FDI and International Portfolio Investment -
Complements or Substitutes?

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Abstract

We show in a dynamic investment setting whether firms choose FDI or international portfolio investment (FPI) in the presence of stochastic productivity taking into account differences in flexibility of both investments. Isolated FPI and FDI investments are compared to combined FPI and FDI investments. FDI requires higher investment specific costs than FPI. Thus, it is not possible to adjust FDI to environmental changes every period. In contrast, FPI bears lower fixed costs and can be adjusted immediately to short-term changes in the environment. Additionally, as a result of the investors’ control position FDI yields a higher return than FPI. Hence, there is a trade-off between flexibility and higher return for firms deciding between FDI and FPI. We explore whether as a consequence of higher investment specific fixed costs and lower flexibility in the case of FDI, small firms prefer FPI and larger firms invest in FDI. We show that a combined strategy dominates the isolated strategy always in time. Further, combined international investment comprises a higher incentive for firms to invest in R&D-investment and consequently firm productivity increase faster than with isolated international investment. Depending on the success-probability and the correlation between the various investment possibilities, even small firms (low productivity) invest in FDI.


1 Introduction

The recent World Investment Report 2006 highlights that Foreign Direct Investment (FDI) flows and growing FDI stocks are now at an unparalleled level with most going to industrial countries. At the same time flows of international portfolio investments (FPI) exceeded FDI flows twice at the beginning of the nineties while more recently FPI growth slowed down and both capital flows converged.\(^1\) What are the motives for firms to invest in one or the other and how are they to be explained?

Previous studies on FDI explained the motives for FDI with differential rates of return, differences in interest rates and risk diversification.\(^2\) Following Andersen and Hainaut (1998) these determinants lost explanatory power and recent theoretical and empirical studies document that FDI is undertaken to exploit cost advantages (vertical FDI)\(^3\) or to serve different markets locally to avoid trade costs (horizontal FDI).\(^4\) If FDI no longer serves risk diversification, does FPI then fill the gap and are these capital flows complements rather than substitutes?

In the present paper we analyse whether firms choose FDI or FPI in the presence of stochastic productivity taking into account differences in flexibility of both investments. In particular FDI is less flexible than FPI and this reduced flexibility entails a higher rigidity of FDI. As FDI requires higher investment specific costs it is not possible to adjust FDI to environmental changes every period.\(^5\) In contrast, FPI bears lower fixed costs and can be adjusted immediately to short-term changes in the environment. However, as a result of the investors' control position FDI yields a higher return than FPI. Hence, there is a trade-off between flexibility and higher return for firms deciding between FDI and FPI. We explore whether as a consequence of higher investment specific fixed costs and lower flexibility in the case of FDI, small firms prefer FPI and larger firms invest in FDI.

We show that the combined investment strategy (FDI and FPI at the same time) always starts the international investment activity earlier in time than the isolated strategy (FDI or FPI). Additionally, with combined international investment, there is a higher incentive for firms to invest in R&D-investment and consequently firm productivity increases faster than with isolated international

\(^1\)See WTO News, October 1996.
\(^2\)See for example Dunning (1973).
\(^3\)Grossman, Helpman, Szeidl (2005) discuss in which states firms decide to outsource or offshore some of their production stages. Acemoglu, Aghion,Griffith and Zilibotti show that vertical integration is more common if the technology intensity differs significantly.
\(^4\)See Helpman, Melitz, Yeaple (2003) for a detailed survey whether firms decide to serve a foreign market through export or FDI. Studies of complex FDI strategies can be found for example in Helpman (2006) or Grossman, Helpmann, Szeidl (2003).
\(^5\)See Goldstein and Razin (2005) for a discussion of the different costs for FDI and FPI.
investment. Depending on the success-probability and the correlation between
the various investment possibilities, even small firms (low productivity) invest
in FDI.

To model firm behaviour we use a monopolistic competition framework with
uncertain firm productivity in combination with a dynamic investment approach
over a finite investment horizon. There are three countries, home and two
foreign countries. The firms are located in the home country and decide to
invest via FDI or FPI in the foreign countries. Thereby, they face uncertainty
about their future productivity and returns on the respective investment. In
particular, firm productivity is endogenous and follows a Poisson process. The
productivity of the different investment opportunities are correlated with each
other. Differences in correlation between FDI and home production account for
different forms of FDI.\(^6\)

The reminder of the paper is organized as follows. In section two, we give
a short overview of the recent literature and emphasize our contribution to
it. Section three outlines the theoretical framework and derives the optimality
conditions for the various investments strategies. Following this, we present the
numerical solution of the model and discuss the results in section four. Finally,
section five concludes.

\section{The Literature}

In the paper we link the information based trade-off literature between FDI and
FPI by Goldstein and Razin (2005) (RG) and Albuquerque (2003) with the firm-
level Export and FDI approaches by Grossman, Helpman and Szeidl (2003) and
Helpman, Melitz and Yeaple (2003). RG analyse the investors’ decision between
FDI and FPI under asymmetric information in a static model.\(^7\) As a result of
the information asymmetry the project revenue from FDI is higher than from
FPI. In the case of FDI the investor is also the manager of the foreign firm.
Hence, he has a higher control over the production processes and can ensure
that the firm is run according to the investors’ interests. If the investor chooses
FPI the investor has no control over the foreign production process and the
expected return is lower. We use these different characteristics shown by RG
to motivate the costs, flexibility and return of the different investment possi-
bilities in the present paper. Additionally, we consider the findings of Chuhan,
Perez-Quiraz and Popper (1996). They provide an empirical analysis on the
different characteristics of short term and long term capital flows. Furthermore,
in contrast to RG, we introduce a long-term investor in a dynamic setting. This
investor has the possibility to adjust his portfolio periodically with rigidity in
FDI-shares. Hence, we also account for the different grades of flexibility of both
investments.

\(^6\)Aizenman and Marion (2001) as well as Markusen and Maskus (2001) show that horizontal
FDI is established in countries similar in size and endowments, while vertical FDI is the
preferred investment in countries with different characteristics as the source country.

\(^7\)See also Razin, Mody and Sadka (2002) and Razin (2002).
Alburquerque (2003) analyses from a country perspective the risk-sharing character of FDI and non-FDI capital flows for countries with different degrees of financial constraints. Thereby, non-FDI flow adjustments arise from shocks in the receiving country. One result is that for financially constrained countries FDI is less volatile than non-FDI flows. With perfect enforcement, the difference in volatility diminishes. We modify this approach by taking the firm perspective and consider shocks on firm level as well as on host country level. Actually, we always find a higher volatility of non-FDI flows (FPI) than FDI flows in our firm-level perspective. The firm reacts to any short-term environment change by adjusting FPI. Precisely, FPI has the main function to smooth risk whereas FDI mainly exploits gains from technology transfers.

