Dynamic Macroeconomics

Problem Set 7

1. **Risk aversion** Consider the following utility functions

$$U(W) = \frac{W^{1-\sigma} - 1}{1-\sigma} \quad \text{(CRRA)}$$
$$U(W) = \log(W) \quad \text{(CRRA)}$$
$$U(W) = -e^{-\rho W}, \quad \text{(CARA)}$$

where W is (financial) wealth.

   (a) Compute the coefficients of absolute risk aversion

$$\text{ARA}(W) = -\frac{U''(W)}{U'(W)} \tag{1}$$

   and the coefficient of relative risk aversion

$$\text{RRA}(W) = -\frac{U''(W)}{U'(W)} W \tag{2}$$

   of each function.

   (b) Choose some parameter value $0 < \alpha, \beta, \gamma, \sigma, \rho < 1$ and plot the functions.

   (c) Which property of the utility functions ensures that the agent is risk avers? Why do we use the ARA and the RRA measures instead?

2. **Certainty equivalent and risk aversion** The utility function of an agent is of the form

$$U(W) = \sqrt{W} \tag{3}$$

   (a) Show that the agent is risk averse by calculating either the coefficient of relative risk aversion or the coefficient of absolute risk aversion. Consider $W \geq 0$.

   (b) What is the certainty equivalent? Assume that the agent faces a lottery with the following discrete probability distribution

$$P(W = 4) = P(W = 16) = 0.5$$
$$P(W \neq 4 \wedge W \neq 16) = 0.$$

   Compute the certainty equivalent C given the distribution and the utility function. Show that the agent is risk averse using the certainty equivalent.

3. **Asset pricing** Consider an agent with the following maximization problem.

$$\max_{c_t, c_{t+1}, a} U(c_t) + \beta E_t U(c_{t+1}) \tag{4}$$

subject to

$$c_t + p_t a = y_t$$
$$c_{t+1} = y_{t+1} + (p_{t+1} + d_{t+1})a,$$

Where c denotes consumption, p the price of the asset a, y is income, and d denote dividends from investing in the asset a. We define $x \equiv p + d$ to be the gross return of asset a.

Period utility is given by

$$U(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma},$$

with $\beta > 0$ and $\gamma > 0$. Furthermore, assume that consumption growth is log-normally distributed, i.e.

$$\log\left(\frac{c_{t+1}}{c_t}\right) \sim \mathcal{N}\left(\mu, \sigma^2\right).$$

Note that the expected value $E(z^\alpha)$ of a log normally distributed variable z is

$$E(z^\alpha) = e^{\alpha\mu + \frac{\alpha^2}{2}\sigma^2},$$

where $\log z \sim \mathcal{N}(\mu, \sigma^2)$.

(a) Compute the first order condition.

(b) Assume that the agent invests in a risk-less asset which has a price $p_t = q_t$. Use the consumption CAPM approach in order to derive an expression for the price q_t of the risk-less asset. Derive also an expression for $\log q_t$.