Risk management edition 1.12

Scenario-based asset allocation

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Contents

Foreword

Dear reader,

We live in a world full of risk and uncertainty. A plethora of factors affect the results of our actions, particularly when it comes to investment. Doing nothing is not an option, nor is only making decisions that are supposedly risk-free.

Pointers are needed for finding the right track when making decisions. Risk models are basic starting points, but they often reach their limits when extreme situations arise. 'Black swans' are more common than models would lead us to expect so decision theory and scenario techniques provide valuable assistance.

This year's Union Investment risk-management study was produced by Professor Arnd Wiedemann and Timo Six of Siegen University. It shows how scenario techniques are used in asset allocation and how an investor's personal attitude to risk can be factored into the optimisation process. Extreme fat-tail events can also be incorporated into the analysis.

Nevertheless, even the best techniques cannot relieve decision-makers of their task, and investors have to think carefully about their attitude to risk, define their investment objectives clearly and decide which scenarios are relevant to them. The study has devised a system for decisions that are often made intuitively—a unique way to create clarity and to give investors a sense of perspective, despite the uncertainty of the future.

I hope this study makes interesting reading.

Alexander Schindler

1 Introduction

The literature on finance over the past few decades has been dominated by the concept of risk/return-based asset allocation formulated by Harry Markowitz in the early 1950s.[1] In the last few years, however, this methodology has been substantially revised and refined, not least in response to the crisis events that have recently occurred in financial markets.

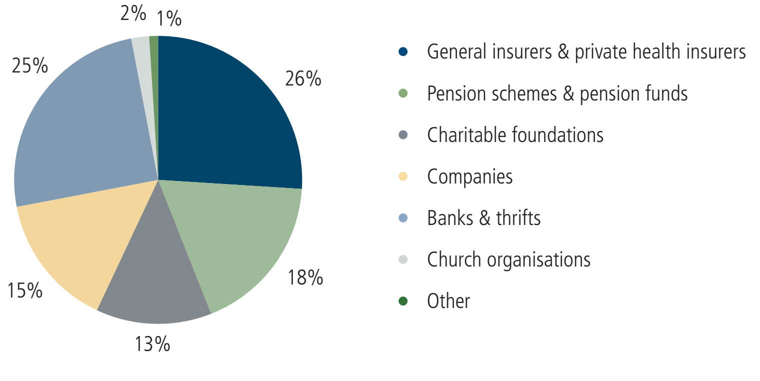
Investors are increasingly keen to be able, among other things, to factor extreme fat-tail events into their asset allocation decisions. The use of scenario techniques enables stress scenarios—in addition to realistic economic performance scenarios—to be factored into such decisions. Scenario-based asset allocation can therefore be an effective tool for constructing stress-resilient portfolios.

The objective of this paper is to illustrate how realistic economic performance scenarios and stress scenarios can be systematically integrated into the asset allocation process. It also demonstrates how investors' particular risk appetites can be factored into scenario-based asset allocation decisions and what impact these appetites have on the future performance of investment strategies.

To this end, chapter 2 starts by outlining the traditional asset allocation approach according to Markowitz and analyses its suitability as a method for determining portfolios' asset allocation based on historical data. Chapter 3 describes the scenario-based asset allocation process and presents alternative methods in detailed individual steps.

This research was undertaken in response to the latest findings of Union Investment's survey of institutional investors in Germany, which it has conducted every year since 2005. The 2013 risk inventory was based on a telephone survey of 104 investors during the period from 2 May to 21 June 2013. Figure 1 gives a breakdown of the sample used in this study.

Figure 1:
Breakdown of the sample used in the 2013 risk management study



The fairly even distribution of the sectors surveyed enables the findings to be representatively evaluated.

A standard example is used to illustrate all the strategies and concepts presented in this paper. The task for investors is to spread a certain investment amount across several asset classes. The investment options available to them are the seven asset classes shown in Table 1. Table 1 also shows the indices used to replicate these as-set classes

[1] See Michaud et al. (2013), p. 6.