Uncertain firm productivity is decisive for the results of our model. This leads to the literature around Melitz (2003) or Grossman, Helpman and Szeidl (2003). They motivate the firms’ choice to export or engage in FDI with differing firm productivity. Melitz (2003) shows that with heterogeneous firms only the large firms (with higher productivity) export. Small firms serve the domestic market only. Furthermore, Helpman, Melitz and Yeaple (2003) extend this and find that firms with higher productivity use higher integrated organisational production structures. They show that less productive firms only serve the domestic market, with increasing productivity firms start to export and finally the most productive firms engage in FDI. In contrast to this literature, in the present paper firm productivity is endogenous. Firms can push their productivity by investing in research and development (R&D). The success of the R&D-investment is uncertain. Moreover, we extend these models by introducing FPI as a new form of investment possibility.

3 Theoretical Framework

The dynamic methodology in the model follows roughly the models of Abel (1973) and Holt (2003).

Firms optimize their investment decisions in a continuous-time model. Inspired by Melitz (2003), the model is based on monopolistic competition with stochastic firm productive. Domestic demand is exogenous and the consumers have Dixit-Stiglitz preferences. There are three countries. Two of these countries are northern countries West (home country) and East (foreign). The third country is a southern country (foreign). In the eastern country, cultural background, production and cost structure are similar to the home country. Hence, the productivities in these countries are positive correlated. On the opposite, the South has different cultural background, production and cost structure than the home and the eastern country. Consequently, the productivity correlations between South and home or South and East are negative.

We consider a setting in which a representative firm faces a choice between performing activities at home (production and R&D-investment) and engaging in two alternative foreign investments: foreign portfolio investment (FPI) or foreign direct investment (FDI). The initial position of each firm is home pro-
duction and home R&D investment. Based on these home activities the firm can additionally choose to invest internationally. Whether a firm decides to invest internationally depends on the firm’s specific productivity $\theta$. In particular, the firm can increase its specific productivity by investing in home research and development (R&D). Whether R&D-investment increases the firm’s productivity is uncertain. The change of $\theta$ through R&D-investment follows a Poisson-Process

$$d\theta = \left(1 - \tau\right) \frac{\phi_t}{K_t} \theta dq.$$  \hspace{1cm} (1)

In (1) $\phi$ is the capital invested in R&D and $K$ is the total stock of capital available to the firm. As obsolete technologies have to be replaced, patent laws are renewed etc., even in case of successful R&D-investment, the growing rate of $\theta$ is smaller than the invested rate of capital. These costs correspond to depreciation and are depicted by $\tau$. Finally $q$ is a random variable that equals 1 with probability $\lambda$ and 0 otherwise. Hence, if R&D-investment is successful, $\theta$ increases by $\left[1 - \left(1 - \frac{\phi}{K}\right)\right] \theta$. With probability $(1 - \lambda)$ R&D-investment fails and $\theta$ stays unchanged.

As every firm, no matter whether it engages in FDI, FPI or not, produces at home and serves the home market, we start with the analysis of the home country.

### 3.1 Home

#### 3.1.1 Production

The firm uses a single factor, capital, to produce output at home $x^h$

$$x^h(\theta) = \theta^h \left(k^h\right)^{\alpha}.$$  \hspace{1cm} (2)

The superscript $h$ states that these are the values in the isolated "Home"-scenario.\(^8\) According to (1), firms also can use capital to invest in R&D and increase their productivity

$$K = k^h + \phi.$$  \hspace{1cm} (3)

As a consequence of monopolistic competition, firms choose the profit maximising-price

$$p_t = \frac{1}{\frac{\phi_t}{\theta_t}}.$$  \hspace{1cm} (4)

Where the rent for capital is set equal to one, $\frac{\phi_t}{\theta_t}$ is the profit maximizing mark up and $\frac{1}{\frac{\phi_t}{\theta_t}}$ are the marginal costs of a firm with productivity $\theta_t$. Furthermore, the firm has fixed costs of home production equal to $f^h$ and costs of R&D-investment equal to $\phi_t$. Hence, the profit of the firm at home in period $t$ is

$$\pi_t(\theta_t) = p_t\theta_t \left(k^h\right)^{\alpha} - \left\{f^h + \frac{x^h_t}{\theta_t} + \phi_t\right\}, \quad 0 < \alpha < 1.$$  \hspace{1cm} (5)

\(^8\)The following scenarios with isolated FPI, FDI and the combined investments are identified by the superscripts $p$, $d$, and $c$ respectively.
The first term on the right hand side equals the revenue from production and sales at the home market, \( r^h_t \). The second term on the right hand side summarizes the costs of home production and R&D investment.

The expected value of firm profits is

\[
V^h_t = \max_{k^h_t, \phi_t} \int_t^T \pi_s(\theta_s) e^{-\rho(s-t)} ds. 
\]

(6)

subject to (2) - (4). Modification of (6) yields:

\[
\rho V^h_t \, dt = \max_{k^h_t, \phi_t} \pi_t(\theta_t) \, dt + E_t \left( dV^h \right) 
\]

(7)

which states that the mean required return of a firm equals the expected return. In period \( t \), the expected return consist of the maximized profit at \( t \) and the expected gain or loss of the future profit flow.

To calculate the expected capital flow, we substitute (1) into \( dV^h \):

\[
E_t \left( dV^h \right) = \lambda \left[ V^h(\gamma \theta) - V^h(\theta) \right] dt 
\]

(8)

with \( \gamma \equiv 1 - \tau \frac{\phi}{K} \). Equation (8) is the expected capital flow. The expected capital flow is a perpetual flow of the difference between the capital flow in case of successful R&D investment \( V^h(\gamma \theta) \) and without successful R&D investment \( V^h(\theta) \) weighted with the success-probability. Substituting (8) back into (7) and divide by \( dt \) leads to

\[
\rho V^h(\theta) = \max_{k^h_t, \phi_t} \left\{ \pi(\theta) + \lambda \left[ V^h(\gamma \theta) - V^h(\theta) \right] \right\}. 
\]

(9)

There are two important features about (9) which one should keep in mind thorough the following analysis. Firstly, all important information about the past concerning current or future decisions are summarized in \( \theta \). How the firm reached the present productivity does not matter at all. Secondly, choosing the optimal production and R&D-investment strategy with respect to the problem starting at the current productivity level \( \theta \) that results from the initial firm strategies, is the optimal strategy no matter what the initial strategy of the firm was.

3.1.2 Optimal Conditions for R&D-Investment and Production Strategies

From (9) we can derive the optimality conditions for firm-strategies for R&D-investment and home production.

\footnote{For a detailed derivation see Appendix A.}
**R&D-Investment**  Deriving the marginal valuation of R&D-investment from (9) yields
\[ \pi_\phi (\theta) + \lambda V_\phi^h (\gamma \theta) = 0. \] (10)
The second part of the brackets of (9) disappears, as \( V^h (\theta) \) does not depend on the current \( \phi \). Rearranging (10) delivers:\(^{10}\)
\[ V_\phi^h (\gamma \theta) = \frac{1}{\lambda} \left[ 1 - \frac{r_\phi^h (\theta)}{\omega} \right]. \] (11)
The marginal valuation of R&D-investment is a perpetual flow equal to one minus the revenue changes caused by \( \phi \), discounted by the probability of successful R&D-investment.

