How to Internalize Pollution Externalities Through 'Excess Burdening' Taxes

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1 Introduction

Taxes are usually called *distortionary* if they are not lump-sum (Ballard and Fullerton 1992). It is well known that if distortionary taxes are introduced in a formerly Pareto efficient perfectly competitive economy they induce an excess burden on the economy. But suppose, the economy is subject to pollution that renders the competitive allocation inefficient in the absence of any taxes. Then it is equally well-known since Pigou that there are taxes on the pollution-generating activity, i.e. emission taxes, allowing to eliminate the allocative inefficiency, provided that an appropriate tax rate is chosen and that the tax revenues are recycled in a non-distortionary way. Such taxes are usually called *corrective* taxes. Obviously, corrective taxes are necessarily distortionary, because it is the tax base eroding effect of increasing the rate of a corrective tax that eventually 'internalizes' the externality.

Partitioning the set of distortionary taxes into two subsets of corrective and non-corrective taxes raises some intriguing questions: What is the distinctive feature of these subsets? Is it true that non-corrective taxes are efficiency reducing and corrective taxes efficiency enhancing? Or do both carry an excess burden because both are distortionary? If the last question is answered in the affirmative - as in the recent literature on ecological tax reform (see below) - we appear to be caught in an inconsistency trap: To see this consider a competitive economy in which no taxes are levied but which is inefficient due to a pollution externality. Suppose an emission tax is introduced in this economy such that its rate is successively raised from zero to its externality-internalizing level. The rising tax rate is then obviously accompanied by an increasing excess burden but, nevertheless, the allocation tends towards Pareto efficiency. The only possible conclusion appears to be that we eliminated the initial allocative inefficiency completely by introducing another inefficiency, the emission tax, which is an obvious contradiction.

It is not the purpose of the present paper to challenge any of the substantial insights of the well-established theory of taxation and externalities. Also, we do not intend to focus on semantic confusion dealing with the appropriateness or inappropriateness of defining of the terms 'neutral', 'distortionary' or 'corrective'. We rather aim at offering a new perspective of looking at distortionary and corrective taxes and their welfare implications based on sound economic reasoning.

An important case for reconsidering the welfare economic implications of distortive and/or corrective taxes is the ‘double dividend’ discussion in the context of analyses of ecological tax reform (Bovenberg and de Mooij 1994, Goulder 1995, Weinbrenner 1999, Pethig 2002a). Since Bovenberg and de Mooij (1994) it has become an established procedure to decompose the total compensating variation induced by such a reform into two effects:[1] The first is the *‘ecological dividend’* identified as the marginal pollution damage avoided by a marginal increase in environmental quality. The second effect is the *‘tax efficiency dividend’* which, if it accrues, consists of a marginal reduction in aggregate excess burden (compensating variation) of *all* taxes - including the emission taxes.[2]

In our view, arguing about taxonomies of partial welfare effects does not offer substantial new insights and should therefore not attract our attention. However, the definition of *‘tax efficiency dividend’* is implicitly based on the claim that by its very nature any corrective tax, and therefore any emission tax, imposes an excess burden. We will demonstrate that this view involves inappropriate welfare economic reasoning, and we offer, at the same time, an analytical concept for avoiding this deficiency.

The basic point of departure of the concept to be developed in the present paper is the insight, aptly expressed by Heller and Starrett (1976, p. 10), e.g., that "one can think of externalities as nearly synonymous with non-existence of markets". In their view an externality is a situation in which the private economy lacks sufficient incentives to create a potential market in some good, and the non-existence of this market results in allocative inefficiencies.

This paper aims at establishing a *common* benchmark for *all* distortionary taxes, corrective and non-corrective ones, by introducing fictitious markets to eliminate the market failure that

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1 This decomposition corresponds to the notion of a ‘strong double-dividend’ in Goulder’s (1995) taxonomy.

2 The advisory board of the German Ministry of Finance also adopted this concept of 'tax efficiency dividend' (Bundesministerium der Finanzen 1997, p. 27n.; translation by the author): "The second dividend (i.e., the tax efficiency dividend, R. P.) could be attained, in particular, by keeping the overall tax revenue constant and reducing those taxes that cause greater excess burdens than the emission taxes." Nota bene, emission taxes cause excess burdens!

gave rise to the pollution externality in the first place. The main conclusion is that emission taxes replace negative producer prices on non-existing markets for waste products (pollutants), and if the emission tax rates fall short of their Pigovian levels (e. g., if they are zero) those producer prices involve truly distortive emission subsidies. Internalization of environmental externalities means to set these virtual emission subsidies equal to zero. It is also shown that distortive taxes on dirty consumer goods may turn out to be corrective if no waste abatement technology is available.

2 The General Model

Consider an economy with the concave technology constraints[3]

$$T^f\left(\boldsymbol{y_f}, \boldsymbol{s_f}, z_f\right) \geq 0, \tag{1}$$

$$T^e(\boldsymbol{s_e}, \boldsymbol{q_e}) = \boldsymbol{Q}(\boldsymbol{s_e}) - \boldsymbol{q_e} = \left[Q^1(\boldsymbol{s_e}),\ldots,Q^k(\boldsymbol{s_e}),\ldots,Q^\ell(\boldsymbol{s_e})\right]^T - \boldsymbol{q_e} \geq \boldsymbol{0}, \tag{2}$$

and the utility functions

$$U^h(\boldsymbol{y_h}, \boldsymbol{q_h}, z_h), \qquad h = 1,\ldots,\bar{h}. \tag{3}$$

The domains of the functions T^f, T^e, U^h are quantities of commodities. In (1) - (3), the following subsets of goods are distinguished:

- $y_h \in \Re^n$ are private goods called *consumer goods*; negative components are labor supplies and nonnegative components are consumer goods proper;
- $y_f \in \Re^n$ are *inputs and outputs* (of consumer goods); negative components are inputs and nonnegative components are outputs;
- $q_e, q_h \in \Re^\ell_+$ are public goods called *environmental quality indicators*;
- $\boldsymbol{s_f} \in \Re^m_+$ and $\boldsymbol{s_e} \in \Re^m_-$ are *waste products* or *pollutants;*

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[3] In this paper we will use the following notational conventions: If $\boldsymbol{x}, \boldsymbol{y} \in \Re^S$ then $\boldsymbol{x}$ and $\boldsymbol{y}$ are understood to be column vectors. For row vectors we write $\boldsymbol{x^T}$ and $\boldsymbol{y^T}$. Multiplication is written as 'xy', if $s = 1$, and as $x^T \cdot \boldsymbol{y} := \sum_{i=1}^{i=s} x_i y_i$ in case of $s > 1$.

- $z_f, z_h \in \Re_+$ is *a public consumption good* in (politically determined) fixed supply $\bar{z} > 0$; it is provided to the consumers and financed by tax revenues;

The function U^h is quasi-concave and strictly increasing in all arguments. The function T^e is an *ecological technology* telling us how waste products, $\boldsymbol{s_e}$, impact on the state of the environment as described by the vector $\boldsymbol{q_e}$ of environmental quality indicators. The function $\boldsymbol{Q}$ is concave and increasing in all arguments, i.e. all components of the $(m \times \ell)$-matrix $\boldsymbol{Q_s} := \left(\boldsymbol{Q_s^1} \ldots \boldsymbol{Q_s^k} \ldots \boldsymbol{Q_s^\ell}\right)$ with $\boldsymbol{Q_s^k} := (Q_{s_1}^k, \ldots, Q_{s_j}^k, \ldots, Q_{s_m}^k)^T$ $(k = 1, \ldots, \ell)$ are non-negative.

