

Dynamic Macroeconomics

Chapter 5: Government: Expenditures and public finances

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Overview

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- ② Financing government expenditures
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The nominal government budget constraint

- The nominal government budget constraint is given by:

$$P_t g_t + P_t h_t + (1 + R_t) B_t = B_{t+1} + \Delta M_{t+1} + P_t T_t \quad (1)$$

- Notation:

- P_t : Price level.
- g_t : Real government expenditure.
- h_t : Real transfers to households.
- B_t : Government “revenue” from issuing bonds in period $t - 1$.
- R_t : Interest rate on government bonds issued in period $t - 1$.
- M_t : Monetary base supplied at the start of period t
($\Delta M_{t+1} = M_{t+1} - M_t$)
- T_t : Total real taxes.

The real government budget constraint

- Dividing the nominal government budget constraint by the price level P_t yields:

$$g_t + h_t + \frac{(1 + R_t) B_t}{P_t} = \frac{B_{t+1}}{P_t} + \frac{\Delta M_{t+1}}{P_t} + T_t \iff \quad (2)$$

$$g_t + h_t + (1 + R_t) b_t = \frac{P_{t+1} B_{t+1}}{P_t P_{t+1}} + \frac{P_{t+1} M_{t+1}}{P_t P_{t+1}} - \frac{M_t}{P_t} + T_t \iff$$

$$g_t + h_t + (1 + R_t) b_t = \frac{(1 + \pi_{t+1}) P_t}{P_t} (b_{t+1} + m_{t+1}) - m_t + T_t \iff$$

$$g_t + h_t + (1 + R_t) b_t = (1 + \pi_{t+1}) (b_{t+1} + m_{t+1}) - m_t + T_t$$

Financing government expenditures: Tax finance

- Assumption: Consider a **permanent** increase in government spending in period t by Δg_t which is financed by an increase in lump-sum taxes, T_t .
- Since the increase is permanent we have:

$$\Delta g_{t+s} = \Delta g_t = \Delta g, \quad \forall s \geq 0 \quad (3)$$

- Moreover, we have:

$$\Delta T_{t+s} = \Delta T_t = \Delta T = \Delta g, \quad \forall s \geq 0 \quad (4)$$

- Since the additional government expenditures are financed by tax increases no additional deficit is generated, i.e. $\Delta b_{t+s} = 0, \forall s \geq 0$.

Financing government expenditures: Tax finance

- The permanent tax increase will reduce the households' available income, $x - T$, from period t on.
- The households' permanent income without the tax increase (denoted by W_t^o) is given by:

$$W_t^o = (1 + R) a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1 + R} \right)^s [x_{t+s} - T_{t+s}] \quad (5)$$

where we assumed that $\pi = 0$ and therefore $r = R$.

- The households' permanent income with the tax increase (denoted by W_t^n) is given by:

$$W_t^n = (1 + R) a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1 + R} \right)^s [x_{t+s} - T_{t+s} - \Delta T] \quad (6)$$

Financing government expenditures: Tax finance

- The households' permanent income with the tax increase can be rewritten as follows:

$$\begin{aligned}
 W_t^n &= (1+R) a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s [x_{t+s} - T_{t+s} - \Delta T] & (7) \\
 &= (1+R) a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s [x_{t+s} - T_{t+s}] - \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s \Delta T \\
 &= (1+R) a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s [x_{t+s} - T_{t+s}] - \frac{1}{1 - \frac{1}{1+R}} \Delta T \\
 &= (1+R) a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s [x_{t+s} - T_{t+s}] - \frac{1+R}{R} \Delta T
 \end{aligned}$$