**Home Production**  Differentiating the right hand side of (9) with respect to \( k^h \), we obtain
\[ \pi_{k^h} (\theta) + \lambda \left[ V_{k^h}^h (\gamma \theta) - V_{k^h}^h (\theta) \right] = 0 \]
\[ \frac{r_{k^h}}{\omega} (\theta) + \lambda \left[ V_{k^h}^h (\gamma \theta) - V_{k^h}^h (\theta) \right] = 0 \]
\[ V_{k^h}^h (\gamma \theta) = V_{k^h}^h (\theta) - \frac{1}{\lambda} \left[ \frac{r_{k^h}^h (\theta)}{\omega} \right]. \] (12)
The subscripts unequal to \( t \) stand for the partial derivation. For simplicity, in the following cases the derivation subscripts are shortened to \( h \) for the derivation with respect to capital invested in home production (respectively \( p \) for investment in FPI, \( d \) for investment in FDI) instead of \( k^h \) (respectively \( k^p, k^d \)). The marginal valuation of production-investment, in the case of R&D-investment equals the marginal valuation of production-investment with no R&D-investment minus the marginal revenue stream resulting from increased capital in production - discounted with the probability of successful R&D-investment. It is \( V_{k^h}^h (\theta) \) minus the revenue stream, as the valuation of \( k^h \) in case of additional investment in R&D is examined. Analysing just the valuation of \( k^h \) without the increased productivity would be \( V_{k^h}^h (\theta \gamma) \) plus the revenue stream.

An optimal strategy requires that the marginal valuation of investment in production equals the marginal valuation of R&D-investment. We can derive an explicit marginal valuation for investment in production by equating (12) and (11), namely
\[ V_{k^h}^h (\theta) = \frac{1}{\lambda} \left[ 1 + \frac{r_{k^h}^h - r_\phi^h}{\omega} \right]. \] (13)
Similar to (11) the marginal valuation of investment in production equals a flow consisting of one plus the difference between the revenue change caused by the two investment decisions. Again, this flow is discounted by the probability of successful R&D investment.

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\(^{10}\)For mathematical details see Appendix B.
There is a trade-off between investing in R&D or not. First of all, investing in R&D reduces the capital available to invest in domestic production. This effect is negative. But secondly, R&D-investment increases productivity and higher productivity enforces the output of the employed production-capital and decreases the variable production costs \( \frac{2}{p} \). Hence, there is also a positive effect of R&D-investment on the marginal valuation of capital invested in home-production. These considerations are reflected in the second part of (13).

### 3.2 Home and Foreign Portfolio Investment

#### 3.2.1 Production and FPI

Now, we analyse the investment decision of the firm and give it an additional investment alternative, namely foreign portfolio investment (FPI). With FPI equation (3) changes to

\[
K_t = k_h^t + k_p^t + \phi_t. \tag{14}
\]

This shows that the total capital available to a firm can be used to invest in domestic production, R&D-investment (the same as in the scenario above) and additionally \( k_p^t \) is the capital invested in FPI. As the firm invests in FPI, it gains ownership on a foreign firm. But the domestic firm has no - or only infinitely small - possibility to exert control over the foreign production and management process. Thus the domestic firm can not directly influence the foreign revenue and the gained dividend

\[
r_t^p = \mu_t \left( k_p^t \right). \tag{15}
\]

\( \mu_t \) is the return rate from FPI (or the productivity of capital invested in FPI). It varies with

\[
\frac{d\mu}{\mu} = \sigma_\mu dz \mu \tag{16}
\]

where \( dz \) is a Wiener process with mean zero and unit variance. Following (15) and (16), the only impact the home firm has on the foreign investment, is the decision of how much capital to invest in FPI.

Investment in FPI requires to buy assets, time to select the appropriate assets, additional administration systems and efforts etc. All these efforts are summarized as fixed costs \( f_p^t \) for this investment. Yet the profit function for the firm (5) changes to

\[
\pi_t (\theta_t) = p_t \phi_t \left( k_h^t \right)^\alpha - \left\{ f_h^t + \frac{2 h}{\theta_t} + \phi_t \right\} + r_t^p - f_p^t. \tag{17}
\]

Following the steps in the home-scenario we get the multi-period optimization problem for the firm

\[
\rho V^p (\theta_t) dt = \max_{k_h^t, k_p^t, \phi_t} \pi_t (\theta_t) dt + E_t (dV^p) \tag{18}
\]

subject to (2), (4) and (14) - (16). As the firm is now in the FPI-scenario, the superscript changed to \( p \) and there is one more control variable, namely \( k_p^t \). The
expected future capital flow depends on two state variables $\theta_t$ and $\mu_t$:

$$dV^p = V^p_\theta d\theta + V^p_\mu d\mu + \frac{1}{2} V^p_{\theta\theta} (d\theta)^2 + V^p_{\theta\mu} (d\theta) (d\mu).$$  \hspace{1cm} (19)

Thus in case of FPI investment, the expectation of the change in the expected capital flow consists of three parts

$$E(dV^p) = \lambda [V^p(\gamma\theta) - V^p(\theta)] dt + \left[\frac{1}{2} \mu^2 \sigma^2 V^p_{\mu\mu} \right] dt + \left[V^p(\gamma\theta)(\sigma_{\mu\mu}) \eta^p \right] dt.$$  \hspace{1cm} (20)

The first part is similar to the expected capital flow in the Home-scenario. Additionally, the variations of the foreign return impacts $V^p$. This impact occurs in the second term. Finally, the third term accounts for common variations of home productivity and foreign productivity that can result from global or industry shocks. The direction of this correlation depends on $\eta^p \equiv (dq)(dz_\mu) \neq 0$. If the firm invests FPI in the East, $\eta^p$ is positive and is negative with FPI in the southern country.

In case of FPI, the present value of the firm profit flows is

$$\rho V^p(\theta) = \max_{k^h, k^s, \phi, \rho} \left[ \pi^p(\theta) + \lambda [V^p(\gamma\theta) - V^p(\theta)] + \varepsilon \right]$$  \hspace{1cm} (21)

with $\varepsilon \equiv \varepsilon^1 + \varepsilon^2$, $\varepsilon^1 \equiv \frac{1}{2} \mu^2 \sigma^2 V^p_{\mu\mu}$ and $\varepsilon^2 \equiv V^p_{\mu\theta}(\gamma\theta)(\sigma_{\mu\mu}) \eta^p$. The uncertain foreign productivity influences the present value of the profit flows twice. Firstly, the isolated variation of the foreign productivity $\varepsilon^1$ enters the capital flows and secondly, the common variation of the home and foreign productivities $\varepsilon^2$ changes the capital flows. Whereas, the home productivity change is a discrete shock and $\varepsilon$ is continuous. Similar to (9), all necessary information for any decision are summarised in $\theta$ and $\mu$. Further, any optimality of future decision on FPI, home production or R&D-investment is independent of the firms’ initial decision.

