Dynamic Macroeconomics

Problem Set 2

The optimal consumption level for the first period (with the utility function $\frac{c^{1-\sigma}-1}{1-\sigma}$) is given by

$$c_1 = \frac{1}{1+(\beta)^{1/\sigma}\left[(1+r)\right]^{1/\sigma-1}}\left[a_0 + y_1 + \frac{y_2}{1+r}\right].$$

When the interest rate increases, three things happen to first period consumption. The **Wealth Effect**, which shows the effect of interest rates on overall life-time wealth, and the **Income Effect** and **Subsituition Effect**, which influences how the share of wealth devoted to first period consumption reacts to changes in the interest rate.

Wealth Effect: When the interest increase, then life-time wealth $\left[a_0 + y_1 + \frac{y_2}{1+r}\right]$ decreases. This tends to lower first consumption.

Substitution Effect: When the interest rate increases, saving becomes more attractive and people reduce first period consumption. Expressed alternatively, a rise in the interest rate increases the relative price of current consumption (relative to future consumption), hence people should substitute away from current consumption and to future consumption.

Income Effect: When the interest rate increases, one can consume more in the second period for any given level of savings. For a given second period consumption level, consumers could thus reduce savings (since the higher interest rate balances the reduction in savings). This would induce a increase in current consumption.

The income and subsitution effect are, in our setup, combined in the expression $\left[(1+r)\right]^{1/\sigma-1}$. If $1/\sigma > 1 \leftrightarrow \sigma < 1$ this expression increase, if r increases. In this case the substitution effect dominates. If $1/\sigma = 1 \leftrightarrow \sigma = 1$ (the case of logarithmic utility) income and substitution effects offset each other (and the share of income devoted to first period consumption becomes independent of the interest rate). If $1/\sigma < 1 \leftrightarrow \sigma > 1$, the income effect dominates and the share of consumption devoted to first period consumption increases if the interest rate increases.