Making a Budget Deficit Attractive.
The Effect of Endogenous Voting

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Discussion Paper No. 125-06

ISSN 1433-058x
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Abstract:
Several two-period models explain budget deficits as a result of a government's incentive to tie its successor's hands under the assumption that elections are exogenous. In this paper voting is endogenized with second period's voting outcome depending on the first period's government's performance. The budgetary outcome of first period's government under endogenous voting is compared to that under exogenous voting. It can be shown that endogenous voting makes a left-wing government raise deficits in an analytical framework that would lead to balanced budgets otherwise.

JEL: H61, H62, H63

* Helpful comments by Rüdiger Pethig are gratefully acknowledged.
1. Introduction

One of the most interesting political-economic explanations of the existence of public debt is as follows: A government manipulates the budget balance to influence its successor's decision on public spending. This strand of research is based on the works of Persson and Svensson (1989), Alesina and Tabellini (1990) and Tabellini and Alesina (1990). These models are confined to two periods and employ the assumption that the budget has to be balanced at the end of the second period. Another common feature of these models is that the voting procedure at the beginning of the second period is exogenously given in the sense that the first period government is unable to influence the election result. The voting outcome is either random (Tabellini and Alesina, 1990) or certain (Persson and Svensson, 1989)\(^1\), and it drives government behaviour in the first period but not vice versa. An important result of these models is that the sign of the budget balance depends on certain properties of the utility functions involved, i.e. surpluses and balanced budgets are possible outcomes as well as budget deficits.

Aghion and Bolton (1990) develop a two-period model in which the (domestic) government debt cannot be used to influence the second period election result, when the budget has to be balanced at the end of the second period. When default is costless, however, situations with the issuance of debt for the sake of re-election may arise. In Milesi-Ferretti and Spolaore (1994) and Persson and Tabellini (2000) the first period government tries to influence voters' behaviour at the beginning of the second period. The more unpopular the incumbent party is the greater is its temptation to raise the budget deficit in order to make the opponent's program appear less attractive. Lockwood, Philippopoulos and Snell (1996) develop a model in which the first period government knows it will not be re-elected in the second period. The government creates a deficit, forcing its successor to make unpopular decisions thus improving its own election chances for the third period. Dealing with that class of models we say that voting is "endogenous".

The present paper contributes to the models with endogenous voting. The first period government can use a budget deficit in order to improve its re-election chances. Contrasting to the papers mentioned above voters' behaviour is retrospective, i.e. the second period voting outcome is a result of first period's government's performance.

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\(^1\) Persson and Svensson (1989) do not explicitly model a voting procedure. Nonetheless it is clear which party will hold office in the second period.
This paper is organized as follows. In chapter 2 I develop a simple linear-logarithmic reference model. If voting is exogenous a balanced budget always occurs. In chapter 3 voting is endogenized with the voters rewarding or punishing the incumbent government’s tax policy. In some cases a budget deficit occurs. Chapter 4 concludes.

2. The reference model
2.1 Basic Assumptions
The formal setup follows Tabellini and Alesina (1990) and Siebel (2005a and 2005b) rather closely, but the focus is different as the problem will be looked at from a party’s or government’s perspective and not from the voters’ perspective.

$g_t$ is the amount of the only public good consumed (=the size of the public sector), which is provided free of charge and $x_t$ is the consumption of a private good. By revealing its preference for the size of the public sector a party also reveals its preferences for private consumption and vice versa. Party $y_t$, characterized by the preference parameter $\delta \in [0, 1]$ has the intertemporal utility

$$W(y_t, g_t, x_t, g_2, x_2) = \delta \ln(g_t) + (1 - \delta) \ln(x_t) + \delta \ln(g_2) + (1 - \delta) \ln(x_2),$$

with $g_t, x_t > 0$. Parties are driven by their preferences. There is no rent for being in office per se. Hence party behaviour is described according to the partisan theory founded by Hibbs (1977). The pre-tax income of all consumer-voters is 1 in each period. Only a single party can hold office in each period. The party in office levies a lump-sum tax $t$, such that the voter’s budget constraint reads

$$x_t = 1 - t,$$

in period $i = 1, 2$. Furthermore $b \in [-1, 1]$ is the budget balance in period 1. Government 1 can borrow ($b > 0$) or lend ($b < 0$) on a foreign capital market with the qualification that the funds borrowed by the government of period 1 have to be paid back or that the public savings from running a budget surplus in period 1 need to be spent in period 2. As the total number of voters is set equal to 1 the government budget constraint in period 1 is

$$g_1 = t_1 + b,$$

and the period 2 budget constraint is

$$g_2 = t_2 - b.$$

Equations (2) - (4) are now inserted in (1) to yield the indirect utility function
\( \bar{W}^2(\delta, t_1, t_2, b) = \delta \ln(t_1 + b) + (1 - \delta) \ln(1 - t_1) + \delta \ln(t_2 - b) + (1 - \delta) \ln(1 - t_2). \)

