A Note on Public Goods: Non-Excludability Implies Joint Consumability

Thomas Christiaans

Department of Economics
University of Siegen

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Abstract: While there are various definitions of public goods, the property of joint consumability has become the main or sole defining characteristic of such goods. Among the other characteristics the property of non-excludability is the one primarily discussed. While it is common in the literature to assert that the latter characteristic is neither necessary nor sufficient for the first to hold, it is shown in the present paper that a reasonable definition of non-excludability implies joint consumability, at least in the framework of the usual timeless and spaceless neoclassical model. Moreover, it is argued that this special case bears some importance for more realistic models involving public goods.

Keywords: Public Goods; Joint Consumability; Non-Excludability

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1. Introduction

According to BLÜMEL, PETHIG and von dem HAGEN (1986, p. 242), the mainstream approach in the theory-oriented literature is defining public goods by the characteristic of joint consumability. BROWN and JACKSON (1978, p. 29) argue that "public goods are non-excludable, and from this property it follows that they are non-rival in consumption".[1] BLÜMEL, PETHIG and von dem HAGEN (1986, p. 248–249) claim that this assertion is wrong. Since there is no disagreement about the non-validity of the reverse implication, the property of non-excludability is considered by these authors neither a necessary nor a sufficient condition for a good to be a public good. Interestingly enough, the statement just quoted does not appear anymore in the 4th edition of BROWN and JACKSON (1990, p. 35).[2] The purpose of the present note is to prove that, at least in the context of the usually employed timeless and spaceless neoclassical model, non-excludable goods (suitably defined) are necessarily jointly consumable (provided the good in question is not a *bad*). Thus, in general the properties of joint consumability and non-excludability are *not* logically independent.

The discussion about the appropriate definition of a public good may be considered as idle by the economic theorist. What really matters is the analysis of (efficient) allocation mechanisms for goods with certain properties, whatever their name is. Nevertheless, the confusion about the notion of *public goods* is an "uncomfortable state of affairs"[3]. For example, what should a politician do, if one of his consultants declares a certain good to be a public good, and another declares that "public goods have to be allocated in a political mechanism", but both of them talk about entirely different things? Similarly, students of economics are easily confused when reading about public goods in different books and journals. Thus, the mainstream approach of defining public goods in terms of non-rivalness (or joint consumability) and non-excludability helps to clarify matters.[4]

The next problem arises, however, if two characteristics are given without correctly specifying the logical dependence among one another. This is to be found in most texts on public finance,[5] where the usual static spaceless neoclassical framework is employed in most of the cases. The reason lies in the fact that excludability is considered a purely technological matter (involving certain exclusion-costs) and rivalness an intrinsic property of the good in question. Consequently, it is asserted

[1] Non-rivalness refers to essentially the same characteristic of a good as joint consumability, at least if there are no *congestion* effects. Cf. the discussion in section 3. below.

[2] The statement is already contradicted in the first edition by "... an example of a non-excludable good which is, however, rival in consumption." (BROWN and JACKSON 1978, p. 29). Thus, the previously quoted passage may have been nothing but a typographical error.

[3] BLÜMEL, PETHIG and von dem HAGEN (1986, p. 245).

[4] This is not the place to provide a comprehensive overview on the various definitions of public goods. Cf. BLÜMEL, PETHIG and von dem HAGEN (1986) for further references. To underline the importance of non-rivalness and non-excludability, however, it should be noted that one (preferably non-rivalness, as in SAMUELSON (1954, p. 387)) or both of them are used to define public goods in every item (except QUINE (1964), of course, and BRINKMANN (1995)) quoted in the references at the end of this note. According to BRINKMANN (1995, p. 4), a good is a public good if it is *non-exclusive.* In contrast, we show that *non-excludability* is a sufficient condition for a good to be a public good, but only since it implies joint consumability. The latter condition is essential for the conditions of efficient allocations.