Table 1: Historical annual returns and annual volatilities of asset classes

Asset classes	Average
Core eurozone sovereigns	BofA Merrill Lynch Euro Government x Greece x Ireland x Italy x Portugal x Spain
Peripheral eurozone sovereigns	BofA Merrill Lynch Greece, Ireland, Italy, Portugal, Spain Government
High-grade credits	BofA Merrill Lynch EMU Corporates
High-yield credits, ex financials	BofA Merrill Lynch Euro High Yield—Constrained/Non-Financials B–BB Fixed & Floating Rate
Emerging market sovereigns	J.P. Morgan EMBI Global Diversified
Global equities	MSCI AC World

The data history being analysed covers the period from 31 December 1997 to 30 August 2013 and is based on monthly price data. Table 2 shows the relevant figures for the seven asset classes. The asset classes' average rates of return—which have been aggregated to give annual values—are compared with their annual volatility. These figures clearly demonstrate that the individual rates of return differ substantially from one asset class to another. There are also considerable differences in the levels of annual volatility.

Table 2: Historical annual returns and annual volatilities of asset classes

Asset classes	Average annual return	Annual volatility
Core eurozone sovereigns	5.28%	3.91%
Peripheral eurozone sovereigns	4.80%	5.16%
High-grade credits	5.16%	3.46%
High-yield credits, ex financials	5.53%	12.61%
Emerging market sovereigns	9.60%	12.54%
Global equities	4.80%	15.90%
Commodities	3.48%	15.59%

In addition, Figure 2 shows the indexed historical performance of the seven asset classes over the period being analysed. The chart supports the figures shown in Table 2. It also clearly illustrates that the individual risk return profiles differ substantially from one analysed asset class to another. For example, the 'emerging market sovereigns' asset class comfortably outperformed all other asset classes during the period being analysed, although it also exhibits one of the highest volatilities.

Figure 2:
Historical performance of the asset classes analysed

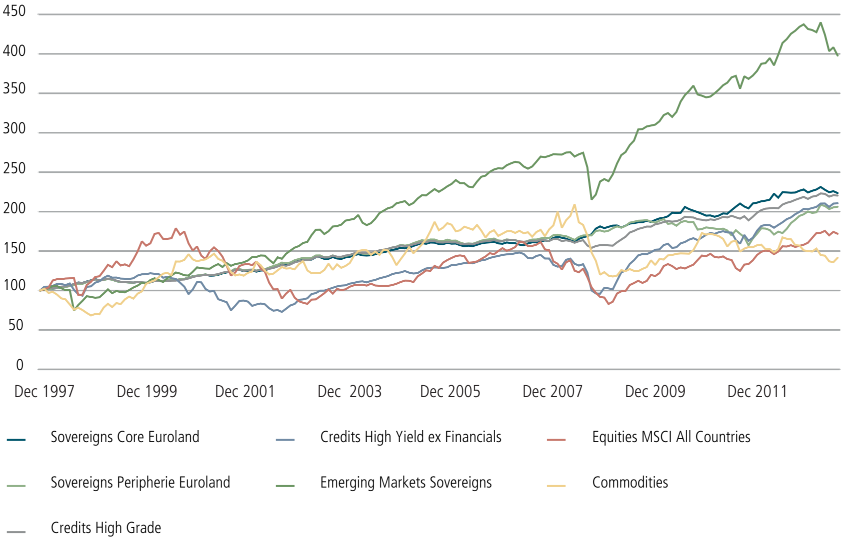


Figure 2 also enables us to draw one or two conclusions about the correlations between these asset classes. For example, the 'commodities' and 'global equities' asset classes are shown to be strongly correlated with each other, whereas there is very little sign of any correlation between these asset classes and the 'high-grade credits' asset class. This is especially clearly illustrated during the financial crisis of 2008. This 'optical' analysis of the data is quantitatively underpinned by the figures shown in Table 3.