In the absence of joint production the technology T^f would be well-behaved if all partial derivatives were non-positive. But modelling non-trivial pollution issues calls for the following assumption of *joint production*:

$$T^f\left(y_f, s_f, z_f\right) = 0 \wedge s_f > 0 \quad \Rightarrow \quad T_{s_j}^f \geq 0 \ \ \forall j; \tag{4}$$

To complete the model we now introduce the economy's endowment of consumer goods, $\boldsymbol{r} \in \Re_+^n$, and list the resource constraints

$$\boldsymbol{y_f} + \boldsymbol{r} = \sum\nolimits_h \boldsymbol{y_h}, \qquad \boldsymbol{s_f} + \boldsymbol{s_e} = \boldsymbol{0}, \tag{5a}$$

$$\boldsymbol{q_e} = \boldsymbol{q_h} \geq \boldsymbol{0}, \quad (h = 1, \ldots, \bar{h}), \text{ and } \ z_f = z_h = \bar{z} \quad (h = 1, \ldots, \bar{h}). \tag{5b}$$

Let *i=1* be a clean consumer good in the sense that $\left(\partial T^f / \partial s_{f_i} \partial y_{f_1}\right) = 0$ for all *i*. Using good *1* as the numeraire we introduce the following notation:

$$MB_k := \sum\nolimits_h \frac{U_{q_k}^h}{U_{y_1}^h} > 0 \quad \text{and} \quad MAC_j := -\frac{T_{s_j}^f}{T_{y_1}^f} > 0 \quad \text{and} \quad MD_j := \sum\nolimits_k MB_k Q_{s_j}^k > 0;$$

$\boldsymbol{MB} := \left(MB_1, \ldots, MB_k, \ldots, MB_\ell\right)^T$ = *marginal benefit of environmental quality*;

$\boldsymbol{MAC} := \left(MAC_1, \ldots, MAC_j, \ldots, MAC_m\right)^T$ = *marginal abatement cost;*

$\boldsymbol{MD} := \left(MD_1, \ldots, MD_j, \ldots, MD_m\right)^T = \boldsymbol{Q_s} \cdot \boldsymbol{MB}$ = *marginal pollution damage.*

Proposition 1: *Suppose the quantities of all commodities are non-zero in efficient allocations of the model (1) - (5).*

(a) In an efficient allocation, the shadow prices of all consumer goods and of all environmental quality indicators are positive, and the shadow prices of all waste products are negative.

(b) Pollutants are efficiently allocated if and only if

$$\boldsymbol{MAC} = \boldsymbol{Q_s} \cdot \boldsymbol{MB} =: \boldsymbol{MD}. \tag{6}$$

(c) Suppose the matrix $\boldsymbol{Q_s}$ *is quadratic* $(\ell = m)$ *and regular. The environmental quality is efficiently allocated if and only if*

$$\boldsymbol{MB} = \boldsymbol{Q}_s^{-1} \cdot \boldsymbol{MAC} =: \boldsymbol{MC}. \tag{7}$$

To prove that proposition consider for any given $\alpha_h \in \Re_{++}$ $(h = 1,\ldots,\bar{h})$ the Lagrangean[4]

$$L = \sum_h a_h U^h(\boldsymbol{y}_h, \boldsymbol{q}_h, \bar{z}) + \boldsymbol{\lambda}_f T^f(\boldsymbol{y_f}, \boldsymbol{s_f}, z_f) + \boldsymbol{\lambda}_z(z_f - \bar{z}) + \boldsymbol{\lambda}^T \cdot (\boldsymbol{y}_f + \boldsymbol{r} - \sum_h \boldsymbol{y}_h) +$$

$$+\boldsymbol{\mu}_q^T \cdot [\boldsymbol{Q}(\boldsymbol{s}_e) - \boldsymbol{q}_e] + (\boldsymbol{\mu}_s^+ - \boldsymbol{\mu}_s^-)^T \cdot (\boldsymbol{s}_f + \boldsymbol{s}_e) + \sum_h \boldsymbol{\mu}_{qh}{}^T \cdot (\boldsymbol{q}_e - \boldsymbol{q}_h).$$

Since an interior solution is assumed to exist, the relevant first order conditions are

$$\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{y_h}} = \alpha_h \boldsymbol{U_y^h} - \boldsymbol{\lambda} = \boldsymbol{0}, \qquad h = 1,\ldots,\bar{h} \tag{8a}$$

$$\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{q_h}} = \alpha_h \boldsymbol{U_q^h} - \boldsymbol{\lambda}_{q_h} = \boldsymbol{0}, \qquad h = 1,\ldots,\bar{h} \tag{8b}$$

$$\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{y}_f} = \lambda_f \boldsymbol{T}_y^f - \boldsymbol{\lambda} = \boldsymbol{0}, \tag{8c}$$

$$\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{s}_f} = \lambda_f \boldsymbol{T}_s^f + \boldsymbol{\lambda}_s^+ - \boldsymbol{\lambda}_s^- = \boldsymbol{0}, \tag{8d}$$

$$\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{s}_e} = \boldsymbol{Q}_s \cdot \boldsymbol{\mu}_q + \boldsymbol{\lambda}_s^+ - \boldsymbol{\lambda}_s^- = \boldsymbol{0}, \tag{8e}$$

$$\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{q}_e} = -\boldsymbol{\mu}_q + \sum_h \boldsymbol{\mu}_{qh} = \boldsymbol{0}. \tag{8f}$$

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[4] We use the following rules of notation. If $\boldsymbol{x}, \boldsymbol{y} \in \Re^s$ then $\boldsymbol{x}$ and $\boldsymbol{y}$ are understood to be column vectors. For row vectors we write $\boldsymbol{x}^T$ and $\boldsymbol{y}^T$. Multiplication is indicated as 'xy', if $s = 1$ and $\boldsymbol{x}^T \cdot \boldsymbol{y} = \sum_{i=1}^{i=s} x_i y_i$ in case of $s > 1$.

Note first that $\boldsymbol{\lambda} > \boldsymbol{0}$ because $\boldsymbol{U}_y^h > \boldsymbol{0}$. Hence $\boldsymbol{\lambda_f} > \boldsymbol{0}$ via (8c) and consequently (8d) and (1) imply $\boldsymbol{\lambda}_s^+ - \boldsymbol{\lambda}_s^- = -\boldsymbol{\lambda}_s^- < \boldsymbol{0}$. With this information, it is straightforward from (8c) and (8d) that $\frac{\lambda_{s_j}^-}{\lambda_1} = -\frac{T_{s_j}^f}{T_{y_1}^f} =: MAC_j$. Moreover, we make use of the equations (8a), (8b), (8e) and (8f) to write $\frac{\lambda_{s_j}^-}{\lambda_1} = \frac{\sum_k \sum_h U_{q_k}^h Q_{s_j}^k}{U_{y_1}^{h'}} = \sum_k MB_k Q_{s_j}^k$. This proves the equation (6) which guides the efficient allocation of pollutants. According to the ecological technology (2) the pollutants are public inputs in the 'generation' of environmental quality indicators. Hence (6) represents the Samuelsonian summation rule for public inputs. A key term specifying the efficient allocation of the 'environmental quality indicators' is $\boldsymbol{MB}$ in (6). The inhomogeneous system of linear equations (6) can be solved for $\boldsymbol{MB}$ under certain conditions only (which will not be explored here in detail). To simplify the exposition we restrict our attention to $\boldsymbol{Q}_s$ being quadratic $(\ell = m)$ and regular[5]. From this assumption follows immediately equation (7) which represents the Samuelson rule for the efficient allocation of environmental quality. Quite obviously, the efficient allocation of pollutants (6) and of environmental quality (7) is determined *uno actu*.