[5] We may again refer to all items (except QUINE (1964)) quoted in the references at the end of this paper.

that there is no logical dependence between both characteristics. *Consumption*, however, is a special kind of *low-cost-technology* that creates a logical dependence, since it is of course of central importance for the definition of joint consumability and it can be an exclusion-technology. Without space and time, every commodity can be consumed at once and no costly stockkeeping or fencing is necessary to protect the opportunity to consume in the future or at other places.

While formal definitions of the properties of joint consumability and non-excludability are hard to find in the literature, BLÜMEL, PETHIG and von dem HAGEN (1986, p. 250) provide a formal definition of the first, assuming that the good in question is not arbitrarily divisible.[6] We will add a formal definition of the second that implies the first. Thus, non-excludability of a good is sufficient for it to be a public good, provided the latter is defined to be a jointly consumable good and one agrees with our definition of non-excludability.

Since the definition of non-excludability is of central importance for the proposition to be proved in the next section, a very strong definition is given which may be controversial for many readers, since the accomplishment of the exclusion is not allowed to create positive net costs. Nevertheless, as discussed below, every milder definition will support the proposition even more. It should be noted, however, that the strong definition employed here is reasonable only in the framework of the timeless and spaceless standard model. In any case, it is important to realize the consequences for the logical dependencies of various definitions that arise under special circumstances. Moreover, we will argue that this special case bears some importance for more realistic models involving public goods. For the sake of simplicity and in order to be specific, we will only consider the case of consumption goods.

2. Definitions and Proof of the Proposition

The first two of our definitions follow those given by BLÜMEL, PETHIG and von dem HAGEN (1986, p. 250, where in (S1*) a slash seems to be out of place), but are formulated differently in order to reveal more clearly that they are contradictions leading to a partition of the set of all goods into private and public goods, since the latter are defined to be jointly consumable goods. As has been noted before, we assume that the good under consideration is not arbitrarily divisible, but is possibly produced and consumed in very small discrete units. We should also note that our definition of non-excludability creates a partition of all goods into excludable and not excludable goods. This does not correspond to the mainstream literature; however, it should be observed that the words *excludable* and *not excludable* as negations of one another clearly imply such a partition. In the following definitions, i, j, and k belong to the same index-set of agents.

[6] The relation of this formal definition to the usually employed private and public scarcity constraints, $\sum_i y^i \leq y$ and $y^i \leq y \ \forall i$, respectively, is discussed in BLÜMEL, PETHIG and von dem HAGEN (1986, p. 250). (Here y denotes the quantity of a given good produced while y^i denotes the quantity consumed by agent i.)

(D1) A good is said to be *not* jointly consumable (i. e. private), if for every allocation of the good it is true that[7]

$$\forall i\ (Y^i \subset Y)\ \wedge\ \forall i \forall j\ [(i \neq j) \rightarrow (Y^i \cap Y^j = \emptyset)],$$

where Y and Y^i (Y^j) denote the non-empty set of all units available of the good in question and the set of units consumed by agent i (agent j) respectively.

(D2) A good is said to be jointly consumable (i. e. public), if there is an allocation of the good for which it is true that[8,9]

$$\exists i\ [\neg(Y^i \subset Y)]\ \vee\ \exists i \exists j\ [(i \neq j) \wedge (Y^i \cap Y^j \neq \emptyset)].$$

(D3) A good is said to be *not* excludable, if for every technology not involving net costs for agent k accomplishing the exclusion, there is an allocation for which it is true that

$$\forall i\ (Y^i \subset Y)\ \wedge\ \exists j\ [(j \neq k) \wedge (Y^j \neq \emptyset)].$$

(D4) A good is said to be excludable, if there is a technology not involving net costs for agent k accomplishing the exclusion, so that for every allocation it is true that

$$\exists i\ [\neg(Y^i \subset Y)]\ \vee\ \forall j\ [(j \neq k) \rightarrow (Y^j = \emptyset)].$$

Let y^i and $\succsim$ denote the number of units consumed by agent i and the preference relation *at least as good as* respectively. The following assumption states essentially, that the good under consideration is really a good and not a *bad*, in the sense that more of the good does not lower the utility of agent i, cf. e. g. VARIAN (1992, p. 96).

