathfrak{u})/\partial T}{\partial \tilde{u}(T,\mathfrak{u})/\partial \mathfrak{u}}.$$

Note that $\partial\tilde{u}(T,\mathfrak{u})/\partial T = (\partial\tilde{u}(\tau;\mathfrak{u})/\partial\tau)|_{\tau=T}$. This derivative has already been shown to be negative by means of Figure 1. Thus, in view of (16), $d\mathfrak{u}/dT > 0$. It is now possible to compute the derivatives of $\tilde{u}(\tau;T) \equiv \tilde{u}(\tau;\tilde{u}(T))$ and $\tilde{L}(\tau;T) \equiv \tilde{L}(\tau;\tilde{u}(T))$, the steady state solutions of the optimization problem, with respect to T:

$$\frac{\partial \tilde{u}(\tau;T)}{\partial T} = \frac{\partial \tilde{u}}{\partial \mathfrak{u}}\frac{d\mathfrak{u}}{dT} > 0, \quad \frac{\partial \tilde{L}(\tau;T)}{\partial T} = \frac{\partial \tilde{L}}{\partial \mathfrak{u}}\frac{d\mathfrak{u}}{dT} > 0 \quad \forall \tau \in [0,T].$$

[6]The validity of changing the order of differentiation after the first equality sign follows from the proof of the theorem on the dependence on initial conditions for ordinary differential equations, cf. e.g. Perko (1996, pp. 80–84, eq. (5)), who does not use the standard term *variational equation*, however.

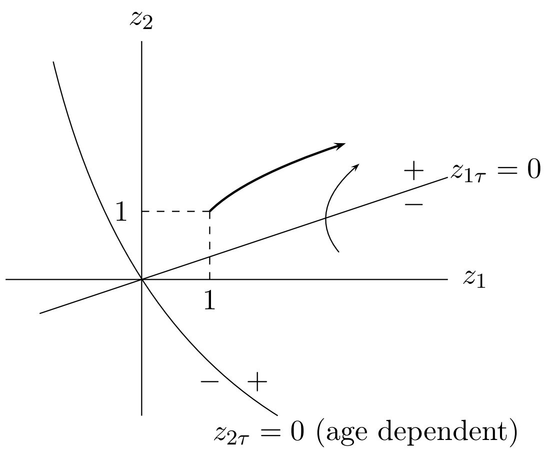

Figure 3. Determining the Signs of z_1 and z_2

The effect of adjustment costs c on hiring or firing and employment can similarly be calculated using the variational equations obtained by differentiating with respect to c. It turns out that the results obtained for the case of constant returns with $a(T) > w(T)$ carry over to the present case. Thus, all comparative dynamics results obtained for the case of constant returns to scale under the assumption that $a(T) > w(T)$ are valid for the steady state under decreasing returns to scale if $a(\tau)$, $w(\tau)$, $c(\tau)$, and $\delta(\tau)$ are constant. In summary:

Proposition 5 *Let $a(\tau)$, $w(\tau)$, $c(\tau)$, and $\delta(\tau) = \delta$, $0 \leqq \delta < 1$, be constant. Then the implications of Propositions 2, 3, and 4 for the case $a(T) > w(T)$ are equally valid with respect to the steady state of the model using production function (10) with decreasing returns to scale.*

Notice that Proposition 5 does not require a condition like $a > w$.

Without aging or an exogenous percentage rate of voluntary quits, the solutions of the models with constant and decreasing returns to scale, respectively, are qualitatively different. Under decreasing returns, the solution approaches a steady state, while under constant returns, the firm exits the market or it grows without bound if $\delta = 0$ (cf. Section 2.2). Proposition 5 shows that, if aging of workers is taken into account, the steady state solutions under constant and decreasing returns are qualitatively equivalent (at least in case of constant parameter values). Thus, constant returns to scale loose much of their special flavor if the model is pushed a step further to reality.

Proposition 5 implies that even strictly convex adjustment costs are an important determinant of steady state labor demand under decreasing returns to scale. Faria and Jellal (2002), who consider efficiency wages in the framework of a dynamic model with neither aging nor voluntary quitting, show that strictly convex adjustment costs do not affect the steady state determination of employment in this setting, whereas linear adjustment costs do. They conclude that "one can infer what type of adjustment costs prevail by investigating empirically whether wages and employment are affected by adjustment costs". The model with aging shows, however, that the proposed empirical test is impossible.

4 Conclusions

The introduction of aging into the dynamic theory of labor demand pushes the model a step towards real life. Among other things, a theoretical analysis of the effects of different spans of working life on employment decisions becomes possible. Under plain-sailing hypotheses it is possible to provide an explanation of empirically observed age-structures of employment. It is remarkable that the cases of constant and decreasing returns to scale turn out to be very similar once aging of workers is taken into account.

Although the analysis requires to deal with partial differential equations, the model introduced in Section 2 is relatively simple and admits a complete analytical solution. As has been shown in Section 3, even the case of decreasing returns to scale is analytically tractable. Other generalizations of the baseline model should therefore be possible. Hence, it is reasonable to expect further applications adding to the understanding of the mechanics of labor demand when workers are getting older, possibly even in a general equilibrium setting or using a framework of imperfect competition.