At the beginning of each period a government is elected via majority voting. Throughout the paper we assume that the first period voting procedure has already taken place, but from a first period viewpoint, the second period election will be taken into account. In view of (5) the activities of the governments can be completely described as follows: The government of period 1 ("government 1") determines \( t_1 \) and \( b \) while the government of period 2 ("government 2") fixes \( t_2 \).

### 2.2 The second period

First, the behaviour of government 2 is analyzed. We examine which policy \( t_2 \) is chosen by party \( \delta \) when a budget deficit \( b \) has to be served. Its utility maximization is

\[
\max_{t_2} \bar{W}^2(\delta, t_2, b) = \delta_2 \ln(t_2 - b) + (1 - \delta_2) \ln(1 - t_2).
\]

The first order condition for an interior maximum is

\[
\bar{W}^2_{t_2}(\delta, t_2, b) = \frac{1 - \delta_2}{t_2 - 1} + \frac{\delta_2}{t_2 - b} = 0.
\]

(7) yields the tax rate, government 2 prefers to all other tax rates\(^3\):

\[
T^2(\delta, b) := \delta_2 + (1 - \delta_2) b.
\]

The partial derivatives of that function are \( T^2_? = 1 - b > 0 \) and \( T^2_b = (1 - \delta_2) > 0 \).

Combining (4) and (8) yields the supply of the public good \( g_2 = G^2(\delta, b) := (1 - b) \delta_2 \). The first derivatives are \( G^2_? = 1 - b > 0 \) and \( G^2_b = -\delta_2 < 0 \).

Invoking (2) and (8) we specify the preferred size of the private sector as \( x_2 = X^2(\delta, b) := (\delta_2 - 1)(b - 1) \) with the first derivatives \( X^2_? = b - 1 < 0 \) and \( X^2_b = \delta_2 - 1 < 0 \).

We have thus established that the amounts of the private and the public good provided depend on the budget balance and party \( \delta \)'s preferences. These functions \( G^2 \) and \( X^2 \) are strictly monotone in all their variables. A popular cliché is that a party with a high preference for the public sector is considered left-wing and a party with little preference for the public sector is

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\(^2\) As the parties' preferences are supposed to be intertemporally stable the time index can also be suppressed. However, we use it in order to make clear which period is affected.

\(^3\) An interior maximum exists, because the second derivative of (7) is \( \bar{W}^2_{t_2}(\delta, t_2, b) = \delta_2 - 1 \) \( (t_2 - 1)^2 \) \( (t_2 - b)^2 \) < 0.
considered right-wing. For ease of exposition we will follow that notation in that we associate 
\( \delta_i < \frac{1}{2} \) with a right-wing government (party), \( \delta_i = \frac{1}{2} \) with a moderate government (party) and 
\( \delta_i > \frac{1}{2} \) with a left-wing government (party) in period \( i = 1, 2 \).

2.3 The first period

In the first period the government derives intertemporal utility from the provision of both 
goods in both periods. But the allocation of the second period is determined by the second 
period's government with preference parameter \( \delta_2 \). By assumption government 1 knows the 
preference parameter \( \beta \) (and the policy of the second period for any given value of \( b \)) and 
thus it solves the optimization problem

\[
\max_{t_i, b} \tilde{W}^1(\delta_1, \delta_2, t_i, b) = \delta_1 \ln(t_i + b) + (1 - \delta_1) \ln(1 - t_i) \\
+ \delta_1 \ln[(1 - b) \delta_2] + (1 - \delta_1) \ln[(\delta_2 - 1)(b - 1)].
\]