- The change in households' permanent income, $\Delta W_t = W_t^o - W_t^n$, is therefore given by:

$$\Delta W_t = W_t^n - W_t^o = -\frac{1+R}{R} \Delta T \quad (8)$$

Financing government expenditures: Tax finance

- Given the reduction in households' permanent income their consumption will also fall.
- Denoting by c_t^o the households' consumption without tax increases, by c_t^n the households' consumption with tax increases and by Δc_t the change in consumption then we have (given the assumptions of chapter 4.2):

$$\begin{aligned}
 \Delta c_t &= c_t^n - c_t^o = \left(\frac{R}{1+R} \right) W_t^n - \left(\frac{R}{1+R} \right) W_t^o & (9) \\
 &= \left(\frac{R}{1+R} \right) (W_t^n - W_t^o) = \left(\frac{R}{1+R} \right) \left(- \left(\frac{1+R}{R} \right) \Delta T \right) \\
 &= -\Delta T
 \end{aligned}$$

⇒ The tax-financed increase in permanent government spending leads to an equally-sized decrease in private consumption.

Financing government expenditures: Tax finance

- The overall effect of the permanent increase in government spending on the economy (denoted by Δy_t) is given by:

$$\Delta y_t = \Delta c_t + \Delta g_t = -\Delta T + \Delta T = 0 \quad (10)$$

⇒ Permanent increase in government spending has no effect on the economy.

Financing government expenditures: Pure bond finance

- Assumption: Consider again a **permanent** increase in government spending in period t by Δg_t which is solely financed by issuing new bonds (i.e. taxes are not affected).
- We further assume that the inflation rate is zero in all periods.
- Since the increase is permanent we have:

$$\Delta g_{t+s} = \Delta g_t = \Delta g, \quad \forall s \geq 0 \quad (11)$$

- To finance the increase of government spending in period t the government must increase bond holdings by

$$\Delta b_{t+1} = \Delta g \quad (12)$$

- In period $t + 1$ bond holdings must increase by

$$\Delta b_{t+2} = \Delta b_{t+1} + R\Delta b_{t+1} = (1 + R)\Delta b_{t+1} = (1 + R)\Delta g \quad (13)$$

Financing government expenditures: Pure bond finance

- In period $n - 1$ bond holdings must increase by

$$\Delta b_{t+n} = (1 + R)^{n-1} \Delta g \quad (14)$$

- Bond holdings in period n are then:

$$\begin{aligned} b_{t+n} &= b_{t+n-1} + \Delta b_{t+n} = b_{t+n-2} + \Delta b_{t+n-1} + \Delta b_{t+n} = \dots \\ &= b_t + \sum_{s=1}^n \Delta b_{t+s} \end{aligned} \quad (15)$$

- Since

$$\Delta b_{t+s} = (1 + R)^{s-1} \Delta g \quad (16)$$

we can write:

$$b_{t+n} = b_t + \sum_{s=1}^n (1 + R)^{s-1} \Delta g = b_t + \Delta g \sum_{s=1}^n (1 + R)^{s-1} \quad (17)$$

Financing government expenditures: Pure bond finance

- The last expression can be rewritten as:

$$\begin{aligned}
 b_{t+n} &= b_t + \Delta g \sum_{s=1}^n (1+R)^{s-1} = b_t + (1+R) \frac{(1+R)^{n-1} - 1}{(1+R) - 1} \Delta g \\
 &= b_t + (1+R) \frac{(1+R)^{n-1} - 1}{R} \Delta g
 \end{aligned} \tag{18}$$

- Dividing both sides of this equation by $(1+R)^n$ yields:

$$\frac{b_{t+n}}{(1+R)^n} = \frac{b_t}{(1+R)^n} + \frac{1}{R} \Delta g - \frac{1}{R(1+R)^{n-1}} \Delta g \tag{19}$$

- Since

$$\lim_{n \rightarrow \infty} \frac{b_{t+n}}{(1+R)^n} = \frac{1}{R} \Delta g \neq 0 \tag{20}$$

the bond-financed increase in government expenditures is not sustainable.