Equation (7) clearly represents the Samuelson rule for the efficient allocation of environmental quality. In (7) the marginal cost of the environmental quality indicator k, MC_k, depends on the vector of marginal abatement costs and on the marginal properties of the ecological technology, $\boldsymbol{Q}_s$, in a fairly complex way. This complexity is reduced by considering the following simplifying specification of $\boldsymbol{Q}_s$:

The matrix $\boldsymbol{Q}_s$ *is quadratic* $(\ell = m)$ *and diagonal or the matrix* $\boldsymbol{Q}_s$ *is a column vector* $(\ell = 1)$.

As a consequence of either specification, waste products are turned from public to private inputs in the generation of environmental quality and the equation (7) reads, respectively,

$$MB_k = \sum_h \frac{U_{q_k}^h}{U_{y_1}^h} = \frac{MAC_k}{Q_{s_k}^k} = MC_k \quad (k=j=1,...,m) \quad \text{and} \quad MB = \sum_h \frac{U_q^h}{U_{y_1}^h} = \sum_j \frac{MAC_j}{Q_{s_j}} = MC.$$

[5] This assumption offers a simple and direct way to think of environmental policy as providing the public goods 'environmental quality (indicators)'.

3 The Lindahl Economy as a Benchmark

The economy studied in the previous section will now be endowed with a full set of competitive markets by considering, in addition to the real markets for consumer goods and for the public good[6], *fictitious* competitive markets for pollutants and for all environmental quality indicators. Since these indicators are public goods, the appropriate concept are Lindahl markets with personalized prices. We add to the complexity of this market model by introducing various taxes and/or subsidies. The notation for quantities, prices and tax rates is listed in table 1 (below). A necessary specification is that the taxes $\boldsymbol{t} \in \Re_+^n$ are assumed to be taxes on proper consumer goods only, i.e. we have $t_i \equiv 0$ if $y_{hi} < 0$.

To clarify the concept of tax-distorted Lindahl economy we now describe the agents' pertaining optimization problems. Since our focus is not on the microeconomics of production we assume that there is only one price-taking and profit maximizing firm for each technology, called firm *f* and eco-firm[7], respectively. Firm *f* solves

$$L^f = \left(\boldsymbol{p}, \pi_s + \tau_f, p_z\right)^T \cdot \left(\boldsymbol{y_f}, \boldsymbol{s_f}, z_f\right) + \beta_f T^f\left(\boldsymbol{y_f}, \boldsymbol{s_f}, z_f\right), \tag{9}$$

and the eco-firm solves

$$L^e = \left(\pi_q + \theta_e, \pi_s + \tau_e\right)^T \cdot \left(\boldsymbol{q_e}, \boldsymbol{s_e}\right) + \beta_e^T \cdot \left[\boldsymbol{Q}(s_e) - \boldsymbol{q_e}\right]. \tag{10}$$

The eco-firm's profit maximizing calculus is a particularly interesting feature of the Lindahl model. Since $\pi_q + \theta_e > \boldsymbol{0}$, $\pi_s + \tau_e < \boldsymbol{0}$, $q_e > \boldsymbol{0}$ and $\boldsymbol{s_e} < \boldsymbol{0}$, the eco-firm receives positive revenues from both selling environmental quality to consumers and buying waste products from firm *f* at negative prices. Therefore the eco-firm is best interpreted as a public enterprise maximizing the value of the natural asset 'environment'. Assuming that the 'environment' is publicly owned, government does not only receive the eco-firm's tax revenue $\theta_e^T \cdot \boldsymbol{q_e} + \tau_e^T \cdot \boldsymbol{s_e}$, but rather its total profit (see below).

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[6] The supply side of the market for the public good **Z** is perfectly competitive while government is assumed to buy the quantity $\bar{z}$ making no use of its monopsony power.

[7] In case of more than one eco-firm it would have been necessary to establish a second Lindahl market for pollutants.

markets		quantities	prices	tax rates and revenues
consumer goods		$\boldsymbol{y}_f, \boldsymbol{y}_h \in \Re^n$	$p \in \Re^n_+,\ p_1 \equiv 1$	$\boldsymbol{t}^T \cdot \sum_h \boldsymbol{y}_h,\ t_1 \equiv 0$
waste products		$\boldsymbol{s}_f \in \Re^m_+,\ \boldsymbol{s}_e \in \Re^m_-$	$\pi_s \in \Re^m_-$	$\tau_f^T \cdot \boldsymbol{s}_f + \tau_e^T \cdot \boldsymbol{s}_e$
environmental quality	consumers	$\boldsymbol{q}_h \in \Re^\ell_+$	$\pi_{qh} \in \Re^\ell_+ \quad \forall h$	$\sum_h \left(\theta_h^T \cdot \boldsymbol{q}_h\right)$
	producers	$\boldsymbol{q}_e \in \Re^\ell_+$	$\pi_q \equiv \sum_h \pi_{qh}$	$\theta_e^T \cdot \boldsymbol{q}_e$
public good Z		$\bar{z} \in \Re_{++}$	$p_z \in \Re_{++}$	—

Table 1: Quantities, prices and taxes in the Lindahl economy

Consumer h solves

$$L^h = U^h\left(\boldsymbol{y}_h, \boldsymbol{q}_h, \bar{z}\right) + \beta_h\left[b_h + g_h - \left(\boldsymbol{p}+\boldsymbol{t}, \pi_{qh} + \theta_h\right)^T \cdot \left(\boldsymbol{y}_h, \boldsymbol{q}_h\right)\right] \tag{11}$$

where $b_h \geq 0$ and $g_h \geq 0$ are the consumer'*s* shares of firm *f's* profit and the government budget surplus, respectively. The latter is

$$\sum_h b_h = b := \left(\pi_q + \theta_e, \pi_s + \tau_e\right)^T \cdot \left(\boldsymbol{q}_e, \boldsymbol{s}_e\right) +$$

$$+\ \boldsymbol{t}^T \cdot \sum_h \boldsymbol{y}_h + \sum_h \theta_h^T \cdot \boldsymbol{q}_h - \tau_f^T \cdot \boldsymbol{s}_f - p_z \bar{z} \geq 0\,. \tag{12}$$

A *Lindahl equilibrium* is constituted by prices and tax rates as shown in table 1 such that the allocation attained by solving (9), (10), and (11) satisfies (5) and (12).