### 3.2.2 Optimality Conditions with FPI

**R&D-Investment**  With FPI the marginal valuation of R&D-investment changes to

$$V^p_\phi(\gamma\theta) = \frac{1}{\lambda} \left[ 1 - \frac{\Phi^h}{\omega} - \Phi \right]$$  \hspace{1cm} (22)

where $\Phi = \frac{\partial(V^p_\mu + \frac{1}{2} \mu^2 \sigma^2)}{\partial \phi} + \frac{\partial(V^p_\mu(\gamma\theta)(\sigma_{\mu\mu}) \eta^p)}{\partial \phi}$. FPI does not have any direct impact on the R&D-investment. In comparison to the pure Home-scenario, the marginal valuation of R&D-investment is reduced by $\Phi$. This effect arises through the common variation of the home and foreign productivities. If the firm invests into closely related industries or even in the same industry (eastern country) then the own risk is not reduced. Thus $\Phi$ is positive and reduces the marginal valuation of R&D-investment slightly but never completely compensates it. Contrary, with investment in a dissimilar industry (South) the risk of R&D failure is diversified. $\Phi$ is negative and increases the valuation of R&D-investment.
Home Production  The direct valuation of home production is unchanged

\[ V^p_h (\gamma \theta) = V^h_h (\theta) - \left[ \frac{1}{\lambda} \frac{r^h_h (\theta)}{\omega} \right]. \] (23)

Following the optimality principle, we can equate the marginal valuation of investment in home production with the marginal valuation of R&D-investment and get

\[ V^p_h (\theta) = \frac{1}{\lambda} \left[ 1 + \frac{r^h_h - r^h_p}{\omega} - \Phi \right]. \] (24)

Similar to (22), the valuation changes by \( \Phi \). Again, the change depends on the industry invested in.

FPI  Optimalty requires that the marginal valuation of FPI also equals the marginal valuation of investment in home production and R&D-investment. Therefore we differentiate (21) with respect to \( k^p \) rearranging delivers

\[ V^p_p (\gamma \theta) = V^p_p (\theta) - \frac{1}{\lambda} \left[ r^p_p + \varepsilon_p \right]. \] (25)

Valuation of FPI is lower with investment located in the East (similar production and cost structure, \( \varepsilon_p > 0 \)) than with investment located in the South (different factor endowment, production and cost structure \( \varepsilon_p < 0 \)). Obviously, the diversification of the risk increases the valuation of the investment abroad.

3.2.3 FPI vs Home

The results from deriving all optimality conditions for FDI are summarized in table 1:

<table>
<thead>
<tr>
<th></th>
<th>( V^p_p (\gamma \theta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D</td>
<td>( \frac{1}{\lambda} \left[ 1 - \frac{r^h_h}{\omega} - \Phi \right] )</td>
</tr>
<tr>
<td>Home</td>
<td>( \frac{1}{\lambda} \left[ \frac{\omega r^p_p - r^h_h - r^h_p}{\omega} - \Phi + \varepsilon_p \right] )</td>
</tr>
<tr>
<td>FPI</td>
<td>( \frac{1}{\lambda} \left[ \frac{r^h_h - \omega r^p_p - r^h_p}{\omega} - \Phi - \varepsilon_p \right] )</td>
</tr>
</tbody>
</table>

Table 1: Optimality Conditions under FPI

Table 1 shows that the effect of FPI on the marginal valuation of investment in home production and R&D is twofold. Additional capital invested abroad reduces capital available for domestic production and R&D-investments. A further effect arises through the exploitation of risk diversification possibilities, \( \Phi \). Investment into countries with closely related industries (East) diminishes the valuation of domestic production, \( \Phi > 0 \). Similar sources of risks are added. Investments in dissimilar countries (South) push the valuation slightly up, \( \Phi < 0 \). In this case, FPI constitutes a hedging instrument for the existing R&D-risk.
Finally, the additional variation of a further unit capital invested in FPI, \( \varepsilon_p \), impacts the valuation of home production. At the same time, \( \varepsilon_p \) affects the valuation of FPI in the opposite direction. The marginal valuation of home production increases with further FPI in the East, \( \varepsilon_p > 0 \) and decreases with additional southern FPI. Eastern FPI delivers additional variation and risk. Home production is valued higher as it is a more secure source of future capital flows.\(^{11}\) FPI in the South hedges existing home risk and consequently the valuation of home production decreases, \( \varepsilon_p < 0 \).\(^{12}\) Additional southern FPI dampens the R&D risk and enforces further R&D investments. The firm withdraws capital from home production and invests the available capacities into southern FPI.

Hence, with isolated investment possibilities the firm will engage in southern FPI.

### 3.3 Home and Foreign Direct Investment

In the case of FDI, the home firm takes ownership as well as control over the foreign firm and thus can influence the profit of its FDI-investment. In the present paper, the firm only transfers capital to the foreign firm. No intermediate goods are traded. However, the choice of the FDI receiving country has a significant impact on the valuation of FDI.

If the home firm decides for FDI it also transfers intangible assets, as for example managerial skills, technology..., to the foreign firm. As a side effect of this asset transfer a part of the home productivity directly enters the return of FDI

\[
r_I^d = \psi \theta \frac{\bar{\sigma}_\theta}{\theta} (k^d_t); \quad 0 < a < 1. \tag{26}
\]

Home productivity \( \theta \) does not impact the foreign investment to the same extent, than home production. This can be caused by country specific conditions or incomplete mobility of some home skills.\(^{13}\) The impact of home productivity on FDI return increases with country similarity. \( \psi \) is the foreign productivity which is stochastic and varies with

\[
d\psi = \psi \sigma \psi d\bar{\psi} \tag{27}
\]

Again, \( d\bar{\psi} \) is Wiener process with mean zero and unit variance. The amount of capital invested in FDI is \( k^d \). Hence equation (3) becomes

\[
K_t = k^h_t + k^d_t + \phi_t. \tag{28}
\]

Further, FDI requires some specific up-front costs like country and market research, a merger or building a new plant. All these activities are costly and summarized in \( f^d \), as the fixed costs arising from FDI. Now the modified profit function of the home firm is

\[
\pi_t (\theta) = p_t \theta \left( k^h_t \right)^\alpha - \left\{ f^h + \frac{x^h_t}{\theta} + \phi_t \right\} + \frac{\bar{r}_t}{\omega} f^d + f_t^d. \tag{29}
\]

\(^{11}\)FPI valuation decreases through the same effect.

\(^{12}\)In this case, the valuation of FPI increases.

\(^{13}\)With \( a \to \infty \), the FDI scenario would be the same than the FPI scenario.
It is important to keep in mind, that the FDI fix costs, $f^d$ exceed the FPI fix costs, $f^p$.

The dynamic optimization problem of the home firm is

$$\rho V^d (\theta) dt = \max_{k^h, k^d, \phi} \left[ \pi^d (\theta) dt + E_t (dV^d) \right]. \quad (30)$$

Equation (30) is a function of the state variables home productivity $\theta$ as well as foreign productivity $\psi$. The control variables are the three investment purses, $k^h, k^d, \phi$. The derivation of the functional equation from (30) is analogue to the steps in the FPI-scenario. Thus, we get

$$\rho V^d (\theta) = \max_{k^h, k^d, \phi} \left[ \pi^d + \lambda \left( V^d (\gamma \theta) - V^d (\theta) \right) + \kappa \right] \quad (31)$$

with $\kappa \equiv \kappa^1 + \kappa^2$, $\kappa^1 \equiv \frac{1}{2} \sigma^2 \psi^2 V^d_{\psi^2}$, $\kappa^2 \equiv V^d (\psi \sigma^2) (\gamma \theta) \eta^d$ and $\eta^d \equiv (dz \sigma)(dq)$.