Table 3:
Empirical correlation matrix

Correlation matrix	Core eurozone sover-eigns	Peripheral eurozone sover-eigns	High grade credits	High yield credits, ex financials	Emerging market sover-eigns	Global equities	Commod-ities
Core eurozone sovereigns	1	0.49	0.61	–0.16	0.04	–0.28	–0.20
Peripheral eurozone sovereigns	0.49	1	0.57	0.08	0.20	–0.11	–0.14
High-grade credits	0.61	0.57	1	0.38	0.35	0.11	0.03
High-yield credits, ex financials	–0.16	0.08	0.38	1	0.52	0.61	0.19
Emerging market sovereigns	0.04	0.20	0.35	0.52	1	0.26	0.21
Global equities	–0.28	–0.11	0.11	0.61	0.26	1	0.31
Commodities	–0.20	–0.14	0.03	0.19	0.21	0.31	1

Table 3 uses the Bravais-Pearson correlation coefficient to map the empirical correlation matrix for the asset classes being analysed. This coefficient has values ranging from -1 to 1. Whereas a correlation coefficient of 1 denotes a perfectly positive correlation, a correlation value of -1 indicates a perfectly negative correlation.[2] The greatest diversification effect during the period being analysed was achieved by euro-denominated sovereign bonds (core eurozone sovereigns), which were negatively correlated with global equities (-0.28), com-

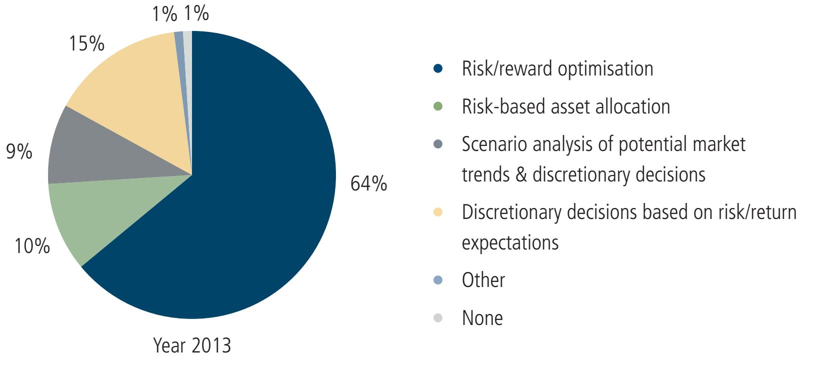
2 See Wiedemann (2013), p. 47 onwards.

modities (-0.20) and high-yield credits, ex financials (-0.16). There were also negative correlations between the 'peripheral eurozone sovereigns' asset class and the two asset classes of global equities (-0.11) and commodities (-0.14).

2 Traditional asset allocation according to Markowitz

More than two-thirds of all institutional investors surveyed as part of the 2013 risk inventory stated—as shown in Figure 3—that they use risk/return optimisation when making investment decisions. By contrast, only 15 per cent of investors make purely discretionary decisions based on their expected levels of risk and return. 10 per cent of investors use risk-based asset allocation, while 9 per cent apply the sort of scenario techniques that are discussed in this research paper. Although combinations and various gradations of these approaches are, of course, possible, this survey was unable to distinguish sufficiently accurately between these applications.

Figure 3:
What approach do you usually adopt when making asset allocation decisions?



Traditional risk/return optimisation is based on the portfolio selection theory published by Harry Markowitz in 1952.[3] This theory states that asset allocation approaches are based on the twin parameters of portfolio return (as measured by the expected value) and portfolio risk (as measured by the portfolio's volatility).

The expected portfolio return μ_P is determined as the average of the expected returns—weighted according to portfolio weights x_i—on the individual asset classes μ_i for N asset classes as shown below.[4]

$$\mu_P = \sum_{i=1}^{N} x_i \cdot \mu_i$$

By contrast, calculations of portfolio volatility also factor in risk-mitigating correlation effects, which arise whenever the returns on the asset classes being analysed are not perfectly positively correlated with each other. Portfolio variance σ_p^2 is determined as shown below, where $\sigma_{i,j}$ denotes the return covariance between asset classes i and j.[5]

