Dynamic Macroeconomics

Problem Set 2

1. **Consumption behavior in a 2-period model** Consider an household who lives for two periods ($t = 1, 2$). At the beginning of the first period, he is endowed with assets a_0 and he receives an exogenously given income y_1. He can use these available resources $(y_1 + a_0)$ for consumption c_1 or for investment in assets a_1, that can be used to save for the second period. In the second period, he receives an exogenously given income y_2 and his assets a_1 and interest income on the assets $r_2 a_1$, where r_2 denotes the (known) interest rate at date $t = 2$. He again can use the available resources for consumption c_2 or for investment in assets a_2.

   The household has a utility function defined over consumption pairs (c_1, c_2) which is given by

   $$u(c_1) + \beta u(c_2) \tag{1}$$

   where $u()$ is a twice continuously differentiable utility function which is strictly increasing and strictly concave. Additionally assume that $\lim_{c \to 0} u'(c_t) = \infty$ and $\lim_{c \to \infty} u'(c_t) = 0$. $\beta \in [0, 1]$ is a discount factor that measures the households impatience.

   a) Formulate the maximization problem of the household.
   b) Argue why $a_2 > 0$ can not be optimal.
   c) Argue why $c_1 = 0$ or $c_2 = 0$ can not be optimal.
   d) Find the first-order conditions and interpret them.

   Now assume that the utility function is given by

   $$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \qquad \text{if } \sigma > 1 \tag{2}$$

   $$u(c_t) = \ln c_t, \qquad \text{if } \sigma = 1 \tag{3}$$

   e) Show that this utility function satisfies the assumptions made above. Plot it for different values of σ and discuss how the shape of the function changes.
   f) State and interpret the first-order conditions for this utility function. Discuss determinants of the optimal c_2^*/c_1^* ratio?
   g) Solve for the optimal consumption level c_1^*. Discuss the economic intuition.
   h) Suppose $r = 0$ and $\beta = 1$. Consider the three cases
      i) $y_1 = 100, y_2 = 0,$
      ii) $y_1 = 50, y_2 = 50,$
      iii) $y_1 = 0, y_2 = 100.$

      Provide solutions for optimal c_1^* and a_1^*. Do you find your results to be a good description of reality?

2. **Borrowing constraints** Consider a consumer that lives for two periods. In the first period he receives a income of 4 units and in the second period an income of 10 units. At the beginning of the first period the consumer does not own any assets (i.e. $a_0 = 0$). The interest rate at which the consumer can borrow or save is r.

   a) Derive the life-time (inter-temporal) budget constraint of the consumer and explain its economic meaning.

   b) Assume that the preferences of the consumer are given by $U(c_1, c_2) = \ln c_1 + \beta \ln c_2$, where $\beta \in [0, 1]$. Solve for the optimal consumption level in the first period. Under what condition will the consumer be a borrower in the first period?

   c) Compare the expression you found for first-period consumption in part b) with a Keynesian consumption function $c_1 = 0.9y_1$.

   d) Suppose that the income in the first period increases by ϵ (while the income in the second period remains constant). Calculate the increase in first-period consumption (Δc_0) and compute the marginal propensity to consume in the first-period (given by $\Delta c_0/\Delta y_0$). Compare the results to those obtained with the Keynesian consumption function from part c).

   e) Suppose that the consumer faces a liquidity constraint, which means that he can save but is unable to borrow in the first-period. Calculate optimal first-period consumption. (You may assume that $r = 0$ and $\beta = 1$). Compare to the results from part b). [Hint: There is no need to use difficult math to answer this question].

   f) Continue to assume that the consumer is liquidity constrained in the first-period. Suppose that the income in the first period increases by ϵ (while the income in the second period remains constant). Calculate the increase in first-period consumption (Δc_0) and compute the marginal propensity to consume in the first-period (given by $\Delta c_0/\Delta y_0$). Compare the results to those obtained with the Keynesian consumption function from part c).