Proposition 2: *Consider a Lindahl equilibrium with non-zero quantities of all commodities.*

(a) The allocation is Pareto efficient, if and only if

(i) $\boldsymbol{t} = \boldsymbol{0}$, *(ii)* $\tau_f = \tau_e$, *(iii)* $\theta_e = \sum_h \theta_h$.

(b) For any given set of tax rates the allocative distortion of pollutants is measured by the internalization gap

$$\boldsymbol{IG}^s := \boldsymbol{MAC} - \boldsymbol{MD} = \tau_e - \tau_f + \boldsymbol{Q}_s \cdot \left(\theta_e - \sum_h \theta_h\right).$$

(c) Suppose the matrix $\boldsymbol{Q}_s$ *is quadratic (*$\ell = m$*) and regular. Then for any given set of tax rates the allocative distortion of environmental quality is measured by the internalization gap*

$$IG^q := MB - MC = \sum_h \theta_h - \theta_e + Q_s^{-1} \cdot \left(\tau_f - \tau_e\right).$$

Proposition 2a is proved by comparing the equations (8a) - (8f) with the first order conditions associated to the solution of (9), (10) and (11). These conditions encompass

$$MRS_y^h := \frac{U_y^h}{U_{y_1}^h} = p + t, \quad MB := \sum_h \frac{U_q^h}{U_{y_1}^h} = \sum_h (\pi_{qh} + \theta_h), \quad \beta_e = \pi_q + \theta_e, \tag{13}$$

$$MAC := -\frac{T_s^f}{T_{y_1}^f} = -\left(\pi_s + \tau_f\right), \quad MRT_y := \frac{T_y^f}{T_{y_1}^f} = p, \quad Q_s \cdot \left(\pi_q + \theta_e\right) = -\pi_s + \tau_e$$

The equations (13) are made compatible with (8a) and (8b) by setting $p = \lambda$, $t = 0$ and $\pi_{qh} + \theta_h = \mu_{qh} \quad \left(h = 1, \ldots, \bar{h}\right)$. We also set $\pi_s + \tau_f = \pi_s + \tau_e = -\lambda_s^- \; \left(hence\; \tau_f = \tau_e\right)$ and $\beta_e = \mu_q$. In view of (8f) and (13) we finally obtain $\beta_e = \pi_q + \theta_e = \sum_h \mu_{qh} = \sum_h \pi_{qh} + \sum_h \theta_h$ and hence $\theta_e = \sum_h \theta_h$.

To prove proposition 2b we differentiate (3) and make use of the first-order conditions associated to (8)

$$\sum_h \frac{du_h}{\beta_h} = (p + t)^T \cdot dy + \left(\pi_q + \sum_h \theta_h\right)^T \cdot dq \tag{14a}$$

where $dy = \sum_h dy_h$, $dq = dq_h = dq_e$ (all h) and $\pi_q = \sum_h \pi_{qh}$. Note that (12) is the marginal welfare change from disturbing an initial Lindahl equilibrium by infinitesimal changes dy and dq (which, in turn, may be caused by some (unspecified) comparative static exercise). Next we differentiate (1) and (2) and exploit the first-order conditions associated to (9) and (10):

$$p^T \cdot dy + \left(\pi_s + \tau_f\right)^T \cdot ds = 0, \quad \text{and} \quad -Q_s^T \cdot ds = dq \tag{15}$$

where $dy_f = dy$ and $ds = ds_f = -ds_e$ is taken into account in (15). Consideration of

$p^T \cdot dy = -\left(\pi_s + \tau_f\right)^T \cdot ds = MAC^T \cdot ds$ from (15) and MB from (13) in (14) leads us to

$$\Sigma_h \frac{du_h}{\beta_h} = t^T \cdot dy + MAC^T \cdot ds + MB^T \cdot dq \tag{14b}$$

The equation $d_q = -Q_s^T \cdot ds$ from (15) allows us to rewrite (14b) as

$$\sum_h \frac{du_h}{\beta_h} = t^T \cdot dy + (MAC - MD)^T \cdot ds = t^T \cdot dy + \left[-\pi_s^T - \tau_f^T - \pi_q^T \cdot Q_s^T - \sum_h \theta_h^T \cdot Q_s^T \right] \cdot ds . \quad (14c)$$

Since $\pi_q^T \cdot Q_s^T = (Q_s \cdot \pi_q)^T$, equation (13) gives us $\pi_q^T \cdot Q_s^T = -(\pi_s \equiv \tau_e + Q_s \cdot \theta_e)^T$. This equation turns (14) into

$$\sum_h \frac{du_h}{\beta_h} = t^T \cdot dy + \underbrace{\left[\tau_e - \tau_f + Q_s \cdot \left(\theta_e - \sum_h \theta_h \right) \right]^T}_{IG^S} \cdot ds \quad (16)$$

Replacing DS in (14b) by DQ via (15) is feasible by assuming that Q_S is quadratic $(\ell = m)$ and regular (which we also presupposed in proposition 1. With this qualification we obtain $ds = -\left(Q_s^T\right)^{-1} \cdot dq = -\left(Q_s^{-1}\right)^T \cdot dq$ and $\pi_q = -\left(Q_s^{-1} \cdot (\pi_s + \tau_e)\right) - \theta_e$ via (13). These equations are substituted in (15) and (14b) to the effect that

$$\sum_h \frac{du_h}{\beta_h} = t^T \cdot dy + (MB - MC)^T \cdot dq = .$$

$$= t^T \cdot dy + \left[\sum_h \theta_h^T - \theta_e^T - \left(Q_s^{-1}(\pi_s + \tau_e)\right)^T + \left((\pi_s + \tau_f)^T \left(Q_s^{-1}\right)^T\right) \right] \cdot dq . \quad (14d)$$

Since $(\pi_s + \tau_f)^T \cdot \left(Q_s^{-1}\right)^T = \left(Q_s^{-1} \cdot (\pi_s + \tau_f)\right)^T$ the equation (14d) is equivalent to

$$\sum_h \frac{du_h}{\beta_h} = t^T \cdot dy + \underbrace{\left[\sum_h \theta_h - \theta_e + Q_s^{-1} \cdot (\tau_f - \tau_e) \right]^T}_{IG^q} \cdot dq \quad (17)$$

Recall that $MB = \pi_q + \sum_h \theta_h$, $MD^T = MB^T \cdot Q_s^T$ and $MAC = -T_s^f / T_{y_1}^f$. Hence in (16) the term in square brackets is $IG^s = MAC - MD$ (where IG stands for ***i**nternalization **g**ap*). Similarly, we find that the square bracked term in (17) is equal to $IG^q = MB - MC$. We know from (6) and (7) that the pollution externality is completely internalized if and only if the internalization gap (of either type) is zero. According to proposition 2b $IG^q = 0$ if $\tau_f = \tau_e$ *and* $\theta_e = \sum_h \theta_h$ holds[8] irrespective of the size of the tax rates t on all consumption goods. Equations (16) and (17) also imply that each single tax τ_f, τ_e, θ_e *or* θ_h can be used to create internalization gaps of any magnitude and sign. For example, we can set $\tau_e = \theta_e \equiv 0$

8 These sufficient conditions are not necessary for $IG = 0$. But we refrain form exploring other combinations of tax rates also leading to $IG = 0$.