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{i,j}$$

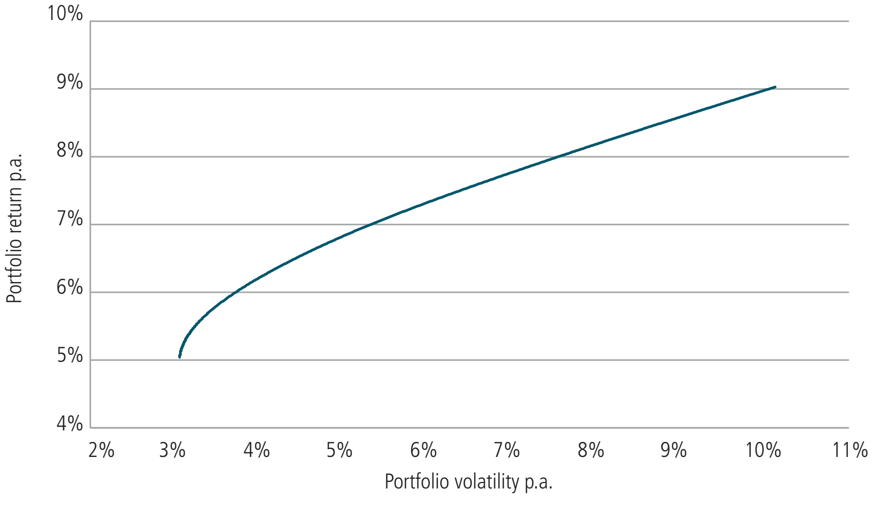
The previous section above showed that although the rates of return on the asset classes being analysed are mainly positively correlated with each other, they are by no means perfectly positively correlated. It therefore makes sense to factor in risk-mitigating correlation effects.

[3] See Markowitz (1952), p. 77 onwards.
[4] See Nöll/Wiedemann (2008), p. 147.
[5] See Markowitz (1952), p. 81.

If the expected portfolio return and the corresponding portfolio volatility are determined for all conceivable portfolio weights applied to the asset classes being analysed, the decision field can be reduced from the total set of possible asset allocation decisions to the set of efficient allocation decisions.[6] An asset allocation decision is deemed to be efficient if there is no other type of asset allocation that would either deliver a higher return for the same level of risk or would yield the same return for a lower level of risk.[7]

If the expected portfolio returns and the corresponding portfolio volatilities are plotted on a risk/return chart, this produces a curve that marks the efficient boundary of the set of all possible allocation decisions. This curve is referred to as the 'efficient frontier'.[8] Figure 4 shows the efficient frontier on an annual basis for the seven asset classes used in the example.

Figure 4:
Efficient frontier for the asset classes used in the example



Although the efficient frontier shown above restricts an investor's decision field to the set of efficient portfolios, it does not produce a decision in favour of a certain type of asset allocation. The optimisation models shown in Table 4 are traditionally used to identify a specific type of asset allocation on the efficient frontier.

6 See Markowitz (1952), p. 82.
7 See Focardi/Fabozzi (2004), p. 474.
8 See Markowitz (1952), p. 82.

Table 4:
Comparison of target functions and constraints of traditional risk/return optimisation

	Target function	Constraints
Model 1	Weight the asset classes so as to maximise the expected portfolio return: $\mu_P = \sum_{i=1}^{N} x_i \cdot \mu_i \rightarrow \max!$	1. $\sum_{i=1}^{N} x_i = 1$ 2. $x_i \geq 0$ 3. $\sigma_P^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{i,j} \leq$ upper limit
Model 2	Weight the asset classes so as to minimise the portfolio risk: $\sigma_P^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{i,j} \rightarrow \min!$	1. $\sum_{i=1}^{N} x_i = 1$ 2. $x_i \geq 0$ 3. $\mu_P = \sum_{i=1}^{N} x_i \cdot \mu_j \geq$ minimum return
Model 3	Weight the asset classes so as to maximise the expected mean-variance utility function: $\text{Utility} = \mu_P = \frac{\gamma}{2}\sigma_P^2 \rightarrow \max!$	1. $\sum_{i=1}^{N} x_i = 1$ 2. $x_i \geq 0$

Under Model 1, the asset classes x_i contained in the portfolio are weighted so as to maximise the expected portfolio return. This optimisation is subject to three constraints. The first constraint ensures that the portfolio's assets are fully allocated, while the second constraint bans negative weights and therefore does not allow short-selling. The third constraint places an upper limit on the level of portfolio risk permitted. This maximum tolerable risk is specified exogenously by the investor.