The sustainability of the fiscal stance

- Starting point: Nominal government budget constraint

$$P_t g_t + P_t h_t + (1 + R_t) B_t = B_{t+1} + \Delta M_{t+1} + P_t T_t \quad (21)$$

- Dividing both sides of this equation by nominal GDP, $P_t y_t$, yields:

$$\frac{P_t g_t}{P_t y_t} + \frac{P_t h_t}{P_t y_t} + \frac{(1 + R_t) B_t}{P_t y_t} = \frac{B_{t+1}}{P_t y_t} + \frac{M_{t+1}}{P_t y_t} - \frac{M_t}{P_t y_t} + \frac{P_t T_t}{P_t y_t} \quad (22)$$

- Rearranging and simplifying yields:

$$\begin{aligned} \frac{g_t}{y_t} + \frac{h_t}{y_t} + (1 + R_t) \frac{b_t}{y_t} &= \frac{T_t}{y_t} + \frac{B_{t+1}}{\frac{1}{(1+\pi_{t+1})} P_{t+1} \frac{1}{(1+\gamma_{t+1})} y_{t+1}} \\ &+ \frac{M_{t+1}}{\frac{1}{(1+\pi_{t+1})} P_{t+1} \frac{1}{(1+\gamma_{t+1})} y_{t+1}} - \frac{m_t}{y_t} \end{aligned}$$

The sustainability of the fiscal stance

- The just derived equation can be simplified as follows:

$$= \frac{T_t}{y_t} + (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \left(\frac{g_t}{y_t} + \frac{h_t}{y_t} + (1 + R_t) \frac{b_t}{y_t} \right) - \frac{m_t}{y_t}$$

where γ_{t+1} denotes the growth rate of real GDP between periods t and $t + 1$.

- The question we want to analyze in this section is whether a given government deficit, $P_t D_t$, is sustainable or not.
- The government deficit in a given period t is given by:

$$P_t D_t = P_t g_t + P_t h_t + R_t B_t - P_t T_t - (M_{t+1} - M_t) \quad (23)$$

The sustainability of the fiscal stance

- Dividing the just derived equation by nominal GDP, $P_t y_t$, yields:

$$\frac{D_t}{y_t} = \frac{g_t}{y_t} + \frac{h_t}{y_t} + R_t \frac{b_t}{y_t} - \frac{T_t}{y_t} - (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{m_{t+1}}{y_{t+1}} + \frac{m_t}{y_t} \quad (24)$$

- From the budget constraint we see that the right-hand side of this equation is given by:

$$\begin{aligned} \frac{g_t}{y_t} + \frac{h_t}{y_t} + R_t \frac{b_t}{y_t} - \frac{T_t}{y_t} - (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{m_{t+1}}{y_{t+1}} + \frac{m_t}{y_t} & \quad (25) \\ & = (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t} \end{aligned}$$

- That is, we have:

$$\frac{D_t}{y_t} = (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t} \quad (26)$$

The sustainability of the fiscal stance

- Note: The difference between the actual deficit $P_t D_t$ and the interest payments on accumulated debt $R_t B_t$ is called (nominal) primary deficit $P_t d_t$ and is given by:

$$P_t d_t = P_t D_t - R_t B_t \quad (27)$$

- Hence the ratio of the primary deficit $P_t d_t$ to GDP $P_t y_t$ is given by:

$$\frac{d_t}{y_t} = \frac{g_t}{y_t} + \frac{h_t}{y_t} - \frac{T_t}{y_t} - (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{m_{t+1}}{y_{t+1}} + \frac{m_t}{y_t} \quad (28)$$

- Now we see from the budget constraint that the right-hand side of this equation is given by:

$$\begin{aligned} \frac{g_t}{y_t} + \frac{h_t}{y_t} - \frac{T_t}{y_t} - (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{m_{t+1}}{y_{t+1}} + \frac{m_t}{y_t} \\ = (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{b_{t+1}}{y_{t+1}} - (1 + R_t) \frac{b_t}{y_t} \end{aligned} \quad (29)$$

The sustainability of the fiscal stance

- That is, we now have:

$$\frac{d_t}{y_t} = (1 + \pi_{t+1}) (1 + \gamma_{t+1}) \frac{b_{t+1}}{y_{t+1}} - (1 + R_t) \frac{b_t}{y_t} \quad (30)$$