and keep $\boldsymbol{IG}^q$ and $\boldsymbol{IG}^s$ constant by substituting the tax rates $(\theta_1,\dots,\theta_h)$ for τ_f and vice versa. In fact, both taxes are equivalent in the sense that they can be completely substituted without allocative disruptions. This observation is made precise in

Proposition 3: *Denote as policy A a tax policy such that* $\tau_e = \tau_f = \theta_e = \boldsymbol{0}$, $\sum_h \theta_{hA} \neq \boldsymbol{0}$ *and as policy B a tax policy characterized by* $\tau_e = \theta_e = \boldsymbol{0}, \theta_h = \boldsymbol{0}$ *for all h,* $\tau_f^B \neq \boldsymbol{0}$. *For every tax policy A [tax policy B] there exists a tax policy B [tax policy A] with* $\boldsymbol{t}_A = \boldsymbol{t}_B$ *such that the respective equilibrium allocations are the same under both policies.*

To prove proposition 3 define, for convenience of notation,

$$\boldsymbol{MRS}_y^h := \frac{\boldsymbol{U}_y^h}{U_{y_1}^h}, \quad \boldsymbol{MRT}_y := \frac{\boldsymbol{T}_y^f}{T_{y_1}^f} \text{ and } \boldsymbol{MAC} := -\frac{\boldsymbol{T}_s^f}{T_{y_1}^f}$$

and suppose, policy A exhibits the prices and taxes $\boldsymbol{\pi}_{sA}$, $\boldsymbol{\pi}_{qA}$, $\boldsymbol{p}_A$, $\boldsymbol{t}_A$ and $\boldsymbol{\theta}_A$. The first-order conditions of solving (9) - (11) enable us to characterize the allocation under policy A by

$$\boldsymbol{MRS}_{yA}^h = \boldsymbol{p}_A + \boldsymbol{t}_A,\ \boldsymbol{MRT}_{yA} = \boldsymbol{p}_A,\ \boldsymbol{MAC}_A = \pi_{sA},\ -\pi_{sA} = \boldsymbol{Q}_{sA}\pi_{qA} \text{ and } \boldsymbol{MB}_A = \boldsymbol{\pi}_{qA} + \boldsymbol{\theta}_A.$$

The allocation under policy B exhibits the same marginal conditions with substituting the subscript B for subscript A and replacing $\boldsymbol{MAC}_A = -\boldsymbol{\pi}_{sA}$ by $\boldsymbol{MAC}_B = -\boldsymbol{\pi}_{sB} - \boldsymbol{\tau}_{fB}$ and $\boldsymbol{MB}_A = \boldsymbol{\pi}_{qA} + \boldsymbol{\theta}_A$ by $\boldsymbol{MB}_B = \boldsymbol{\pi}_{qB}$.

Define (i) $\pi_{qB} := \pi_{qA} + \theta_A$, (ii) $\tau_{fB} := \boldsymbol{Q}_{sA}\theta_A$, (iii) $\boldsymbol{\pi}_{sB} + \boldsymbol{\tau}_{fB} = \boldsymbol{\pi}_{sA}$, (iv) $\boldsymbol{p}_B = \boldsymbol{p}_A$ and (v) $\boldsymbol{t}_B = \boldsymbol{t}_A$. We want to show that the prices and tax rates $\boldsymbol{\pi}_{sB}, \boldsymbol{\pi}_{qB}, \boldsymbol{p}_B, \boldsymbol{t}_B$ and $\tau_{fB}(\textit{with } \theta_B = \boldsymbol{0})$ support the allocation under policy A as an equilibrium under policy B. Condition (i) yields $\boldsymbol{MB}_A = \pi_{qA} + \theta_A = \pi_{qB} = \boldsymbol{MB}_B$. The conditions (iv) and (v) give us immediately $\boldsymbol{MRS}_{yB}^h = \boldsymbol{MRS}_{yA}^h$ as well as $\boldsymbol{MRT}_{yB} = \boldsymbol{MRT}_{yA}$. Observe that (iii) implies

$$-\boldsymbol{MAC}_A = \boldsymbol{\pi}_{sA} = \boldsymbol{\pi}_{sB} + \boldsymbol{\tau}_{fB} = -\boldsymbol{MAC}_B.$$

Note finally that in view of (iii) and $\pi_{qA} = \pi_{qB} - \theta_A$ from (i) the equation $\boldsymbol{Q}_{sA}\pi_{qA} = -\pi_{sA}$ is turned into $\boldsymbol{Q}_{sA}\pi_{qB} - \boldsymbol{Q}_{sA}\theta_A = -\pi_{sB} - \tau_{fB}$ and hence into $\boldsymbol{Q}_{sA}\pi_{qB} = -\pi_{sB}$. Using the definitions of π_{sB}, π_{qB}, $\boldsymbol{p}_B$, $\boldsymbol{t}_B$ *and* τ_{fB} it can be shown that shifting from policy A to policy B

leaves unaffected the profit of firm *f*, the government's budget surplus and the consumer's budget constraint.

In an analogous and straightforward way one can show that an equilibrium allocation under policy *B* can be supported by some policy *A* with appropriate assignment of prices and tax rates.

Proposition 3 allows us to restrict our attention to either policy. Our subsequent analysis will be based on policy B (while policy A is ignored), because policy B constitutes an important link to real-world emission charges. With policy B (13) simplifies to $\tau_f = -\boldsymbol{I}\boldsymbol{G}^s$ and therefore τ_f is readily interpreted as measuring the marginal inefficiency of the pollution externality. We demonstrated, in fact, that *the Lindahl economy can be viewed as transforming the externality into a tax distortion.* τ_f is a distortionary non-corrective tax as are the indirect taxes $\boldsymbol{t}$, since $\tau_f = \boldsymbol{0}$ and $\boldsymbol{t} = \boldsymbol{0}$ ensure Pareto efficiency as pointed out in proposition 2a. As a consequence, the internalization gap turns out to be a *tax wedge* in complete analogy to the tax wedges $\left(\boldsymbol{t}^T \cdot \boldsymbol{dy}\right)$ on existing markets. Recall from (8) that $\pi_s^f := \pi_s + \tau_f$ are producer prices of pollutants which are typically nonpositive. Since $\pi_s < 0$, $\boldsymbol{\tau}_f$ are subsidies [taxes] on pollutants, if $\tau_f < 0$ $\left[\tau_f > 0\right]$. We refer to $\tau_f > 0$ as *virtual emission subsidies*.