By contrast, Model 2 follows exactly the opposite sequence. A minimum expected return (third constraint) is specified exogenously, and the asset classes contained in the portfolio are weighted so as to minimise the portfolio risk. The first and second constraints are identical to those imposed by Model 1.

The same procedure applies to Model 3, which—with the exception of the first two conditions—does not impose any additional constraints. The utility function shown in Table 4 is based on the general utility function described below. The target value is based on a negative exponential utility function, which describes the utility in terms of the return r:[9]

$\text{Utility}(r) = 1 - e^{\gamma \cdot r}$

This utility function is strictly concave, which reflects investors' risk aversion. Coefficient γ is referred to as a 'risk aversion parameter'. The higher it is, the greater an investor's risk aversion is.[10] Freund (1956) demonstrated that the target function presented in optimisation model 3 (see Table 4) maximises the concave utility function described above.[11]

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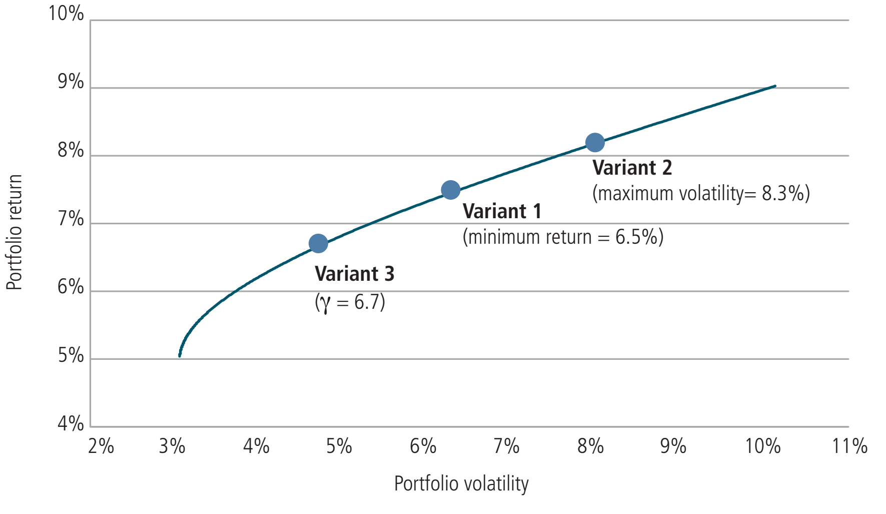
9 See Freund (1956), p. 255.
10 See Freund (1956), p. 255.
11 The methods used to arrive at this conclusion can be found in Freund (1956), p. 255.

Figure 5 illustrates the risk/return combinations of the asset allocation decisions reached on the basis of the three models on the risk/return chart. A maximum portfolio volatility of 6.5 per cent and a minimum expected return of 8.3 per cent were specified exogenously for Variant 1 and Variant 2 respectively. The value of risk aversion parameter γ required for Variant 3 is determined by how risk averse the investor is. A value of 6.7 is specified for γ in the example.[12] It is, of course, possible to generate the same optimum portfolio from all three models by selecting appropriate parameters.