Weak monotonicity. *Iff* $\hat{y}^i \geq \bar{y}^i$, *then* $\hat{y}^i \succsim \bar{y}^i$.

Proposition. *If weak monotonicity holds for the good in question, then non-excludability implies joint consumability.*

Proof: Since (D2) is the negation of (D1) and (D4) is the negation of (D3), it follows from the rule of the contrapositive that [(D3) $\Rightarrow$ (D2)] is equivalent to [(D1) $\Rightarrow$ (D4)]. By weak monotonicity, it is worth to consume the whole quantity for every arbitrary consumer k: $Y^k = Y$. If the good is not jointly consumable, it follows from (D1) that $Y^j \cap Y = \emptyset\ \forall j \neq k$ and thus $\forall j\ [(j \neq k) \rightarrow (Y^j = \emptyset)]$ is true. Consequently (D4) holds. □

[7] The logical symbol $\rightarrow$ designates the *conditional*, that is, an actual condition instead of a logical deduction. The *truth assignment* of the conditional is usually defined by using a *truth table*. (D1) is equivalent to $\forall i\ (Y^i \subset Y) \wedge \forall i \forall j\ [\neg(i \neq j) \vee (Y^i \cap Y^j = \emptyset)]$, where $\forall$, $\wedge$, $\vee$, and $\neg$ denote the universal quantifier, the logical *and* (the *conjunction*), the logical *or* (the *disjunction*), and the logical *negation*, respectively. (We denote the existential quantifier by $\exists$.) The *conditional* should not be confused with the *logical implication* $\Rightarrow$ that designates a logical conclusion. The implication is given, if the corresponding conditional is universally valid. Cf. QUINE (1964) as a textbook on formal logic; we employ, however, other symbols which are more usual in applications.

[8] The negation of a *closed sentence* involving the quantifiers $\forall$ and $\exists$ is obtained by negating every *open sentence* and replacing all universal quantifiers by existential quantifiers (et vice versa). E. g., the negation of the open sentence $[\neg(i \neq j) \vee (Y^i \cap Y^j = \emptyset)]$ in (D1) is $[(i \neq j) \wedge (Y^i \cap Y^j \neq \emptyset)]$ by one of de MORGAN's laws and the negation of the the closed sentence $\forall i \forall j\ [\neg(i \neq j) \vee (Y^i \cap Y^j = \emptyset)]$ is therefore $\exists i \exists j\ [(i \neq j) \wedge (Y^i \cap Y^j \neq \emptyset)]$. The complete closed sentence in (D1) is similarly negated by the other law of de MORGAN.

[9] Since the condition $\exists i\ [\neg(Y^i \subset Y)]$ is false for every real good and the *disjunction* $A \vee B$ is defined to be true whenever at least one of A, B is true, (D2) is actually equivalent to $\exists i \exists j\ [(i \neq j) \wedge (Y^i \cap Y^j \neq \emptyset)]$.

The proof amounts to nothing more than stating that any agent j could be excluded from consumption of a not jointly consumable good by simply allowing agent k to consume the whole quantity available, which by weak monotonicity cannot put him at a disadvantage and thus creates no net costs (at least if there is neither space nor time). It follows in turn that the good must be jointly consumable, if such an exclusion is not possible.

Note that every milder definition of non-excludability that allows for positive exclusion-costs will support the proposition even more. To prove this, simply observe that excludability at some (including no) cost encompasses excludability at no cost by purely logical reasoning. Thus, the possibility of excludability at no cost implies the possibility of excludability at some cost. It follows in turn that non-excludability at some cost implies non-excludability at no cost, that is (D3) and therefore (D2) in light of the proposition.