4 Real emission charges and virtual emission subsidies

We now leave the world of artificial markets for waste products and environmental quality indicators. The eco-firm is inactive and consumers enjoy environmental quality free of charge. Any quantity of waste released by industry simply deteriorates the environment via the ecological technology (2). Firm *f* no longer sells its waste products on the market (at price $\pi_s^f := \pi_s + \tau_f$), but it is now assumed to be subject to the emission charges[9] $\boldsymbol{t}_s \in \Re^m$. Hence it solves

$$L^f = \left(\boldsymbol{p},\ \boldsymbol{t}_s,\ p_z\right)^T \cdot \left(\boldsymbol{y}_f, \boldsymbol{s}_f, z_f\right) + \beta_f T^f\left(\boldsymbol{y}_f, \boldsymbol{s}_f, z_f\right). \tag{18}$$

Consumers face a more significant change since there is no market for environmental quality any more. Now they simply take the 'prevailing' environmental quality as given and solve

9 The relevant case is $t_s \in \Re_-^m$, since waste products are negative-priced outputs.

$$L^h = U^h\left(\boldsymbol{y_h}, \boldsymbol{q_h}, \bar{z}\right) + \beta_h\left[b_h + g_h - \left(\boldsymbol{p}+\boldsymbol{t}\right)^T \cdot \boldsymbol{y_h}\right]. \tag{19}$$

The government budget surplus is now given by

$$b := \ \boldsymbol{t}^T \cdot \sum\nolimits_h \boldsymbol{y}_h - t_s^T \cdot \boldsymbol{s}_f - p_z \bar{z} \geq 0 \, . \tag{20}$$

To distinguish the model of the present section from the Lindahl economy of section 3 we refer to it as the *competitive economy*. An equilibrium of the competitive economy is constituted by prices $\boldsymbol{p}$ and tax rates t_s such that the resultant allocation satisfies (5) and the solution to (18) and (19).

Proposition 4: *Consider an equilibrium of the competitive economy with non-zero quantities of all commodities.*

(a) For any tax policy $\left(\boldsymbol{t}, \boldsymbol{t_s}\right)$ *in the competitive economy there is a tax policy* $\left(\boldsymbol{t}, \tau_f\right)$ *in the associated Lindahl economy such that the respective equilibrium allocations are the same under both policies.*

(b) Let $\left(\boldsymbol{t}_B, \tau_{fB}\right)$ *be the tax policy in the Lindahl economy that is equivalent to the tax policy* $\left(\boldsymbol{t}_M, \boldsymbol{t}_{sM}\right)$ *in the competitive economy according to proposition 4a.*

Then $\boldsymbol{t}_B = \boldsymbol{t}_M$ *and* $\pi_{sB} + \tau_{fB} = \boldsymbol{t}_{sM} = -\boldsymbol{MAC}_M = -\boldsymbol{MAC}_B$.

Solving (18) and (19) yields $\boldsymbol{MRS}_{yM}^h = \boldsymbol{p}_M + \boldsymbol{t}_M$, $\boldsymbol{MAC}_M = -\,\boldsymbol{t}_{sM}$ and $\boldsymbol{MRT}_{yM} = \boldsymbol{p}_M$. Assign (i) $\boldsymbol{MB}_{hB} = \boldsymbol{MB}_{hM} = \left(\boldsymbol{U}_q^h / U_{y_1}^h\right)_M$ [with $\boldsymbol{U}_q^h = \boldsymbol{L}_q^h$ from (19)], (ii) $\boldsymbol{p}_B = \boldsymbol{p}_M$, (iii) $\pi_{qhB} = \boldsymbol{MB}_{hB}$, (iv) $\pi_{SB} + \tau_{fB} = \boldsymbol{t}_{sM} = -\,\boldsymbol{MRT}_{sM}$, (v) $\boldsymbol{t}_B = \boldsymbol{t}_M$ and (vi) $p_{zB} = p_{zM}$. We want to show that the prices and tax rates $\pi_{sB}, \pi_{qB}, \boldsymbol{p}_B, \boldsymbol{t}_B$ and τ_{fB} in the Lindahl economy support the equilibrium allocation in the competitive economy with tax policy $\left(\boldsymbol{t}_M, \boldsymbol{t}_{sM}\right)$. It is straightforward that the equations $\boldsymbol{MB}_{hB} = \boldsymbol{MB}_{hM}, \boldsymbol{MRS}_{yB}^h = \boldsymbol{MRS}_{yM}^h, \boldsymbol{MRT}_{sB} = \boldsymbol{MRT}_{sM}$ and $\boldsymbol{MRT}_{yB} = \boldsymbol{MRT}_{yM}$ are implied by the conditions (i) - (v). Owing to (ii), (iv) and (vi) firm *f's* profit is the same under either regime: $g_{hB} = g_{hM}$. Next we substract (20) from (12) to obtain $b_B - b_M = \sum\nolimits_h \pi_{qh} \cdot \boldsymbol{q}$. When the government transfers the amount $b_{hB} := b_{hM} + \pi_{qh} \cdot \boldsymbol{q}$ to consumer h $\left(h = 1, \ldots, \bar{h}\right)$, then her budget constraint reads

$$b_{hB} + g_{hB} - \left(\boldsymbol{p}_B + \boldsymbol{t}_B, \pi_{qbB}\right)^T \cdot \left(\boldsymbol{y}_h, \boldsymbol{q}\right) = b_{hM} + g_{hM} - \left(\boldsymbol{p}_M + \boldsymbol{t}_M\right)^T \cdot \boldsymbol{y}_h .$$

In terms of the formal analysis, proposition 4 is similar to proposition 3 and hence can be proved along the same lines. Proposition 4b offers an important characterization of the emission charges $\boldsymbol{t}_s$ in the competitive economy: These charges are identified as substitutes for the prices on the missing markets for pollutants. More precisely, they substitute market prices, distorted by an emission tax $\left(\tau_f < \boldsymbol{0}\right)$ or an emission subsidy $\left(\tau_f > \boldsymbol{0}\right)$.

Now we are well equipped to investigate the distortionary potential of $\boldsymbol{t}_s$ by deriving, after some straightforward calculations, the pendant of (16):

$$\sum_h \frac{du_h}{\beta_h} = \underbrace{\boldsymbol{t^T \cdot dy - t_s^T \cdot ds}}_{TED^O} - \underbrace{\boldsymbol{MD^T \cdot ds}}_{ED^O} = \underbrace{\boldsymbol{t^T \cdot dy}}_{TED^*} - \underbrace{\boldsymbol{(t_s^T \cdot ds - MD^T \cdot ds).}}_{ED^*} \tag{21}$$

To fix our ideas consider an initial equilibrium in the competitive economy with $\boldsymbol{t}_s \leq \boldsymbol{0}$ and suppose the pollution control is tightened by increasing the emission charges $\left(\boldsymbol{dt}_s < \boldsymbol{0}\right)$. Assume further that this change results in a reduction of pollutants $\left(\boldsymbol{ds}_f < \boldsymbol{0}\right)$ - which can be expected as normal. In view of (21) the welfare impact $\left(\boldsymbol{t}_s + \boldsymbol{MD}\right)^T \cdot \boldsymbol{ds}_f$ of $\boldsymbol{ds}_f < \boldsymbol{0}$ is ambiguous in sign. Recall that $\boldsymbol{MD} = \boldsymbol{Q}_s \cdot \pi_q = -\pi_s$. It follows that

$$\boldsymbol{t}_s + \boldsymbol{MD} = \boldsymbol{t}_s + \boldsymbol{Q}_s \cdot \pi_q = \boldsymbol{t}_s - \pi_s = \tau_f . \tag{22}$$

which is confirmed by (16) for $\tau_e = \theta_e = \sum_h \theta_h = \boldsymbol{0}$. We conclude that the welfare impact of $ds_f < 0$ is positive, if and only if virtual emission subsidies, $\tau_f > 0$, are placed on waste products.