The first order condition for maximizing \( \tilde{W}^1 \) with respect to \( t_i \) is

\[
\tilde{W}_t^1(\delta_1, \delta_2, t_i, b) = \frac{\delta_1 - 1}{1 - t_i} + \frac{-\delta_1}{b + t_i} = 0
\]

and the first order condition for maximizing \( \tilde{W}^1 \) with respect to \( b \) is

\[
\tilde{W}_b^1(\delta_1, \delta_2, t_i, b) = \frac{b - \delta_1 + b \delta_1 + t_i}{(b - 1)(b + t_i)} = 0.
\]

Solving (10) and (11) with respect to \( t_i \) leads to \( b = 0 \) and \( t_i = T^1(\delta_i) = \delta_i \). As \( \tilde{W}^1_{\delta_1} < 0 \),
\( \tilde{W}^1_{\delta_2} < 0 \) and \( \tilde{W}^1_{\delta_1} \tilde{W}^1_{\delta_2} - (\tilde{W}^1_{\delta_2})^2 > 0 \) (see appendix), the equations (10) and (11) characterize a 
maximum. (2) and (3) yield \( g_1 = G^1(\delta_1) = \delta_1 \) and \( x_1 = X^1(\delta_1) = 1 - \delta_1 \), whereas the second 
period allocation is \( g_2 = G^2(\delta_2) = \delta_2 \) and \( x_2 = X^2(\delta_2) = 1 - \delta_2 \) with the tax rate being 
\( t_2 = T^2(\delta_2) = \delta_2 \). Thus the maximum utility is

\[
\tilde{W}^1(\delta_1, \delta_2) = \delta_1 \ln(\delta_1) + (1 - \delta_1) \ln(1 - \delta_1) + \delta_1 \ln(\delta_2) + (1 - \delta_1) \ln(1 - \delta_2).
\]
If the election outcome is exogenous, government 1 always balances its budget. It has no incentive to run a deficit. The allocation in the first period depends on government 1’s preferences only and the second period’s allocation on government 2’s preferences only.

3. Endogenous election outcome

Now the election outcome in the second period is assumed to be endogenous from government 1’s viewpoint. Two parties, denoted ‘party α’ and ‘party β’ compete for government in a simple majority vote and we assume that party α holds office during the first period, i.e. δ₁ = α. The party which secures more than half of the votes holds office in the second period. If both parties achieve a voting share of 50 per cent, government 1 stays in office. Voters assess government 1’s performance rewarding it with re-election or punishing it by voting for the competing party. Government 1 then has an incentive to signal its competences – an idea which was put forward by Rogoff (1990) and Rogoff and Sibert (1988).

It is assumed that voters only care about the tax rate. The share of votes government 1 receives is (1−t₁) or (1−α).

Then a right-wing or moderate government 1 with preference parameter \( \alpha \leq \frac{1}{2} \) will always be rewarded with re-election, if it pursues its most preferred policy.

In contrast, a left-wing government 1 is confronted with a trade-off: On the one hand it has the option to set \( b = 0 \) and \( t₁ > \frac{1}{2} \) according to its ideological preferences for the first period policy. But in doing so it will lose the election and the successor will implement a policy that differs from the left-wing government 1’s preferences for the second period. On the other hand the left-wing government 1 can set a tax rate \( t₁ = \frac{1}{2} \) to ensure re-election.

\(^4\) A balanced budget occurs, because the concavity index of the logarithmic utility functions is constant. The concavity index is defined as \( \lambda (k) := -\frac{u\prime\prime_n (k)}{[u\prime_n (k)]^2} \) for \( k = g, x \). See Tabellini and Alesina (1990) and Siebel (2005a and 2005b).

\(^5\) The assumption that a tax rate of 50% or less always ensures re-election is of course an oversimplification. Without changing the basic results of the present model, an additional (exogenous) parameter \( \phi \) for the incumbent party’s competence in other fields of policy could be introduced. Government 1’s share of votes would then be \( \min \left[ 1, 1 - t₁ + \phi \right] \).
The natural question to ask is which of these two options is in the left-wing party's best interest. We know already that, if the left-wing party follows its preferences (and is not re-elected), its utility is given by (12). However, if it follows the re-election strategy (9) simplifies to

\[
\max_b \hat{W}^l\left(\alpha, \beta, \frac{1}{2}, b \right) = \alpha \ln \left( \frac{1}{2} + b \right) + (1 - \alpha) \ln \left( \frac{1}{2} \right) + \alpha \ln \left[ (1 - b) \alpha \right] + (1 - \alpha) \ln \left[ (\alpha - 1)(b - 1) \right]
\]

with the first order condition\(^6\) \(\hat{W}_b^l\left(\alpha, \beta, \frac{1}{2}, b \right) = \frac{1 - 4\alpha - 4b^2(1 + \alpha) + 4b(2\alpha - 1)}{(b - 1)^2(1 + 2b)^2} = 0\).