3. Discussion of the Proposition

The proposition depends strongly on the above definition of non-excludability as well as on the definition of joint consumability. We will therefore discuss these definitions in some detail.

Starting with joint consumability, it should be noted that this is not exactly the same as *non-rivalness* in consumption. While the former only states that the consumption of one unit of a given good by more than one person is in principle possible, the latter states that "...one person's consumption does not reduce the amount available to other consumers."[10] If jointly consumable goods are subject to congestion – as crowded roads for example – there is a rising degree of rivalness with the introduction of new consumers. Such goods are often quoted as examples of non-excludable, but rival goods.[11] However, as has been noted by BLÜMEL, PETHIG and von dem HAGEN (1986, p. 247), "...the maximum degree of rivalry does not characterize only private goods but also public goods at the upper bound of joint consumption, ...". Since the term *non-rivalness* is frequently not used on an all or nothing basis but as a matter of degree, the use of *joint consumability* instead of *non-rivalness* as the defining characteristic of public goods has the advantage of partitioning the set of all goods into two classes, thereby ruling out " 'impure' public goods"[12]. At the same time, the use of *joint consumability* creates a clear-cut logical dependence between public goods and the property of *non-excludability* since the impossibility of joint consumption provides an easy way of exclusion by simply consuming all units available. On the other hand, *rivalness* (understood as a matter of degree) does not imply that others are excluded from consumption by the consumption of any agent. E. g., the use of a crowded road has elements of rivalry, but up to the point of complete traffic congestion, no one is excluded. On the other hand, if there is only space left for one more user, any person that uses this space will exclude any other person from its use at the same time. Thus, a congested road is clearly excludable and should not be called a non-excludable good.

[10] VARIAN (1992, S. 414). This statement coincides with SAMUELSON's (1954, p. 387) definition of public goods, where "...each individual's consumption of such a good leads to no subtraction from any other individual's consumption ...".
[11] Cf. e. g. VARIAN (1992, S. 415).
[12] SANDMO (1986 , p. 1061).

Moreover, it is well known that even empty roads are excludable, although not by one person using it. This example leads us to a more thorough treatment of the definition of non-excludability. As has been said before, the proof of the proposition in the last section implicitly presupposes the timeless and spaceless standard model of neoclassical allocation theory. It has been assumed that the exclusion can be accomplished by simply consuming all available units of the good in question. This is not possible for the example of an empty road; clearly, the concept of a road involves the notion of space. Similarly, it is not possible to exclude someone without costs from the consumption of a good by consuming it, if the available units are too much for the moment, that is, if the good ought to be stored for later use. As a consequence, our proposition on the logical dependence of joint consumability and non-excludability is, strictly speaking, only valid for this special case. But since most of the models employed in the context of public goods belong to this static and spaceless framework, it seems to be important to recognize and indicate the logical dependencies that exist in this case. Moreover, it is the author's conviction that the core of the notion of non-excludability is exactly what is captured by these simple models. It is not the height of the exclusion-costs that matters, but the general possibility to exclude someone from using a good or not. Only if a good is jointly consumable, an exclusion in use can be generally impossible. Our discussion of the example of a road shows that the core of the proposition accords well with more general cases involving time and space.

In passing we note that the proposition accords also with the practice of interpreting the scarcity constraint $y^i = y \; \forall i$ as indicating that the good in question is non-exclusive (or non-rejectable), where y denotes the number of units of the good available. Obviously, this form of the scarcity constraint implies joint consumability, and even more obviously, non-excludability implies non-exclusiveness.