- This is a first order difference equation that determines the evolution of b_t/y_t .
- Solving for b_{t+1}/y_{t+1} we get:

$$\frac{b_{t+1}}{y_{t+1}} = \frac{1}{(1 + \pi_{t+1}) (1 + \gamma_{t+1})} \left[\frac{d_t}{y_t} + (1 + R_t) \frac{b_t}{y_t} \right] \quad (31)$$

- This equation can be stable or unstable depending on whether

$$\frac{1 + R_t}{(1 + \pi_{t+1}) (1 + \gamma_{t+1})} \leq 1 \quad (32)$$

The sustainability of the fiscal stance

- Why?
- Implications?
- Assuming that inflation π , output growth γ and the interest rate R are constant, the difference equation is given by

$$\frac{b_{t+1}}{y_{t+1}} = \frac{1}{(1 + \pi)(1 + \gamma)} \frac{d_t}{y_t} + \frac{1 + R}{(1 + \pi)(1 + \gamma)} \frac{b_t}{y_t} \quad (33)$$

- Stable case:

$$\frac{1 + R}{(1 + \pi)(1 + \gamma)} < 1 \quad (34)$$

- Define $\mathcal{A} := [(1 + \pi)(1 + \gamma)]$.

The sustainability of the fiscal stance

- Then the difference equation can be *solved backwards* as follows:

$$\begin{aligned}
 \frac{b_t}{y_t} &= \frac{1+R}{\mathcal{A}} \frac{b_{t-1}}{y_{t-1}} + \frac{1}{\mathcal{A}} \frac{d_{t-1}}{y_{t-1}} & (35) \\
 &= \frac{1+R}{\mathcal{A}} \left(\frac{1+R}{\mathcal{A}} \frac{b_{t-2}}{y_{t-2}} + \frac{1}{\mathcal{A}} \frac{d_{t-2}}{y_{t-2}} \right) + \frac{1}{\mathcal{A}} \frac{d_{t-1}}{y_{t-1}} \\
 &= \left(\frac{1+R}{\mathcal{A}} \right)^2 \frac{b_{t-2}}{y_{t-2}} + \frac{1}{\mathcal{A}} \frac{1+R}{\mathcal{A}} \frac{d_{t-2}}{y_{t-2}} + \frac{1}{\mathcal{A}} \frac{d_{t-1}}{y_{t-1}} \\
 &= \dots \\
 &= \left(\frac{1+R}{\mathcal{A}} \right)^T \frac{b_{t-T}}{y_{t-T}} + \frac{1}{\mathcal{A}} \sum_{s=0}^{T-1} \left(\frac{1+R}{\mathcal{A}} \right)^s \frac{d_{t-1-s}}{y_{t-1-s}}
 \end{aligned}$$

- Denoting today's period as period t , this equation implies that the deficit in the future period $t+n$ is given by:

$$\frac{b_{t+n}}{y_{t+n}} = \left(\frac{1+R}{\mathcal{A}} \right)^n \frac{b_t}{y_t} + \frac{1}{\mathcal{A}} \sum_{s=0}^{n-1} \left(\frac{1+R}{\mathcal{A}} \right)^{n-s-1} \frac{d_{t+s}}{y_{t+s}} \quad (36)$$

The sustainability of the fiscal stance

- Since $\frac{1+R}{\mathcal{A}} < 1$ we have:

$$\lim_{n \rightarrow \infty} \left(\frac{1+R}{\mathcal{A}} \right)^n \frac{b_t}{y_t} = 0 \quad (37)$$

- Then:

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{b_{t+n}}{y_{t+n}} &= \frac{1}{\mathcal{A}} \sum_{s=0}^{\infty} \left(\frac{1+R}{\mathcal{A}} \right)^{n-s-1} \frac{d_{t+s}}{y_{t+s}} \\ &= \frac{1}{(1+\pi)(1+\gamma)} \sum_{s=0}^{\infty} \left(\frac{(1+R)}{(1+\pi)(1+\gamma)} \right)^{n-s-1} \frac{d_{t+s}}{y_{t+s}} \end{aligned} \quad (38)$$