Analogue to the FPI scenario, the uncertainty of the foreign productivity has two impacts on the present value of the profit flows: the variation of the foreign productivity $\kappa^1$ and the common variation of the foreign and the home productivity $\kappa^2$. All necessary information for any decision is included in $\theta$ and $\psi$.

### 3.3.1 Optimality Conditions with FDI

**R&D-Investment** Following the same steps as in the two previous scenarios we get the marginal valuation of additional R&D-investment

$$V^d (\gamma \theta) = \frac{1}{\lambda} \left[ 1 - \frac{r^h_{\phi}}{\omega} \phi - \xi \right]. \quad (32)$$

First, there is a additional impact of FDI on the marginal valuation of R&D-investment. It is a very small positive effect through a slight increase in the foreign revenue. In comparison to the isolated home-scenario, this marginal change in $r^d$ again increases the marginal valuation of R&D-investment.

Secondly, the influence of $\phi$ on the foreign productivity is included in $\xi \equiv \frac{\partial \kappa^1}{\partial \phi} + \frac{\partial \kappa^2}{\partial \phi}$. The sign of $\xi$ is not definite. The degree ($\frac{1}{2}$) of the home productivity influence on foreign revenue is decisive for $\xi$.

**Proposition 1** If $a$ is sufficient high (low control over foreign firm - low impact of $\theta$ on $r^d$), then $\xi$ in the case of eastern FDI $\xi < 0$ and with FDI in the South $\xi > 0$.

The overall effect of eastern FDI increases the valuation of domestic R&D-investment, southern FDI decreases it. Horizontal FDI is mostly undertaken among industrial countries (East). For these countries, production structure, factor endowments and business-culture are relatively similar. Thus, increased productivity at home transfers very easily to the foreign affiliate. On the other hand, technology transfer with horizontal FDI in countries with differing production and cost structures is rather complicated and depends strongly on the
cost structure of the different countries.\textsuperscript{14} Therefore, the implementation of new technologies - developed for domestic production - is not as easy with FDI in the South as in the case of eastern FDI.

**Proposition 2** If $a$ is sufficient low (high control over foreign firm - high impact of $\theta$ on $r^d$), then $x$ in the case of eastern FDI $x > 0$ and with FDI in the South $x < 0$.

A strong control position and high skills facilitate the entering of new markets even in the south. High control is not necessary for FDI in a very similar investment location.

**Home Production** As expected from the previous section, home production stays unchanged again

$$V^d_h(\gamma \theta) = V^d_h(\theta) - \frac{1}{\Lambda} \left[ \frac{r^h_h(\theta)}{\omega} \right]. \quad (33)$$

Substituting equation (32) into the marginal valuation of investment in home production delivers

$$V^d_h(\theta) = \frac{1}{\Lambda} \left[ 1 + \frac{r^h_h - r^h_\phi - \omega r^d_\phi}{\omega} - x \right]. \quad (34)$$

The changes in $\phi$ affect directly the FDI revenue and indirectly the variations of the productivity of FDI. The reduction of the marginal valuation of the investment in home production is not as high as under FPI. In the current case, R&D-investment does not only diminish the capital available for FDI it also increases the productivity of capital invested in the foreign firm. Further, the sign of $x$ depends on the FDI location.

**FDI** To derive the optimality condition for FDI, we differentiate (31) with respect to $k^d$. This yields

$$V^d_d(\gamma \theta) = V^d_d(\theta) - \frac{1}{\Lambda} \left[ r^d_d + \kappa_d \right]. \quad (35)$$

Equation (35) shows that the marginal valuation of FDI in case of successful R&D-investment depends again on the FDI location. If the firm invests in eastern FDI then the term in the brackets remains positive and hence reduces the valuation. On the other hand, if the firm undertakes southern FDI the sign of $\kappa$ changes. But the indirect effect through $\kappa$ is weaker than the direct effect of the changed revenue. Thus the valuation is still reduced but not as much as in the case of FDI in the East. Generally, we observe a decreasing marginal product of capital either invested in domestic production or invested in foreign production. With a negative correlation between domestic and foreign productivity at hand, the decrease of the marginal product invested in FDI is damped.

\textsuperscript{14}Grossman, Helpman and Szeidl (2003) show that under different cost structures in the observed countries, firm strategies changes from horizontal to vertical FDI and vice versa.
### 3.3.2 FDI vs Home

Table 2 summarizes the optimality conditions with FDI:

<table>
<thead>
<tr>
<th>Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D</td>
<td>( V^d_\phi (\gamma \theta) = \frac{1}{\bar{\alpha}} \left( 1 + \frac{\omega r^d_{d} - r^h_{d} - \kappa}{\omega} \right) )</td>
</tr>
<tr>
<td>Home</td>
<td>( V^d_h (\gamma \theta) = \frac{1}{\bar{\alpha}} \left( 1 + \frac{\omega (r^d_{d} + r^d_{h}) - r^h_{d} - \kappa}{\omega} + \kappa_d \right) )</td>
</tr>
<tr>
<td>FDI</td>
<td>( V^d_d (\gamma \theta) = \frac{1}{\bar{\alpha}} \left( 1 + \frac{r^h_{d} + \omega (r^d_{d} - r^d_{h}) - r^h_{d} - \kappa}{\omega} - \kappa_d \right) )</td>
</tr>
</tbody>
</table>

Table 2: Optimality Conditions under FDI

As discussed above, FDI pushes the marginal valuation of R&D-investment (row one of table 2) up. This effect depends on the impact degree of the domestic productivity on the foreign revenue. With low domestic impact on \( r^d \) FDI in the East increases the valuation of R&D. Southern FDI decreases it. High domestic impact on \( r^d \) has the opposite effects on R&D-valuation.

Further, these effects carry over to the valuation of investment in home-production with respect to R&D-investment. From the second row of table 2, we see more closely how the valuation of capital invested in home production depends on the different effects of FDI. Additional capital invested abroad decreases the marginal revenue of FDI no matter if the firm undertakes southern or eastern FDI. This effect increases the valuation of investment in domestic production. Similar to R&D, FDI impacts the valuation of domestic production indirectly by the common variation of foreign and home productivity \( \kappa \).

The last parameter in the home valuation stands for the variation of one additional unit capital invested in FDI. Analogue to the FPI scenario, FDI in the East adds additional variations. The valuation of home production increases with further eastern FDI. In this case, home production is a very close substitute for FDI and even a more secure source for future capital flows. FDI in the south adds variations not common to the home variations. Thus, home production is not a close substitute for southern FDI as additional - even though only minor - gains on risk diversification arise with southern FDI.

The marginal valuation of FDI with respect to R&D (row three, table 2) equals a perpetual flow of the difference of changed revenues through additional capital invested in FDI and R&D, discounted by the probability of successful R&D-investment. The hedging components impact the FDI valuation in the same way as the R&D-valuation.

Finally, with isolated FDI the preferred location depends on the impact degree of home productivity on foreign revenue - or rather on the control degree of the investor. High control investors prefer southern FDI and low control investors engage in eastern FDI.