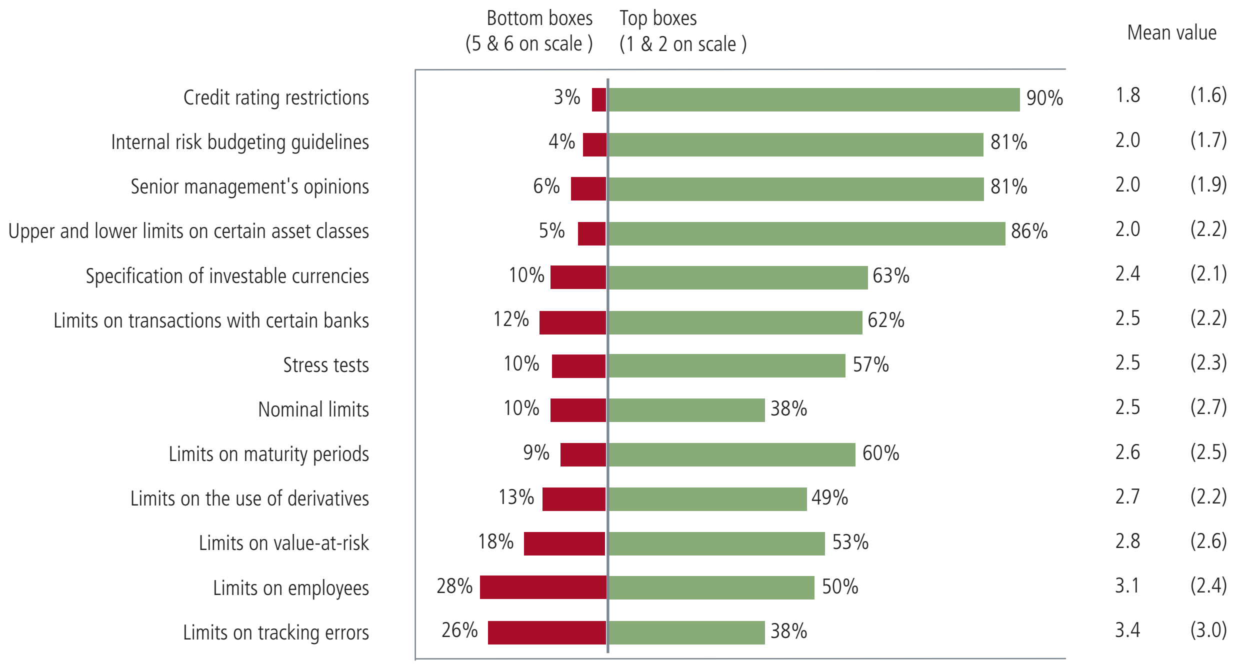
Figure 5:
Comparison of risk/return combinations of asset allocation decisions



In addition to the constraints mentioned above, the optimisation models may contain further investor-specific constraints. Figure 6 and Figure 7 illustrate the importance of internal and external restrictions on institutional investors based on the findings of the 2013 risk inventory.

[12] For the purposes of our example, this value was calculated from the historical data as the excess return on the market portfolio compared with the risk-free return in relation to the variance in the mar-ket portfolio.

Figure 6:
Importance of internal restrictions on institutional investors

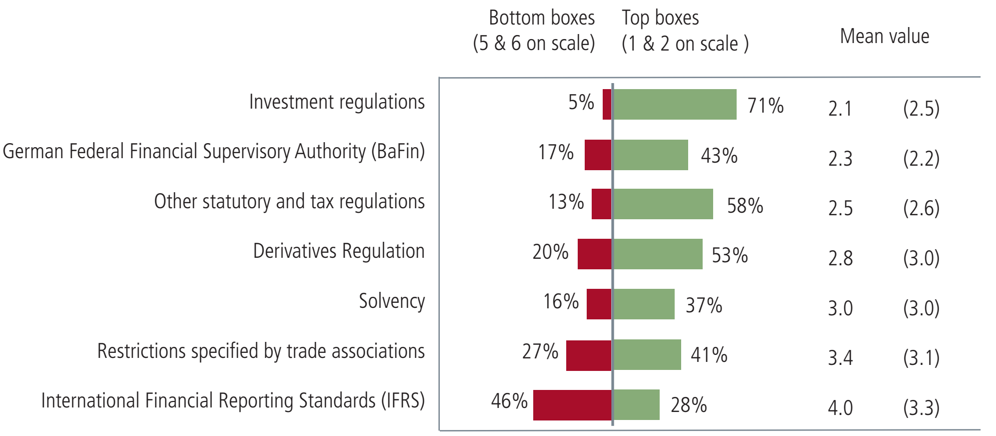


Question 7; all respondents (n = 104); scale of 1 = 'extremely important' to 6 = ' not at all important'; 2012 values in brackets

Credit rating restrictions, internal risk budgeting guidelines, senior management's opinions, and upper and lower limits on certain asset classes are the most important internal restrictions on institutional investors. By contrast, Figure 6 shows that the limits placed on value-at-risk, on employees and on tracking errors are much less important.