Suppose next $\boldsymbol{t}_s \leq \boldsymbol{0}$ and a (marginal) environmental tax reform is carried out, i.e. under the condition of revenue neutrality the emission charges are raised ($\boldsymbol{dt}_s \leq \boldsymbol{0}$) resulting in a reduction of waste products $\left(\boldsymbol{ds} < \boldsymbol{0}\right)$. According to the conventional view, the excess burden associated with the emission charge increase[10] is $t_s^T \cdot ds < 0$ in (21). But taking the Lindahl economy as the relevant benchmark shows unambiguously that the excess burden of raising $\boldsymbol{t}_s$ (and reducing $\boldsymbol{s}$) is $IG^{sT} \cdot ds = -\tau_f^T \cdot ds > t_s^T \cdot ds > 0$.

[10] That is the welfare impact of the emission charce increase via the tax base eroding effect.

Following Bovenberg and de Mooij (1994), the literature on environmental tax reform usually defines TED^o as the '***t**ax **e**fficiency **d**ividend*' (*TED*) and ED^o as the '***e**nvironmental **d**ividend*' (*ED*). But we conclude from our analysis that the correct assignment is $TED = TED^*$ and $ED = ED^*$, because (22) reveals that in the light of the Lindahl economy the emission charges $\boldsymbol{t}_s = \pi_s + \tau_f$ turn out to be (real) substitutes for virtual producer prices on fictitious markets for waste products distorted by virtual emission subsidies (in the relevant case of $\tau_f > \boldsymbol{0}$). Protagonists of environmental protection use to demand prices for pollutants and environmental quality to 'tell the ecological truth'. From the viewpoint of our analysis, this requirement may be interpreted to mean that virtual subsidies on pollutants ought to be abolished.

Proposition 5:

(a) In a competitive economy (without fictitious markets) the emission charges $\boldsymbol{t_s}$ *are corrective [non-corrective] if and only if in the associated Lindahl economy the implied levies on waste products are emission subsidies* $(\tau_f > 0)$ *[emission taxes* $(\tau_f < 0)$*]. This is so because increasing the emission charges in the competitive economy is tantamount to reducing the emission subsidy [to increasing the emission tax] in the associated Lindahl economy.*

(b) If there is no pollution control at all in the competitive economy $(\boldsymbol{t_s} = \boldsymbol{0})$*, the implied levies on waste products are heavy emissions subsidies. These subsidies are fixed in such a special way* $(\tau_f = -\pi_s > 0)$ *that the suppliers of waste products face zero prices on the markets for waste products in the associated Lindahl economy.*

5 The Special Case of Rigid Joint Production

The literature on revenue-neutral ecological tax reform has been largely based on a model (notably Bovenberg and the Mooij, 1994) with rigid joint production in the sense that the amounts of pollutants generated are uniquely determined by the quantities of useful commodities produced. To compare our analysis to this literature we now replace the production technology (1) and (4) by a simple linear technology. To specify this technology it is convenient to distinguish three types of commodities: Let $N_L = \{1\}$, N_C and $\boldsymbol{N_D}$ be a partition of the set *N* of all consumer goods and define $n := \# N$, $n_C := \# N_C$ and $n_D := \# N_D$. Good 1 is labor, the numeraire. Good $i \in N_D$ $[i \in N_C]$ is called a dirty [clean] good, because [no] pollution is generated during the process of its production. Also, define $\boldsymbol{y_f} := (y_{f1}, \boldsymbol{y_C}, \boldsymbol{y_D}) \in \Re_-^1 \times \Re_+^{n_C} \times \Re_+^{n_D}$. Finally we simplify by setting $n_D = m$ and by assuming that each dirty commodity generates a single pollutant only. With these qualifications, consider the technology

$$\boldsymbol{1_n^T} \cdot \boldsymbol{y_f} + z_f \leq 0 \text{ and } \boldsymbol{s_f} - \boldsymbol{S} \cdot \boldsymbol{y_D} \geq \boldsymbol{0}, \tag{23}$$

where $\boldsymbol{1_n^T} = (1, \ldots, 1) \in \Re_+^n$ and $\boldsymbol{S} = (\sigma_i)$ is a $(n_D \times n_D)$ diagonal matrix. The diagonal elements $\sigma_i > 0$ of $\boldsymbol{S}$ are interpreted as the amount of pollutant *i* per unit output of consumer good $i \in N_D$.

In this modified model, Pareto efficiency is still characterized by proposition 1, in principle. $\boldsymbol{MD} = \boldsymbol{Q_s} \cdot \boldsymbol{MB}$ from (6) can be shown to carry over in the new model while *MAC* is now given by

$$MAC_i = \frac{U_{yi}^h}{U_{y1}^h} - \frac{1}{\sigma_i} \text{ for all } i \in N_D.$$

Like in proposition 1c we will draw on the simplifying assumption that $\boldsymbol{Q_s}$ is quadratic and rectangular. When combined with $\boldsymbol{S}$ being diagonal this assumption implies $\ell = m = n_D$. The next step is to briefly review the tax distortions in the Lindahl economy. We restrict our attention to policy *B* (see proposition 3), i. e. we set $\tau_e = \boldsymbol{0}$ and $\theta_h = \boldsymbol{0}$ for all *h*. To characterize a Lindahl equilibrium the optimization problems (10) and (11) carry over. Accounting for (23), firm *f's* profit is

$$g_f = p_z z_f + \boldsymbol{p^T} \cdot \boldsymbol{y_f} + (\pi_s + \tau_f)^T \cdot \boldsymbol{s_f} = p_z z_f + \boldsymbol{p^T} \cdot \boldsymbol{y_f} + (\pi_s + \tau_f) \cdot \boldsymbol{S} \cdot \boldsymbol{y_D}$$

$$= p_z z_f + y_{1f} + \boldsymbol{p_C^T} \cdot \boldsymbol{y_C} + \left(\boldsymbol{p_D^T} + (\pi_s + \tau_f)^T \cdot \boldsymbol{S} \right) \cdot \boldsymbol{y_D} \tag{24}$$

It is well known that with a linear technology a necessary equilibrium condition is $g_f = 0$. In view of (23) this condition is satisfied if and only if $p_z = 1, \boldsymbol{p_C} = \boldsymbol{1_C}$ and $\boldsymbol{p_D} = \boldsymbol{1_D} - \boldsymbol{S} \cdot (\pi_s + \tau_f)$. Moreover, $g_f = 0,\ z_f = \bar{z}$ and $dg_f = 0$ yield

$$\boldsymbol{p^T} \cdot \boldsymbol{dy_f} = -(\pi_s + \tau_f)^T \cdot \boldsymbol{ds_f}. \tag{25a}$$

We also use $\boldsymbol{dq} = -\boldsymbol{Q}_{\boldsymbol{s}}^{\boldsymbol{T}} \cdot \boldsymbol{ds}_{\boldsymbol{f}}$ and $\pi_s = -\boldsymbol{Q}_{\boldsymbol{s}} \cdot \pi_q$ to write

$$\pi_q^T \cdot \boldsymbol{dq} = -\pi_q^T \cdot Q_s^T \cdot ds_f = \pi_s^T \cdot \boldsymbol{ds}_{\boldsymbol{f}} . \tag{25b}$$

With (25), equation (14a) is transformed into

$$\sum_h \frac{du_h}{\beta_h} = \boldsymbol{t}^{\boldsymbol{T}} \cdot \boldsymbol{dy}_{\boldsymbol{f}} - \tau_f^T \cdot \boldsymbol{ds}_{\boldsymbol{f}} . \tag{26}$$