### 3.4 Home and Combined International Investment

Finally, we analyse a combined international investment strategy for the firm. Besides the usual home activities of the firm, it invests in FPI as well as in FDI.
at the same time. Because there are four different investment alternatives for capital, (3) changes to
\[ K_t = k^h_t + k^p_t + k^d_t + \phi_t. \] (36)
The return functions of the international investments are similar to the return functions under isolated international investment. Hence, the firms’ profit function with combined international investment is\(^{15}\)
\[ \pi^c_t(\theta_t) = p_t\theta_t\left(\frac{k^h_t}{\theta_t}\right)^\alpha - \left\{ f^h_t + \frac{x^h_t}{\theta_t} + \phi_t \right\} + r^p_t - f^p_t + r^d_t - f^d_t. \] (37)
and the dynamic firm problem is:\(^{16}\)
\[ \rho V^c(\theta) dt = \max_{k^h, k^p, k^d, \phi} [\pi^c(\theta) dt + E_t(dV^c)]. \] (38)
The control variables in the dynamic combined optimization problem are the various investment purposes: investment in domestic production \(k^h\), R&D-investment \(\phi\) and the two international investment alternatives FPI \(k^p\) and FDI \(k^d\). Further, in the combined scenario the present value of the firms capital flows is a function of the three state variables: home productivity \(\theta\), productivity of the portfolio investments \(\mu\) and the productivity of the direct investment \(\psi\). These three variables summarize all the necessary information for an optimal investment-decision in the present period. We need the functional equation of the optimizing problem (38) to derive the optimality conditions. Again, the steps are very similar to the isolated investment strategies and therefore, we neglect them and directly turn to the functional equation
\[ \rho V^c(\theta) = \max_{k^h, k^d, \phi} [\pi^c + \lambda [V^c(\gamma\theta) - V^c(\theta)] + \varepsilon + \kappa + \xi] \] (39)
where \(\xi \equiv V^c_{\mu\psi}(\mu, \sigma, \mu, \psi)\eta^c\) and \(\eta^c \equiv (dz_\mu)(dz_\psi)\). In (39) we have the investment effects of the isolated international strategies combined. Additionally, the common variation of the two international investments is included through \(\xi\).

3.4.1 Optimality Conditions with Combined International Investment

**R&D-Investment** Following (39), the optimality condition for R&D-investment changes slightly in comparison to the isolated scenarios:
\[ V^c_\phi(\gamma\theta) = \frac{1}{\lambda} \left[ 1 - \frac{\lambda^h - \lambda^d}{\omega} - \Phi - \kappa - \xi_\phi \right]. \] (40)
The first part of the bracket stays unchanged. Also, the isolated effects of the different investment possibilities, \(\Phi\) and \(\kappa\), are the same as above. But the
\(^{15}\)We have to keep in mind that \(f^d > f^p\) still holds.
\(^{16}\)We will keep the detailed transforming-steps very short as the necessary steps for the transformation are similar to the steps undertaken in the previous isolated section.
interaction of FPI and FDI changes the impact of the isolated investment effects. The only new term is $\xi_\phi$. Its impact depends on the international investment interaction, too. Table 3 summarizes the effects from the isolated strategies and adds the common effects in case of CII.

<table>
<thead>
<tr>
<th></th>
<th>impact FPI</th>
<th>impact FDI</th>
<th>impact FDI</th>
<th>common</th>
</tr>
</thead>
<tbody>
<tr>
<td>eastern country FPI / eastern FDI</td>
<td>$\Phi &gt; 0$</td>
<td>$\pi &lt; 0$</td>
<td>$\pi &gt; 0$</td>
<td>$\xi_\phi &gt; 0$</td>
</tr>
<tr>
<td>southern country FPI / southern FDI</td>
<td>$\Phi &lt; 0$</td>
<td>$\pi &gt; 0$</td>
<td>$\pi &lt; 0$</td>
<td>$\xi_\phi &gt; 0$</td>
</tr>
<tr>
<td>eastern country FPI / southern FDI</td>
<td>$\Phi &gt; 0$</td>
<td>$\pi &gt; 0$</td>
<td>$\pi &lt; 0$</td>
<td>$\xi_\phi &lt; 0$</td>
</tr>
<tr>
<td>southern country FPI / eastern FDI</td>
<td>$\Phi &lt; 0$</td>
<td>$\pi &lt; 0$</td>
<td>$\pi &gt; 0$</td>
<td>$\xi_\phi &lt; 0$</td>
</tr>
</tbody>
</table>

Table 3: Impact of different International Investment Possibilities

From table 3, we can emphasize two cases. The first case is a domestic firm with low influence on the foreign revenue (or low productivity). According to table three, a combined strategy with FPI in an southern country and FDI in the East dominates all other investment combination with respect to the impact of additional R&D-investment on the firms capital flows. With FPI in an unrelated country, the firm secures risk diversification and with eastern FPI the technology transfer is facilitated. Further, both investment possibilities are negatively correlated and this pushes the marginal valuation of R&D-investment additionally.

The second case is a firm with high influence on the foreign revenue (or high productivity). The preferred FPI location stays unchanged; whereas FDI switches to the South. As in the former case, FPI still serves as diversification instrument for domestic risk. It does not hedge FDI-location risk anymore. But with the increasing domestic productivity and its higher impact on foreign revenue, the remaining share of FDI location specific risk diminishes.

**Home Production**  Analogue to the isolated investment possibilities, the impact of home production does not change

$$V^c_h(\gamma \theta) = V^c_h(\theta) - \frac{1}{\lambda} \left[ \frac{r^{hc}_h(\theta)}{\omega} \right].$$  \hspace{1cm} (41)

In combination with the marginal valuation of R&D-investment, the impact of CII on the home production valuation becomes clear

$$V^c_h(\theta) = \frac{1}{\lambda} \left[ 1 + \frac{r^{hc}_h - r^{dc}_\phi}{\omega} - \pi - \Phi - \xi_\phi \right].$$  \hspace{1cm} (42)

We see from table 3, that the optimal investment combinations with respect to R&D-investment are the optimal combinations with respect to the valuation of home production in combination with R&D. But we still cannot generalize this optimal investment combination.
First, we have to examine the effects on the various international investments and the combination of all effects. As there are all derived similarly to the isolated strategies, table 4 just summarizes the results.

\[
V^c_\phi (\gamma \theta) = \frac{1}{\lambda} \left[ 1 - \frac{r^{hc}_e - r^{dc}_d}{\omega} - \Phi - \kappa - \xi_\phi \right]
\]

\[
V^c_h (\gamma \theta) = \frac{1}{\lambda} \left[ 1 + \frac{r^{hc}_e - r^{dc}_d - r^{hc}_h - r^{hc}_e}{\omega} - \kappa - \Phi + \kappa - \xi_\phi + \xi_d \right]
\]

\[
V^c_d (\gamma \theta) = \frac{1}{\lambda} \left[ 1 + \frac{r^{hc}_e - r^{dc}_d + \omega (r^{dc}_d - r^{dc}_p)}{\omega} - \kappa - \Phi - \xi_\phi - \xi_d \right]
\]

\[
V^c_p (\gamma \theta) = \frac{1}{\lambda} \left[ 1 + \frac{r^{hc}_e - r^{hc}_p + \omega (r^{dc}_d - r^{dc}_p)}{\omega} - \kappa - \Phi - \xi_\phi - \xi_p \right]
\]

Table 4: Optimality Conditions under CII.