Appendix: Solution of Equation (1b)

This appendix sketches the *method of characteristics* for the solution of a linear partial differential equation (PDE) such as (1b) with boundary conditions (1c).[7] Using the optimal control $\tilde{u}(\tau) := u(t, \tau)$ (suppressing the variable T used in the text), this equation reads

$$L_t(t,\tau) + L_\tau(t,\tau) = \tilde{u}(\tau) - \delta(\tau)L(t,\tau), \quad L(0,\tau) = L_0(\tau), \quad L(t,0) = 0. \tag{A1}$$

The first step is to represent t and τ parameterically as functions $t(\sigma)$ and $\tau(\sigma)$ with $t'(\sigma) = \tau'(\sigma) = 1$ (note that t and τ are measured in the same units). Substituting into (A1) reduces the PDE to an ordinary differential equation (ODE)

$$\frac{d\hat{L}}{d\sigma} = \tilde{u}(\tau(\sigma)) - \delta(\tau(\sigma))\hat{L}(\sigma), \tag{A2}$$

where $\hat{L}(\sigma) := L(t(\sigma), \tau(\sigma))$. Choosing the initial point (t_0, τ_0), it follows that $t = t_0 + \sigma$ and $\tau = \tau_0 + \sigma$. Therefore, solving the linear ODE (A2) one gets

$$\begin{aligned}\hat{L}(\sigma) &= e^{-\int_0^\sigma \delta(\tau_0+\rho)\, d\rho} \left[\hat{L}(0) + \int_0^\sigma \tilde{u}(\tau_0 + s) e^{\int_0^s \delta(\tau_0+\rho)\, d\rho}\, ds\right] \\ &= e^{-\int_{\tau_0}^{\tau_0+\sigma} \delta(\rho)\, d\rho} \left[L(t_0, \tau_0) + \int_{\tau_0}^{\tau_0+\sigma} \tilde{u}(s) e^{\int_{\tau_0}^s \delta(\rho)\, d\rho}\, ds\right],\end{aligned} \tag{A3}$$

which together with $t = t_0 + \sigma$ and $\tau = \tau_0 + \sigma$ is called a *characteristic* of (A1).

With respect to the boundary conditions two cases must be distinguished. (a) If $t < \tau$ and choosing the initial point $(t, \tau) = (0, \tau_0)$, the condition $L(0, \tau) = L_0(\tau)$ implies that

$$\hat{L}(\sigma) = e^{-\int_{\tau_0}^{\tau_0+\sigma} \delta(\rho)\, d\rho} \left[L_0(\tau_0) + \int_{\tau_0}^{\tau_0+\sigma} \tilde{u}(s) e^{\int_{\tau_0}^s \delta(\rho)\, d\rho}\, ds\right].$$

[7] For a general account of this method, which can be used to solve all linear PDEs appearing in the present paper, cf. any textbook on PDEs. Keyfitz and Keyfitz (1997) comprehensively discuss the solution of the famous *McKendrick PDE*, which is a prototype of PDEs appearing in population studies. The method works even in the more general case of *quasi-linear* PDEs.

For the initial point $(0, \tau_0)$ and $(t, \tau) = (0 + \sigma, \tau_0 + \sigma)$, the definition of $\hat{L}$ implies $\hat{L}(\sigma) = L(\sigma, \tau_0 + \sigma) = L(t, \tau)$. Using $\tau - t = \tau_0 + \sigma - 0 - \sigma = \tau_0$, it follows that

$$\begin{aligned} L(t,\tau) &= e^{-\int_{\tau-t}^{\tau} \delta(\rho)\, d\rho} \left[L_0(\tau - t) + \int_{\tau-t}^{\tau} \tilde{u}(s) e^{\int_{\tau-t}^{s} \delta(\rho)\, d\rho}\, ds \right] \\ &= L_0(\tau - t) e^{-\int_{\tau-t}^{\tau} \delta(\rho)\, d\rho} + \int_{\tau-t}^{\tau} \tilde{u}(s) e^{-\int_{s}^{\tau} \delta(\rho)\, d\rho}\, ds \quad \text{if} \quad t < \tau. \end{aligned}$$

(b) If $t \geqq \tau$, choosing the initial point $(t, \tau) = (t_0, 0)$ and using the conditions $L(t, 0) = 0$, $\tau = \tau_0 + \sigma = \sigma$ in (A3) immediately implies that

$$L(t,\tau) = e^{-\int_0^{\tau} \delta(\rho)\, d\rho} \int_0^{\tau} \tilde{u}(s) e^{\int_0^{s} \delta(\rho)\, d\rho}\, ds = \int_0^{\tau} \tilde{u}(s) e^{-\int_s^{\tau} \delta(\rho)\, d\rho}\, ds \quad \text{if} \quad t \geqq \tau.$$

It follows from the general properties of the method of characteristics that the solution of the boundary value problem (A1) is unique, because the boundary conditions in (A1) are specified along a so-called *non-characteristic curve*, the non-negative t- and τ-axes, being transversal to the projection of the characteristics into the (t, τ)-plane.

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