# Dynamic Macroeconomics

## Problem Set 6

1. **Ordinary difference equations** Consider the following difference equations

$$y_t = a y_{t-1} \tag{1}$$
$$y_t = a y_{t-1} + b, \tag{2}$$

   where $a$ and $b$ are constants and $y_0$ is given.

   a) Solve both difference equations. This means, find $y_t$ as a function of $y_0$. What happens as $t \to \infty$?

2. **Sustainability of the fiscal stance** Recall from lecture that

$$\frac{b_{t+1}}{y_{t+1}} = \frac{1}{\pi + \gamma} \frac{b_t}{y_t} + \frac{1}{\pi + \gamma} \frac{D_t}{y_t}. \tag{3}$$

   Assume for simplicity that $D_{t+s}/y_{t+s} = 0$ for all $s \geq 0$.

   a) How long does it take for a country to arrive at

$$\frac{b_{t+s}}{y_{t+s}} = 0.6$$

   given its current value of $b_t/y_t$?

3. **Dynamics of Government Debt** The budget constraint of the government is given by

$$B_{t+1} - B_t = G_t - T_t + r B_t$$

   where $r$ is the interest rate on government debt. $B_t$ denotes outstanding government debt in period $t$, $G_t$ denotes government expenditure in period $t$ and $T_t$ are tax revenues in period $t$.

   a) Using the definition $D_t \equiv G_t - T_t$ show that the ratio of debt to GDP ($b_t = B_t/Y_t$) evolves according to

$$b_{t+1} - b_t = \frac{r - g}{1 + g} b_t + \frac{1}{1 + g} d_t$$

   where $g$ is the growth rate of nominal GDP and $d_t = D_t/Y_t$.

   b) Simulate the behaviour of $b_{t+i}, i = 1, 2, \cdots$ given the following parameter values $b_t = 0.73$, $r = 0.047$ and $g = 0.019$. Also assume that $d_t$ is constant.[1]

   c) What is the share of government debt to GDP in some period $T$ (given a starting value $t = 0$ of $b_0$ and assuming that $d$ is constant)?

   d) How large has $d$ to be, if the government wants to achieve a deficit of $b_T$ in period $t$? Discuss how this $d$ depends upon $r$ and $g$.

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[1] The parameter values are roughly those that Germany faced in April 2010, see http://www.bundesbank.de/download/volkswirtschaft/mba/2010/201004mba_verschuldung.pdf. Further data on government debt can be found in the *Fiscal Monitor* of the IMF available at http://www.imf.org/external/pubs/ft/fm/2011/02/fmindex.htm or from the OECD available at http://stats.oecd.org/Index.aspx?datasetcode=GOV_DEBT