From table 4, we see that each marginal valuation increases with negative correlation of the home industry and the chosen industry for FPI ($\Phi < 0$). The risk of unsuccessful R&D-investment at home can be propped up by the short term portfolio-investment.

To detect the preferred FDI location, again we have to distinguish two cases. With low productivity and low control, $\kappa < 0$ pushes the respective valuation up. This negative sign for $\kappa$ arises under eastern FDI. Overall, with a facilitated technology transfer under eastern FDI, R&D impacts the expected capital flows of the firm positively. The variation effects between the international investments ($\xi_\phi$, $\xi_p$, $\xi_d$) either support or decrease the strength of the direct impact but never compensates it. Hence, both international investments are mostly favoured with FPI in South and FDI in the eastern country. For FPI, the risk diversification is the stronger effect with the highest impact on the firm decision. In particular, FPI is the more flexible investment and can be adjusted almost costless. Therefore, it is the appropriate instrument to diversify a firms’ risk. On the other hand, FDI reacts more sensitive to productivity changes and thus, is the favourable instrument to exploit productivity gains internationally.

With high domestic control over the foreign firm the preferred FDI location switches from the East to the South. High control (low $a$) reduces the share of location or industry specific risk and facilitates the technology transfer from home to the South.\(^{17}\) FPI looses its function of direct-hedging FDI location specific risk. But, FPI still works as hedging instrument for R&D risk. With the increasing domestic control - and therefore higher impact of $\theta$ on the FDI revenue, this role even gains on importance. Higher domestic productivity requires higher R&D-investments and this in turn stipulates a more intensive risk hedging. Concluding, FPI looses its impact as direct hedging instrument with respect to FDI but with respect to domestic production and thus indirectly to FDI, the hedging necessity increases.

In CII, FPI can prop up the risk from home production and FDI. The relations between the home country and the recipient countries are unchanged to the isolated investment scenarios for FPI as well as for FDI. Hence, we expect\(^{17}\) A high home productivity $\theta$ has the equivalent consequences.

\(^{17}\)
in CII the share of FPI to adjust to short-term environment changes whereas FDI stays unchanged. It is not possible to derive an explicit analytical solution for the respective international investment shares. The definite shares of FPI and FDI will be derived numerically.

4 Optimal Investment Strategies

As for both FDI investor scenarios - low and high control on the foreign firm, the results emphasize that FPI works as diversification instrument and the firm uses FDI as a technology transfer channel. These findings are valid for the isolated strategies as well as for the combined strategy. To proof or reject these findings clearly in the following analysis we consider FDI in the East and FPI in the South.

Unfortunately, the problem has no tractable closed form solution. Hence, the solution must be approximated by numerical methods. We use recursive policy function iteration. The first run computes the solution for the isolated international investment strategies and determines the cut-offs at which the firm changes from one strategy to another (home, FPI or FDI). In the second run, we repeat the same steps for the combined international investment strategy. Precisely, with CII the firm changes its strategy only once: from isolated home production to FPI and FDI at the same time.

We derive a benchmark case with a depreciation of $\tau = 0.3$. A higher depreciation pushes the start of international activity backwards in time and a lower depreciation pulls it forward. The general results stay the same. Further, the probability for successful R&D-investment varies and shows a significant impact on the firms’ decision to invest internationally or not.

4.1 Isolated International Investment

4.1.1 Start of International Activity

The first international activity of the firm is FPI. As expected, FPI requires lower cut-off productivity than FDI. However, the firm starts investing in FPI not until the probability of successful R&D is 0.3 or higher. Figure 1 shows for example the investment choices and changes for a R&D-success probability of $\lambda = 0.3$ and $\lambda = 0.5$. Even with $\lambda = 0.5$, the international activity starts very late in time. With increasing R&D-probability, the firm undertakes international investment at an earlier stage. This is very intuitive, as with high success-probability the productivity increases more quickly. All these results confirm the findings in the recent literature. Firms with low productivity stay isolated at the home market. With a slight increase in productivity the first small international steps are made and finally, firms with a remarkable high productivity invest in FDI.

\footnote{For detailed discussion and mathematical background see Adda and Cooper (...), Dixit and Pindyck (1994).}
4.1.2 Variation of Foreign Productivity

Firstly, we observe changes in the FPI productivity. Table 5 in Appendix C shows that neither the productivity cut off nor the cut-off time change with variations in FPI productivity. One might have expected that with higher foreign productivity the firm engages earlier in international investment. This is not the case. The firm first secures the home production process and then goes abroad.

Further, the firm does not reduce or increase its share in FDI. Only the FPI shares increase with higher FPI productivity. This might seem intuitive, as only the FPI-productivity changes. Hence, the FDI shares are independent of the FPI productivity. Let’s take a closer look on FDI-productivity changes, to see whether this independence also hold in the opposite direction and we can confirm FPI as the more flexible instrument.

Table 6 in Appendix C shows that with a high FDI-productivity the firm engages in international investment at an earlier stage in time, than with a lower FDI-productivity. Further, the productivity cut-off is lower than with the benchmark productivity. The only exceptions are the cases with a very high success probability of R&D investment. For these cases the cut-offs are the same as for the benchmark case and the high FDI-productivity.

Finally, with varying FDI-productivity both international shares change in comparison to the benchmark case. In particular, the FPI shares do not only vary in comparison to the benchmark case. They also change between the various cases of high FDI-productivity while the FDI shares stay almost the same. Again, only with the high R&D-probability the FDI shares change between the different cases, but they don’t change with respect to the benchmark case. So, we find again FPI as the flexible instrument adjusting to short-term changes while FDI reacts more sluggishly. These are only results for the isolated investment scenario.

4.2 Combined International Investment

In contrast to the isolated international investment strategy, the firm starts its international activities with both investment alternatives FPI and FDI at the
same time. Figure 2a shows that even with a low R&D-probability, the firm engages in its first international investment. However, we have to distinguish between the first international investment and the investment in FDI. In both cases, the first international activity under CII (CII FDI and FPI vs isolated FPI) takes place at an earlier date in time than the first international firm activity under an isolated international investment strategy. Additionally, the first international activity at all requires a lower R&D-probability under CII than for the isolated international investments.

At a moderate probability, the firm switches from home to international investment (isol. FPI and combined FPI-FDI respectively at the same time). With increasing probability the isolated investment even dominates the combined strategy in time. We have to keep in mind, that we are comparing the firm starting isolated FPI against the firm starting combined FPI and FDI under CII.

Now, we turn to the comparison of isolated FDI and the combined international investment.

For the switch to FDI the picture changes as shown in figure 2b. Under CII the firm switches from home production to international investment at a lower R&D-probability and at an earlier stage in time. Further, with increasing success-probability, CII still dominates the isolated investments in time.