External legislation and regulations also limit institutional investors' freedom to make investment decisions. Figure 7 lists a number of external restrictions and shows the extent to which they impact on investment decisions.

Figure 7:
Importance of external restrictions on institutional investors



All respondents (n = 104); scale of 1 = 'extremely important' to 6 = ' not at all important'; 2012 values in brackets

Irrespective of how the target function and the pertinent constraints are configured in individual cases, the traditional form of risk/return optimisation devised by Markowitz has certain weaknesses. One significant drawback is that the results are largely determined by the inputs, which in the case of our example were calculated from historical data.[13] Even minor changes in these inputs—especially in the expected returns—can cause asset allocation decisions to vary substantially.[14] The results obtained purely from the optimisation methods devised by Markowitz are therefore not very robust.[15] What's more, these methods occasionally produce extreme forms of asset allocation in portfolios.[16]

A further problem with the traditional method of portfolio optimisation is the return-distribution assumptions on which it is implicitly based. What this means in practice is that this method only analyses the first two moments of any distribution, i.e. the ex-pected value and the variance, while higher moments such as skewness and kurtosis are ignored. Traditional risk/return optimisation therefore assumes that returns follow a normal distribution.[17]

Empirical studies demonstrate, however, that normal distribution—especially during periods of market turbulence—significantly underestimates the likelihood of strongly negative returns.[18] As Table 5 shows, some of the returns generated by the asset classes analysed in our example exhibit values that deviate considerably from a normal distribution.

Table 5: Empirical skewness and kurtosis of returns on the asset classes being analysed

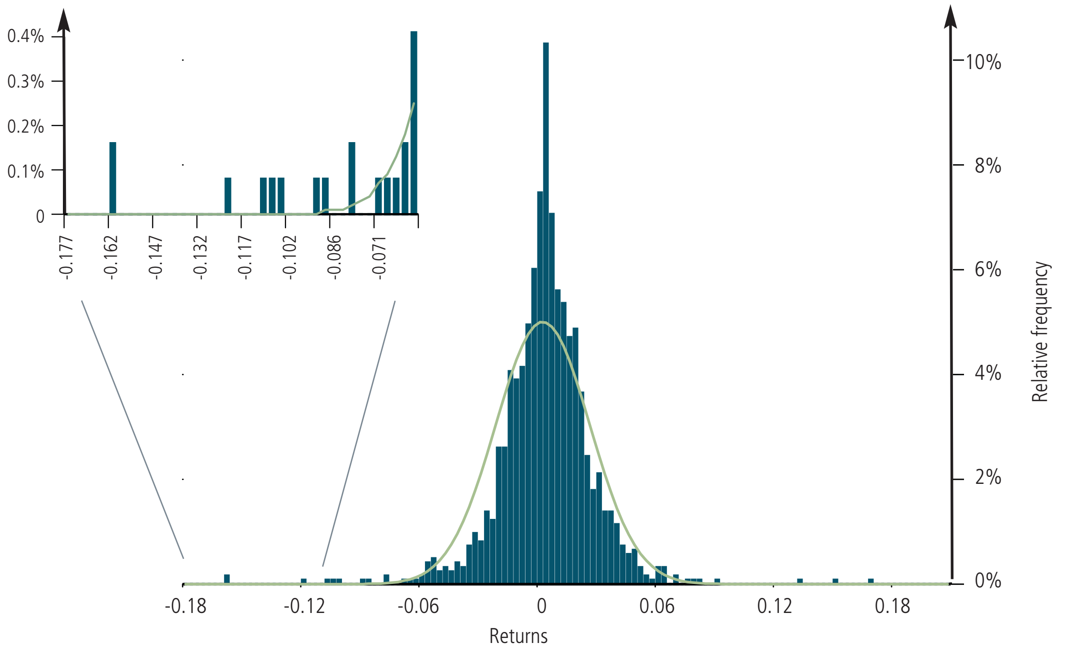
	Core eurozone sovereigns	Peripheral eurozone sovereigns	High grade credits	High yield credits, ex financials	Emerging market sovereigns	Global equities	Commodities
Skewness	0.22649	–0.38255	–0.54915	–0.83418	–3.38066	–0.63590	–0.09178
Kurtosis	3.20946	6.20021	4.88168	7.97153	27.48780	4.35501	3.46809

The kurtosis of the returns on the 'emerging market sovereigns', 'peripheral eurozone sovereigns' and 'high-yield credits, ex financials' asset classes exhibits much fatter distribution tails than in the case of normal distribution.