Government 1 now pursues a policy different from that in section 2.3, setting \(b = B(\alpha) = \frac{2\alpha - 1}{2(1 + \alpha)} > 0 \) with \(B'(\alpha) = \frac{3}{2(1 + \alpha)^2} > 0\), \(g_1 = \frac{3\alpha}{2(1 + \alpha)} > \alpha\) and \(x_1 = \frac{1}{2}\). In the second period it chooses an allocation \(t_2 = \frac{5\alpha - 1}{2(1 + \alpha)} > \alpha\), \(g_2 = \frac{3\alpha}{2(1 + \alpha)} > \alpha\) and \(x_2 = \frac{3(1 - \alpha)}{2(1 + \alpha)} < 1 - \alpha\). Note that the deficit is the greater the more left-wing government 1 is.

Furthermore government 1 smoothes out the size of the public sector across both periods, leaving the intertemporal distortion to the despised private sector. Associated to this strategy government 1's maximum utility is

\[
\hat{W}^l(\alpha) = \alpha \ln \left[ \frac{3\alpha}{2(1 + \alpha)} \right] + (1 - \alpha) \ln \left( \frac{1}{2} \right) + \alpha \ln \left[ \frac{3\alpha}{2(1 + \alpha)} \right] + (1 - \alpha) \ln \left[ \frac{3(1 - \alpha)}{2(1 + \alpha)} \right].
\]

In order to examine which alternative serves the left-wing government's interests better, we have to compare (12) for \(\delta_1 = \alpha\) and \(\delta_2 = \beta\) to (13). Clearly, the left-wing government chooses

\[
(14) \quad b\left(\hat{\alpha}\right) > 0, \text{ if and only if } D(\alpha, \beta) = \hat{W}^l(\alpha) - \bar{W}^l(\alpha, \beta) > 0.
\]

Due to the involved algebra of (12) and (13) I have not been able to determine the exact 'critical' values of \(\beta\) leading to \(D(\alpha, \beta) = 0\) for any given value of \(\alpha \in \left[\frac{1}{2}, 1\right]\), when government 1 is indifferent between both options. However, progress can be made by

\(^6\) \(\hat{W}_b^l = 0\) characterizes a maximum since \(\hat{W}_b^l\left(\alpha, \beta, \frac{1}{2}, b \right) = \frac{-1}{(b - 1)^2} - \frac{4\alpha}{(1 + 2b)^2} < 0\).
differentiating $D(\alpha, \beta)$ with respect to $\beta$: $D_{\beta}(\alpha, \beta) = \hat{W}_\beta(\alpha) - \hat{W}_\beta(\alpha, \beta) = -\frac{\alpha}{\beta} + \frac{1-\alpha}{1-\beta}$.

We observe that

(15) $\text{sign}\{D_{\beta}(\alpha, \beta)\} = \text{sign}\{\beta - \alpha\}$.

For any given $\alpha$, government 1's utility difference is smallest, when both parties have the same preferences. The more divergent the parties' interests are, the greater is the utility difference. Furthermore we conclude that $D(\alpha, \beta)$ is strictly convex in $\beta$, since

(16) $D_{\beta\beta}(\alpha, \beta) = \frac{1-\alpha}{(1-\beta)^2} + \frac{\alpha}{\beta^2} > 0$.

Hence we cannot have more than two values for $\beta$ solving $D(\alpha, \beta) = 0$, when $\alpha$ is fixed. If two solutions exist, one must be smaller than $\alpha$ and the other must be greater than $\alpha$.

**Proposition 1:**

If both parties have exactly the same preferences, the left-wing government leaves a balanced budget and will be voted out of office.

**Proof:**

Assume that $\alpha = \beta$. Then $D(\alpha, \beta)$ of (14) turns into $D(\alpha, \alpha) < 0$ for all $\alpha > \frac{1}{2}$ (see appendix).

The intuition behind that result is as follows: Government 1 knows that the other party will implement exactly the same policy in the second period. On the other hand there is no utility for being in office per se and any positive value of $b$ will cause intertemporal distortions.