The sustainability of the fiscal stance

- The fiscal stance is sustainable if the expression $\frac{b_{t+n}}{y_{t+n}}$ remains finite for $n \rightarrow \infty$.
- The sustainability of the fiscal stance therefore depends on the combinations of the following expressions:
 - $1 + R$
 - $(1 + \pi)(1 + \gamma)$
 - $\frac{d_{t+s}}{y_{t+s}}$ (for $s \geq 0$).
- From this we can derive the following implications:
- Assume that the primary deficit is expected to remain constant over time at its current level, i.e. $\frac{d_{t+s}}{y_{t+s}} = \frac{d_t}{y_t}$ for $s \geq 0$.

The sustainability of the fiscal stance

- Then:

$$\begin{aligned}
 \lim_{n \rightarrow \infty} \frac{b_{t+n}}{y_{t+n}} &= \frac{1}{(1+\pi)(1+\gamma)} \sum_{s=0}^{\infty} \left(\frac{(1+R)}{(1+\pi)(1+\gamma)} \right)^{n-s-1} \frac{d_t}{y_t} \\
 &= \left(\frac{1}{(1+\pi)(1+\gamma)} \right) \left(\frac{d_t}{y_t} \right) \left(\frac{1}{1 - \left(\frac{(1+R)}{(1+\pi)(1+\gamma)} \right)} \right) \\
 &= \frac{1}{(1+\pi)(1+\gamma) - (1+R)} \left(\frac{d_t}{y_t} \right) \simeq \frac{1}{\pi + \gamma - R} \frac{d_t}{y_t}
 \end{aligned}$$

⇒ If the government manages to keep the primary deficit ratio constant, the fiscal stance is sustainable.

- But also: If $\pi + \gamma > R$ then b_t/y_t is finite regardless of the value of d_t/y_t .

The sustainability of the fiscal stance

- The government may want both b_t/y_t and d_t/y_t to be constant over time. This would imply that

$$\frac{b_t}{y_t} \geq \frac{1}{\pi + \gamma - R} \frac{d_t}{y_t}$$

⇒ The government can find b/y for any constant value of d/y and any constant value of $\pi + \gamma - R$ that is positive.

- Unstable case:

$$\frac{1 + R}{(1 + \pi)(1 + \gamma)} > 1 \quad (39)$$

- Define again $\mathcal{A} := [(1 + \pi)(1 + \gamma)]$.

The sustainability of the fiscal stance

- Then the difference equation must be *solved forward* resulting in:

$$\frac{b_t}{y_t} = \frac{\mathcal{A}}{1+R} \frac{b_{t+1}}{y_{t+1}} - \frac{1}{1+R} \frac{d_t}{y_t} \quad (40)$$

$$= \left(\frac{\mathcal{A}}{1+R} \right)^n \frac{b_{t+n}}{y_{t+n}} - \frac{1}{1+R} \sum_{s=0}^{n-1} \left(\frac{\mathcal{A}}{1+R} \right)^s \frac{d_{t+s}}{y_{t+s}} \quad (41)$$

- Since $\frac{\mathcal{A}}{1+R} < 1$ we have (no-Ponzi condition):

$$\lim_{n \rightarrow \infty} \left(\frac{\mathcal{A}}{1+R} \right)^n \frac{b_{t+n}}{y_{t+n}} = 0 \quad (42)$$

The sustainability of the fiscal stance

- Then:

$$\frac{b_t}{y_t} \leq \frac{1}{1+R} \sum_{s=0}^{\infty} \left(\frac{(1+\pi)(1+\gamma)}{1+R} \right)^s \frac{-d_{t+s}}{y_{t+s}} \quad (43)$$

