Quantitative methods in macroeconomics

Rules of differentiation

Guenter W. Beck

University of Siegen

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Rules of differentiation

- $\frac{d}{dx}(c) = 0$ (c a constant)
- $\frac{d}{dx}(x^n) = nx^{n-1}$ (n a real number)
- $\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}\left[f(x)\right]$ (c a constant)
- $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$
- $\frac{d}{dx}\left[f(x) * g(x)\right] = \frac{d}{dx}\left[f(x)\right] * g(x) + f(x) * \frac{d}{dx}\left[g(x)\right]$ (product rule)
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]*g(x) - f(x)*\frac{d}{dx}[g(x)]}{[g(x)]^2}$ (quotient rule)

Rules of differentiation

- $\frac{d}{dx}\left[g(f(x))\right] = g'(f(x)) * f'(x)$ (chain rule)

- Higher derivatives:

$$f'' = \frac{d^2y}{dx^2} \tag{1}$$

$$\ldots \tag{2}$$

$$f^{(n)} = \frac{d^ny}{dx^n} \tag{3}$$

Rules of differentiation

- Implicit differentiation: Example

$$y^3 + xy = 3x + 1 \tag{4}$$

$\Longrightarrow$ Use $y = f(x)$ and write

$$f(x)^3 + xf(x) = 3x + 1 \tag{5}$$

$\Longrightarrow$ Compute first differences and solve for f'(x) to obtain:

$$f'(x) = \frac{3 - f(x)}{3f(x)^2 + x}. \tag{6}$$

Taylor approximations

- First-order Taylor approximation of $F(x)$ around x_0

$$F(x) \approx F(x_0) + \frac{\partial F(x_0)}{\partial x}(x - x_0). \tag{7}$$

- Second-order Taylor approximation of $F(x)$ around x_0

$$F(x) \approx F(x_0) + \frac{\partial F(x_0)}{\partial x}(x - x_0) + \frac{1}{2}\frac{\partial^2 F(x_0)}{\partial x^2}(x - x_0)^2. \tag{8}$$