**Proposition 2:**

A left-wing government, which is confronted with a moderate or right-wing opponent, leaves a deficit thus securing re-election.
Proof:

If $\alpha > \frac{1}{2}$ and $\beta = \frac{1}{2}$, we have $D\left(\alpha, \frac{1}{2}\right) > 0$ (see appendix). Due to (15) this leads to $D(\alpha, \beta) > 0$ and $b > 0$ for any combination of $\alpha > \frac{1}{2}$ and $\beta \leq \frac{1}{2}$.

Proposition 3:

A left-wing government facing a more moderate left-wing party strives for re-election and issues debt, if the ideological differences between the two parties are too large.

Proof:

Invoke (15), propositions 1 and 2 and note that the difference function $D(\alpha, \beta)$ is continuous for all $\alpha, \beta \in \left]0, 1\right[$. For any $\alpha > \frac{1}{2}$ there exists a value $\beta = \beta' < \alpha$ such that $D(\alpha, \beta') = 0$ and this value lies in the open interval $\left]\frac{1}{2}, \alpha\right[$. If $\frac{1}{2} < \beta < \beta' < \alpha$, we have $D(\alpha, \beta) > 0$ connected with $b > 0$.

Now consider government 1 being extremely left-wing (i.e. $\alpha$ is arbitrarily close to 1). $D(\alpha, \beta)$ turns into $\lim_{\alpha \to 1^{-}} D(\alpha, \beta) = -2 \ln \left(\frac{4}{3}\right) - \ln(\beta)$ and invoking (14) we conclude that $b > 0$ for $\beta \geq \frac{9}{16}$.

Proposition 4:

A left-wing government challenged by a more left-wing party, uses a budget deficit to ensure re-election, if the political preferences of both parties diverge too much.

Proof:

According to (15) we have $D_{\beta}(\alpha, \beta) > 0$, if and only if $\beta > \alpha$ and it can be easily shown that $\lim_{\beta \to 1^{-}} D(\alpha, \beta) = \infty$. Regarding proposition 1 again, we conclude that there is a solution $\beta = \beta' > \alpha$, solving $D(\alpha, \beta') = 0$. If $\frac{1}{2} < \alpha < \beta' < \beta$ this means $D(\alpha, \beta) > 0$.
Figure 1: Graph of $D(\alpha, \beta)$ for $\alpha \in \left[ \frac{1}{2}, 1 \right]$ and $\beta \in \left[ \frac{1}{2}, 1 \right]$.

Figure 1 as well as several numerical examples calculated for various values of $\alpha \in \left[ \frac{1}{2}, 1 \right]$ illustrate our result, that the left-wing government strives for re-election, i.e. $D(\alpha, \beta) > 0$, if the other party has significantly different preferences.

Figure 2: A left-wing government’s incentive to run a deficit if $\alpha = \frac{7}{10}$.

Figure 2 illustrates how government 1’s choice of strategy for the next election depends on the competing party’s preference parameter $\beta$, if its own preference parameter is set equal to $\alpha = \frac{7}{10}$. The graph shows the value of $D\left(\frac{7}{10}, \beta\right)$ that determines government 1’s choice of
strategy. As long as \( D\left(\frac{7}{10}, \beta\right) > 0 \), i.e. as long as the utility of being re-elected exceeds the utility of being voted out of office, the left-wing government ensures re-election by limiting the tax to \( t_i = \frac{1}{2} \) and leaving a deficit. If \( \beta \) is in the interval between the two hatched lines in figure 2 we have \( D\left(\frac{7}{10}, \beta\right) < 0 \) and thus the left-wing government prefers to resign, because the other party's preferences are quite similar to its own. On the right-hand side of the hatched lines the competing party is even more left-wing than the incumbent. The ideological difference between the two parties is too large again, making the left-wing government prefer re-election.

4. Conclusions
If the election outcome is independent of government behaviour under the current framework a balanced budget arises, independent of the first period government's political preferences. But a left-wing first period government having the opportunity to influence the second period election outcome, may have an incentive to create a budget deficit. If a deficit emerges it is not used to prevent the succeeding government from making the 'wrong' decisions on public spending, but to prevent an electoral victory of a party with significantly divergent preferences. The deficit is the greater the more left-wing government 1 is. Resources are transferred from the second to the first period. Whereas the size of the public sector is smoothed out between both periods, the size of the private sector suffers from intertemporal distortion.