But, CII does not always dominate isolated FDI in productivity. Particularly, the productivity cut-offs for FDI under CII do not always lie below the cut-offs for isolated FDI. The productivity cut-offs are analysed according to the $\theta$ at which the firm switches from one strategy (for example home) to another strategy (for example FDI). We compare the marginal impact of $\theta$ on the different discounted capital flows under a given productivity level. Figure 3 shows the different productivities for FDI cut off with isolated FDI (iso) and CII-FDI (cii).
The domination of the isolated investment strategy might be unexpected. CII implicates a higher incentive to invest in R&D. This in turn pushes domestic productivity up and also the cut off productivities.

4.2.1 Variation of Foreign Productivity

First of all, minor changes in the foreign productivities relation may diminish any international investment under isolated strategy completely. If both productivities are very low or at least the FPI-productivity is very low then the firm does not invest abroad. On the other hand, these changes do not reduce international investment under CII totally. The productivity cut-off changes as both foreign productivities drift apart (negatively correlated) or move together (positively correlated). The following figure 4 shows the variation of FPI and FDI shares in dependence on their productivity relation.

Precisely, the cut-off is lower with very different foreign productivities and
raises as the productivities converge. This is shown in figure 5.

If the foreign productivities move very close to each other then FPI does not hedge FDI specific risk anymore. Though, FPI gains on importance to hedge domestic productivity risk. This in turn increases again the incentives to invest in R&D and boosts domestic productivity up. Hence, productivity cut-offs are higher but FDI investment is less risky.

Overall, the share of FPI varies more thorough the changed productivities than the FDI shares. The latter are more or less stable. Additionally, FPI shares under CII fluctuate even more than under isolated international investment. Whereas, CII FDI is more stable than isolated FDI. Figure 6 shows the variation of both investments under the respective strategy with varying investment cut off conditions.

Hence with CII, the firm reacts to short-term changes in its environment by adjusting FPI and keeping FDI stable. Thus, FPI does not necessarily increase with FDI, but adjusts according to R&D-probability, depreciation and variation in home and both foreign productivities. These results confirm again the risk-adjusting task of FPI and the more sluggish technology transfer FDI instrument.
5 Conclusion

We show in a dynamic investment setting that the relation of FPI and FDI is rather complementary. Isolated FPI and FDI investments are compared to combined FPI and FDI investments. The combined investment strategy dominates the isolated investments always in time. Further, CII comprises a higher incentive to invest in R&D. The risk diversifying effect from additional FPI pushes the marginal valuation of R&D investment above the valuation with isolated investment strategies. As a consequence, home productivity increases much faster and without smaller relative opportunity costs than under isolated investment strategies. Finally, this leads to a higher productivity cut off for FDI but at an earlier date in time. The significant higher CII R&D investment than isolated FDI R&D investment confirms this observations. Surprisingly, this is not only the case with a combination of horizontal FDI in a country with similar structure and FPI in a country with dissimilar structure than the home country, but also with both international investments in a dissimilar country structure than the home structure.

Furthermore, we also find that firms adjust to short-term changes via FPI and keep FDI stable. FPI can prop up small and medium sized changes and therefore, the valuation of FDI with combined FPI is higher than of isolated FDI.

Hence, a combined FPI and FDI investment strategy increases the firms’ flexibility. The consequences are earlier international activity for medium sized as well as small sized firms and better adjustment to environment changes, especially trade liberalization.

6 References


7 Appendix

7.1 Appendix A

7.1.1 Derivation of Expected Capital Flow

The value of the firm in the case without international investment is a function of the state variable $\theta$ (productivity).

$$dV^h = V^h_{\theta} d\theta$$  \hspace{1cm} (43)

The state variable follows a Poisson process with $q = 1$ with prob. $\lambda dt$ and $q = 0$ with prob. $(1 - \lambda dt)$:

$$d\theta = \left(1 - \tau \frac{\phi}{K}\right) \theta dq$$  \hspace{1cm} (44)

$$\Rightarrow E(dV^h) = \lambda \left[V^h \left(\left(1 - \tau \frac{\phi}{K}\right) \theta \right) - V^h(\theta)\right] dt$$  \hspace{1cm} (45)$$

change of capital flow caused by increased $\theta$ weighted with the probability

$$+ \frac{(1 - \lambda dt) \left[V^h(\theta) - V^h(\theta)\right]}{(1 - \lambda dt) \left[V^h(\theta) - V^h(\theta)\right]}$$  \hspace{1cm} (46)

change of capital flow in the case of unchanged $\theta$ weighted with respective probability

$$\Rightarrow E(dV^h) = +\lambda \left[V^h(\gamma \theta) - V^h(\theta)\right] dt$$  \hspace{1cm} (47)

with

$$\gamma = \left(1 - \tau \frac{\phi}{K}\right)$$  \hspace{1cm} (48)

For a general discussion of Poisson processes in continuous time see Dixit and Pindyck (1994).

7.2 Appendix B

7.2.1 Derivation of the Profit Function with Variable Revenue

Domestic consumers have Dixit-Stiglitz preferences for differentiated goods with elasticity of substitution $\omega = \frac{1}{1+\phi} > 1$. The price index for the home country is

$$P = \left[\int_{j \in J} p(j)^{1-\omega} dj\right]^{\frac{1}{1-\omega}}$$  \hspace{1cm} (49)

and the demand level is

$$A = \left[\int_{j \in J} x(j)^{\phi} dj\right]^{\frac{1}{\phi}}.$$  \hspace{1cm} (50)

From (49) and (50) we derive the demand function

$$x_i = \lambda p_i^{-\omega}$$  \hspace{1cm} (51)
for each good variety produced by firm \( i \). In the following the firm index \( i \) is neglected, as we just analyse one representative firm.

According to (5) the profit of the firm in period \( t \) equals

\[
\pi_t(\theta_t) = r_t^h - J_t^h - \frac{x_t^h}{\theta_t} - \phi_t. \tag{52}
\]

Revenue equals supply multiplied by the price we can rearrange (52) to

\[
\pi_t(\theta_t) = r_t^h - f_t^h - \frac{px_t^h}{\theta_t} \frac{1}{p} - \phi_t \tag{53}
\]

\[
\pi_t(\theta_t) = r_t^h - J_t^h - \frac{r_t^h}{\theta_t} \frac{1}{v^h} - \phi_t \tag{54}
\]

\[
\pi_t(\theta_t) = r_t^h (1 - \varphi) - J_t^h - \phi_t.
\]

7.3 Appendix C

7.3.1 Foreign Productivity Variation

Isolated International Investment - FPI

<table>
<thead>
<tr>
<th>Period fpi</th>
<th>Productivity fpi</th>
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<th>Period fdi</th>
<th>Productivity fdi</th>
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</thead>
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<td>0.352</td>
<td>6</td>
<td>1.36</td>
</tr>
<tr>
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Table 5: Productivity Cut-Offs and Changing Investment Shares under FPI Productivity Variation
### Isolated International Investment - FPI

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Table 6: Productivity Cut-Offs and Changing Investment Shares under FDI Productivity Variation
## Combined International Investment - FPI

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Table 8: Productivity Cut-Offs and Changing Investment Shares under FPI Productivity Variation
Combined International Investment - FPI

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Table 9: Productivity Cut-Offs and Changing Investment Shares under FDI Productivity Variation