A potential objection to the view proposed in this note might come from another often quoted example of a class of goods, namely the so-called *common property resources*. These are natural resources that are *non-exclusive*. An early example is given by David Hume, where a number of neighbors have access to a meadow that "...is common property in the sense that each individual has the right to graze his cattle on it but no single individual has the right to sell the meadow"[13]. This common property resource is considered a non-excludable but rival good by Brown and Jackson (1990, pp. 35–36). This coincides with the statement of Blümel, Pethig and von dem Hagen (1986, p. 258), that only "...some of these common property resources are public goods ...", i. e. are jointly consumable. These authors, however, give no example of a *non-excludable* good that is not jointly consumable, but only of *non-exclusive* goods, where the lack of exclusiveness follows from the absence of exclusive property rights or property rights enforcement.

Obviously, this example involves time and space, and the simple argument of our proposition cannot be applied directly. In any case, the example does not contradict the proposition. The meadow is considered non-excludable since the law of common property prevents the exclusion of any of the neighbors. Clearly this kind of non-excludability is no intrinsic property of the meadow but arises from institutional arrangements; thus, it should not be called non-excludable but non-exclusive. Nevertheless it may be objected that it is impossible to exclude others from con-

[13] Brown and Jackson (1990, p. 30).

sumption by simply consuming all units available. It follows that the meadow should be regarded a public good if our definition of non-excludability and the proposition are applied to this example. A major distinction is in order at this point, however. The meadow as a *subsystem* of nature has to be distinguished from the goods or services provided by this subsystem – the grass and the right to walk across the meadow in the present example. Blümel, Pethig and von dem Hagen (1986, p. 258) choose to regard only the provided goods as *natural resources*. Consequently, only the provided goods or services are called *common property resources* if they are non-exclusive. Some of these non-exclusive goods are considered to be public goods in the sense of joint consumability while others are not.

Clearly, the right to walk across the meadow is a public good, that is, however, subject to the possibility of congestion. Every good that is subject to congestion is excludable by completely crowding it (with time and space, this kind of exclusion is more complicated than in the timeless and spaceless neoclassical model). This does not contradict the above proposition, since non-excludability is not said to be a necessary condition for joint consumability. On the other hand, the grass provided by the meadow is clearly a private good, since it is not possible that two cows eat the same bushel of grass. In accordance with the proposition, it follows here that this good is excludable, since it is not jointly consumable. Note that some authors – Brown and Jackson (1990), e. g. – nevertheless consider such goods to be non-excludable, perhaps due to the relatively high exclusion-costs caused by fencing. However, it should be observed that everybody else *is* actually excluded from the consumption of any given unit of a private good such as a bushel of grass, if it is consumed by some person. At least in this sense, there seems to be no doubt on the validity of our proposition, even if time and space are taken into account.

Non-excludability should refer to a quality of the goods considered and should therefore be distinguished from the definition and enforcement of private property rights, which involves costs for a wide variety of given private and public goods. Non-excludability may be interpreted as the possibility to define private property rights on principle. If a good is not jointly consumable and thus private, it is possible to define private property rights on principle; this is why (D1) implies (D4) and therefore why (D3) implies (D2). This point of view seems to be implicit in the section on social wants in the classical book of Musgrave (1959, pp. 9–12), where it is claimed that "... we must combine the condition of joint consumption with that of inapplicability of the exclusion principle." Although it is not expressed explicitly, this section's tenor is that, while there are goods that are jointly consumable but excludable, there are no goods that are non-excludable and not jointly consumable.[14] Similarly, Milleron (1972, pp. 420–425) defines public goods by the criterion of *no exclusion in use*. His discussion of the scarcity constraint indicates that this is essentially the same as joint consumability; his use of the term *exclusion in use* can be interpreted in the spirit of the present note: If a good is non-excludable, it is jointly consumable and therefore a public good.

[14] Cf. e. g. footnote 1 on p. 10 in Musgrave (1959), where it is claimed that "... the condition of equal consumption must apply to all, ...", from which it is concluded that the inapplicability of the exclusion principle must hold for social wants. In other words, the inapplicability of the exclusion principle is *defined* by the condition of equal consumption applying to all.

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