# Dynamic Macroeconomics

## Problem Set 5

1. **Budget constraints** Consider a household, who lives from period $t = 0$ to $t = \infty$. At date $t = 0$ he has assets $a_0$ and in each period of his life, he faces a period budget constraint of the form

$$c_t + a_{t+1} = (1 + r)a_t + y_t, t = 0, 1, 2, \dots, \tag{PB}$$

where $c_t$ denotes consumption, $a_t$ denotes assets, and $y_t$ is income in period $t$. Show that you can obtain the following condition form the constraints in (PB)

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t c_t = (1 + r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t. \tag{LB}$$

and provide an interpretation of (LB). (Also discuss any assumptions you may have to make to reach equation (LB)).

2. **Euler Equation** The representative household maximizes

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

subject to (PB). The period utility function is given by

$$u(c_t) = c_t - \frac{\alpha}{2} c_t^2, \tag{2}$$

where $\alpha > 0$ is a parameter.

a) Derive the Euler equation for the consumers optimization problem.

b) Discuss determinants of the growth rate of consumption.

3. **Permanent income hypothesis** Consider the objective function (1), the period utility function (2) and the budget constraint (LB). Assume that $\beta(1 + r) = 1$.

a) Discuss the economic meaning of the assumption $\beta(1 + r) = 1$.

b) Use the Euler equation derived above and the lifetime budget constraint (LB) to show how consumption depends on lifetime wealth.

c) Assume income follows the process

$$y_{t+1} = (1 - \rho)\bar{y} + \rho y_t + u_{t+1}, \quad 0 < \rho \leq 1, \tag{3}$$

where $\bar{y}$ is the mean income and $u_t$ a shock to income in period $t$. Also assume that $a_0 = 0$.

i) First suppose $y_0 = \bar{y}$ and $u_t = 0, t = 0, 1, \dots$. Solve for savings.

ii) Then consider the case $y_0 = \bar{y}$ and $u_0 = 1$ and $u_t = 0, t = 1, 2, \dots$. Discuss how this affects consumption and saving patterns.

4. **Reading Exercise: Tax Rebates** Read the article *Consumer Response to Tax Rebates* by Matthew D.Shapiro and Joel Slemrod (published in the American Economic Review, Vol. 93, 2003, pages 381 - 396), which you can find at `www.jstor.org/stable/3132182`. (You have to be connected to the University Network to access the paper). How does it relate to the questions on this problem set?