- with $-d_t$ as the primary surplus.
- All implications of the stable case hold with opposite interpretation.
- exception: inequality sign; future primary surpluses must be large enough to meet current debt liabilities.
- Assuming that the deficit-GDP ratio remains constant over time the equation for the dynamics of the debt-GDP ratio (equation (26)) represents a stationary difference equation.
- Thus, the debt-GDP ratio will converge to a (constant) steady-state level and we will have in equilibrium:

$$\frac{b_{t+1}}{y_{t+1}} = \frac{b_t}{y_t} = \frac{b}{y} \quad (44)$$

The sustainability of the fiscal stance

- Using the equation for the dynamics of the debt-GDP ratio (26) with constant growth rates π and γ we obtain:

$$\begin{aligned} \frac{b_{t+1}}{y_{t+1}} &= \frac{1}{(1+\pi)(1+\gamma)} \frac{D_t}{y_t} + \frac{1}{(1+\pi)(1+\gamma)} \frac{b_t}{y_t} \\ \frac{b}{y} &= \frac{1}{(1+\pi)(1+\gamma)} \frac{D}{y} + \frac{1}{(1+\pi)(1+\gamma)} \frac{b}{y} \\ \frac{(1+\pi)(1+\gamma) - 1}{(1+\pi)(1+\gamma)} \frac{b}{y} &= \frac{1}{(1+\pi)(1+\gamma)} \frac{D}{y} \\ \frac{b}{y} &= \frac{1}{(1+\pi)(1+\gamma) - 1} \frac{D}{y} \end{aligned}$$

- Since $(1+\pi)(1+\gamma) \approx 1 + \pi + \gamma$ we get:

$$\frac{b}{y} = \frac{1}{\pi + \gamma} \frac{D}{y} \quad (45)$$

The sustainability of the fiscal stance

- Implications of the analysis: Assume that the current debt-GDP ratio is given by $\frac{\bar{b}}{\bar{y}}$.
- Then:

- If the deficit-output ratio is kept constant over time and we have:

$$\frac{1}{\pi + \gamma} \frac{D}{y} < \frac{\bar{b}}{\bar{y}} \quad (46)$$

then the debt-GDP ratio will fall.

- If the deficit-output ratio is kept constant over time and we have:

$$\frac{1}{\pi + \gamma} \frac{D}{y} = \frac{\bar{b}}{\bar{y}} \quad (47)$$

then the debt-GDP ratio will remain constant.

- If the deficit-output ratio is kept constant over time and we have:

$$\frac{1}{\pi + \gamma} \frac{D}{y} > \frac{\bar{b}}{\bar{y}} \quad (48)$$

then the debt-GDP ratio will increase.

The stability and growth pact

- The stability and growth pact requires that the following rates hold:

$$\frac{D_t}{y_t} \leq 0.03 \quad (49)$$

$$\frac{b_t}{y_t} \leq 0.60 \quad (50)$$

- What implications does this have for fiscal sustainability?

$$\frac{b}{y} \geq \frac{1}{\pi + \gamma} \frac{D}{y} \quad (51)$$

- Thus

$$60 \geq \frac{1}{\pi + \gamma} 3 \rightarrow \pi + \gamma \geq \frac{3}{60} = 5 \quad (52)$$

The stability and growth pact

- Even if the above ratios are met the growth rate of nominal GDP has to exceed 5 percent.
- \implies The Stability and Growth Pact is not sufficient for fiscal sustainability.
- If the above ratios are not met there are growth rates of nominal GDP that would be consistent with fiscal sustainability.
- \implies The Stability and Growth Pact is not necessary for fiscal sustainability.
- Example: France in 2002 had $b/y = 59.1$, $D/y = 3.1$, $\pi = 1.9$ and $\gamma = 1.2$.
- $\frac{1}{\pi+\gamma} \frac{D}{y} = 100$ did not meet the condition for fiscal sustainability although ratios were nearly met.
- The growth rate of nominal GDP was too low.
- Nonetheless people still held government debt and continue to do so in France. Why?