In other words, proposition 2 holds for the modified model of the present selection, too, and so does proposition 4. Consequently, we have $t_s = \pi_s + \tau_f$ (proposition 4) and therefore (21). With this information at hand we now use (23) again to rewrite (26) as follows:

$$\sum \frac{du_h}{\beta_h} = \boldsymbol{t}^{\boldsymbol{T}} \cdot \boldsymbol{dy}_{\boldsymbol{f}} - \tau_f^T \cdot \boldsymbol{ds}_{\boldsymbol{f}} = \boldsymbol{t}_{\boldsymbol{C}}^{\boldsymbol{T}} \cdot \boldsymbol{dy}_{\boldsymbol{C}} + \left(\boldsymbol{t}_{\boldsymbol{D}}^{\boldsymbol{T}} \cdot \boldsymbol{S}^{-1} - \tau_f^T \cdot \boldsymbol{ds}_{\boldsymbol{f}} \right)$$

$$\boldsymbol{t}_{\boldsymbol{C}}^{\boldsymbol{T}} \cdot \boldsymbol{dy}_{\boldsymbol{C}} + \left(\boldsymbol{t}_{\boldsymbol{D}}^{\boldsymbol{T}} - \tau_f^T \cdot \boldsymbol{S} \right) \cdot \boldsymbol{dy}_{\boldsymbol{D}} = \boldsymbol{t}_{\boldsymbol{C}}^{\boldsymbol{T}} \cdot \boldsymbol{dy}_{\boldsymbol{C}} \left(\tau_f^T - \boldsymbol{t}_{\boldsymbol{D}}^{\boldsymbol{T}} \cdot \boldsymbol{S}^{-1} \right) \cdot \boldsymbol{Q}^{\boldsymbol{T}-1} \cdot \boldsymbol{dq} \tag{27}$$

Proposition 6: *Suppose assumption (2) is replaced by (23) and the tax system* $\left(t, t_s\right) \in \Re_+^n \times \Re^{n_D}$ *prevails in an equilibrium of a competitive economy (as defined in section 4). The equilibrium allocation is Pareto efficient, if and only if*

(i) $\boldsymbol{t}_{\boldsymbol{C}} = \boldsymbol{0}$ *and* *(ii)* $\tilde{\boldsymbol{t}}_{\boldsymbol{s}} := \boldsymbol{t}_{\boldsymbol{s}} - \boldsymbol{S}^{-1} \cdot \boldsymbol{t}_{\boldsymbol{D}} = \boldsymbol{MD}$ *or* $\tilde{\boldsymbol{t}}_{\boldsymbol{D}} := \boldsymbol{t}_{\boldsymbol{D}} - \boldsymbol{S} \cdot \boldsymbol{t}_{\boldsymbol{s}} = \boldsymbol{S} \cdot \boldsymbol{MD}$.

Recall that $\boldsymbol{t}$ is nonnegative while $\boldsymbol{t}_{\boldsymbol{S}}$ is nonpositive. The term $-\boldsymbol{t}_{\boldsymbol{D}} \cdot \boldsymbol{S}^{-1}$ represents the transformation of the tax rates $\boldsymbol{t}_{\boldsymbol{D}}$ on dirty consumer goods into tax rates on pollutants. Therefore $\tilde{\boldsymbol{t}}_{\boldsymbol{s}}$ are the tax rates on pollutants that are equivalent to the actual tax rates $\boldsymbol{t}_{\boldsymbol{s}}$ on pollutants *and* the tax rates $\boldsymbol{t}_{\boldsymbol{D}}$ on consumer goods. Similarly, $\tilde{\boldsymbol{t}}_{\boldsymbol{D}}$ from (23) are the tax rates on consumer goods that are the equivalent to the actual to the actual tax rates $\boldsymbol{t}_{\boldsymbol{D}}$ on dirty consumer goods *and* the tax rates $\boldsymbol{t}_{\boldsymbol{s}}$ on pollutants. It follows that taxes $\boldsymbol{t}_{\boldsymbol{D}}$ and $\boldsymbol{t}_{\boldsymbol{s}}$ are perfect substitutes since any given $\tilde{\boldsymbol{t}}_{\boldsymbol{D}}$ or $\tilde{\boldsymbol{t}}_{\boldsymbol{s}}$ can be secured with arbitrary combinations of $\boldsymbol{t}_{\boldsymbol{D}}$ and $\boldsymbol{t}_{\boldsymbol{s}}$.

Proposition 6 adds a further interesting insight into the relationship of corrective and distortionary taxation: If joint production is rigid distortionary commodity taxation and corrective environmental taxation need not be contradictory if dirty commodities are taxed. If there is no tax on pollutants ($\boldsymbol{t}_{\boldsymbol{s}} = 0$) but $\boldsymbol{t}_{\boldsymbol{D}}$ is nonzero and satisfies $\boldsymbol{t}^{\boldsymbol{T}} \cdot \boldsymbol{S}^{-1} \leq \boldsymbol{MD}$ then the distortionary commodity taxes are clearly corrective.[11] This is obviously due to the special technological conditions that render taxation of consumer goods a perfect substitute for taxing pollutants. However, if $\boldsymbol{t}^{\boldsymbol{T}} \cdot \boldsymbol{S}^{-1} > \boldsymbol{MD}$ for some of its components, say in component *i*, then the tax on commodity *i* is distortionary and non-corrective.

6 Concluding Remarks

Distortionary and non-corrective commodity taxes interfere with the otherwise well-functioning market by driving a wedge between producer and consumer prices. In contrast, distortionary and corrective taxes serve as an auxiliary arrangement to replace the missing market, and the replacement is the better, the closer the tax rate is set to its Pigovian level. The conventional analysis underlying the concept of 'tax efficiency dividend' counts and lumps together all non-corrective and corrective taxes and claims that all these taxes contribute to the overall excess burden of taxation disregarding the fact that the benchmark for measuring the excess burden of distortionary and non-corrective taxes is the undistorted competitive market while the benchmark for measuring the excess burdens of distortionary and corrective taxes is the absence of some market. Taking one and the same benchmark for all taxes, namely the economy with a full set of competitive markets, reveals that the (real) emission tax is a price on a (fictitious) market for pollutants which typically contains, however, a subsidy. It is this virtual emission subsidy rather than the real emission tax that measures exactly the inefficiency in the allocation of pollutants. Thus we made precise the notion that an environmental policy being too lax or even absent amounts to implicitly placing a subsidy on the emission of pollutants.

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[11] It is interesting to observe that Bovenberg and de Mooij (1994) analyze an economy where a tax on labor is levied in addition to an emission tax whereas in the present paper commodity taxes were applied. Since a tax on labor is equivalent to a uniform tax on all consumer goods Bovenberg and de Mooij's implicit assumption is $\boldsymbol{t}_C > \boldsymbol{0}$ (in terms of our notation). Hence in view of (23) a revenue-neutral ecological tax reform would, in fact, exhibit a negative tax efficiency dividend, equal to $\boldsymbol{t}_C^T \cdot \boldsymbol{dy}_C$, if the tax reform turns out to imply $\boldsymbol{dy}_C < \boldsymbol{0}$. See also Weinbrenner (1999).

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