A shortcoming of the model is the fact, that it cannot explain deficits when voting is exogenous, as I use logarithmic utility functions. Government 1 does not alter the budget balance to influence government 2's spending decisions. It would be very interesting to find out, whether the deficit is larger when government 1 intends to influence the election outcome or when it wants to influence its successor's spending decisions. To do so utility functions with a decreasing concavity index like the CES-function are necessary, but these functions cause severe analytical problems in the current framework, which could not yet be overcome.
Appendix

1. Sufficient condition for a maximum of $\bar{W}^1$ in (9)

In order to find out whether the solutions of (10) and (11) characterize a maximum, the second derivatives of (9) have to be examined. Those second derivatives are

$$
\bar{W}_{\alpha\alpha}^1(\delta_1, \delta_2, t_i, b) = \frac{\delta_1 - 1}{(1-t_i)^2} - \frac{\delta_1}{(b+t_i)^2} < 0 \quad \text{and}
$$

$$
\bar{W}_{bb}^1(\delta_1, \delta_2, t_i, b) = \frac{\delta_1 - 2b\delta_i + b^2(1+\delta_i) + 2bt_i + t_i^2}{(b-1)^2(b+t_i)^2} < 0.
$$

For the solutions $b = 0$ and $t_i = \delta_i$ of (10) and (11) they can be modified to

$$
(A1) \quad \bar{W}_{\alpha\alpha}^1(\delta_1, \delta_2, \delta_i, 0) = \frac{\delta_1 - 1}{(1-\delta_i)^2} - \frac{1}{\delta_i} < 0
$$

and

$$
(A2) \quad \bar{W}_{bb}^1(\delta_1, \delta_2, \delta_i, 0) = -\frac{1+\delta_i}{\delta_i} < 0.
$$

$$
\bar{W}_{\alpha b}^1(\delta_1, \delta_2, t_i, b) = \frac{-\delta_i}{(b+t_i)^2} < 0 \quad \text{is the cross derivative, which becomes}
$$

$$
(A3) \quad \bar{W}_{\alpha b}^1(\delta_1, \delta_2, \delta_i, 0) = -\frac{1}{\delta_i} < 0.
$$

Combining (A1) – (A3) gives the determinant of the Hesse matrix, which is

$$
\bar{W}_{\alpha\alpha}^1(\delta_1, \delta_2, \delta_i, 0) \cdot \bar{W}_{bb}^1(\delta_1, \delta_2, \delta_i, 0) - [\bar{W}_{\alpha b}^1(\delta_1, \delta_2, \delta_i, 0)]^2 = \frac{2}{\delta_i(1-\delta_i)} > 0.
$$

With the second order conditions being fulfilled, the optimization of (9) leads to a maximum.

2. Determination of sign $\{D(\alpha, \alpha)\}$

For $\delta_1 = \delta_2 = \alpha$ regard (12), (13) and (14) and define a function $\bar{D}(\alpha) := D(\alpha, \alpha)$. It can be easily shown that $\lim_{\alpha \to \frac{1}{2}^+} \bar{D}(\alpha) = 0$. The first derivative of $\bar{D}(\alpha)$ can be simplified to

$$
\bar{D}_\alpha(\alpha) = \ln \left[ \frac{3(1-\alpha)}{1+\alpha} \right].
$$

Hence, $\bar{D}_\alpha(\alpha) < 0$ as well as $D(\alpha, \alpha) = \bar{D}(\alpha) < 0$, if and only if

$$
\alpha \in \left[ \frac{1}{2}, 1 \right].
$$
3. Determination of sign $\left\{ D\left(\alpha, \frac{1}{2}\right) \right\}$

Regarding (12), (13) and (14) shows that $\lim_{\alpha \to \frac{1}{2}^+} D\left(\alpha, \frac{1}{2}\right) = 0$, whereas the first derivative of $D\left(\alpha, \frac{1}{2}\right)$ with respect to $\alpha$ is $D_{\alpha}\left(\alpha, \frac{1}{2}\right) = \ln\left(\frac{3\alpha}{1+\alpha}\right)$, implying $D_{\alpha}\left(\alpha, \frac{1}{2}\right) > 0$ and $D\left(\alpha, \frac{1}{2}\right) > 0$, if and only if $\alpha > \frac{1}{2}$.

References


Siebel, Jens Peter: Budget Deficit, Size of the Public Sector and Majority Voting, *Volkswirtschaftliche Diskussionsbeiträge*, Vol. 120-05, University of Siegen, 2005a.


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