Figure 8 shows the empirical distribution of historical returns on the 'high-yield credits, ex financials' asset class compared with normal distribution, whose two parameters—'average' and 'standard deviation'—correspond to those of the empirical return distribution. The left distribution tail has been magnified in order to illustrate the fat distribution tails. This clearly shows that normal distribution significantly underestimates the likelihood of strongly negative returns.

[13] See Gosling (2010), p. 53.
[14] See Best/Grauer (1991), p. 332, and Michaud (1989), p. 35.
[15] See Michaud et al. (2013), p. 6.
[16] See Black/Litterman (1992), p. 28, and He/Litterman (1999), p. 3.
[17] See Gosling (2010), p. 53.
[18] See, for example, Mandelbrot (1962), p. 394 onwards.

Figure 8:
Empirical distribution of returns on the 'high-yield credits, ex financials' asset class compared with normal distribution



Empirically, therefore, we can observe strongly negative returns for which the normal distribution hypothesis would predict only an infinitesimal probability. Empirical return distributions are not symmetric and exhibit fat tails.

One way of resolving the aforementioned problems facing the traditional asset allocation approach is to incorporate forward-looking performance scenarios. This method enables investors to factor fat-tail events and asymmetric expected returns into their asset allocation decisions.

[19] For a detailed discussion of this subject see Nöll/Wiedemann (2008), p. 152 onwards.

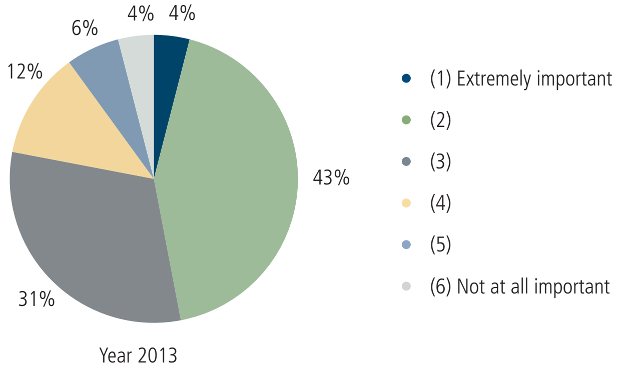
3 Scenario-based asset allocation

3.1 Scenario-based asset allocation process

As indicated in the previous section, scenario techniques can be used to integrate fat-tail events and asymmetric expected returns into the asset allocation process. This is also what a growing number of institutional investors would like to do.

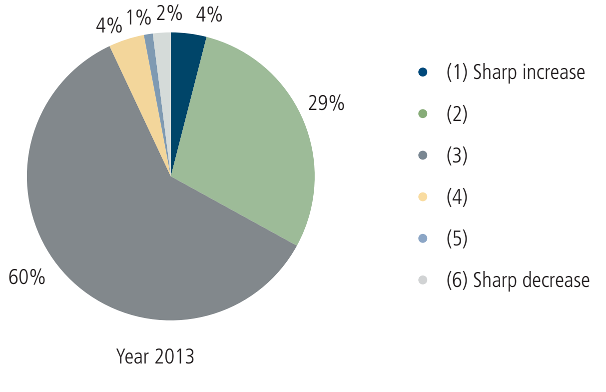
Most of the institutional investors surveyed as part of the 2013 risk inventory stated that the use of scenario techniques was highly important (see Figure 9), with 29 per cent of those surveyed claiming that the use of scenario techniques in their investment decisions had actually become more important (see Figure 10).

Figure 9:
Importance of the use of scenario techniques by institutional investors



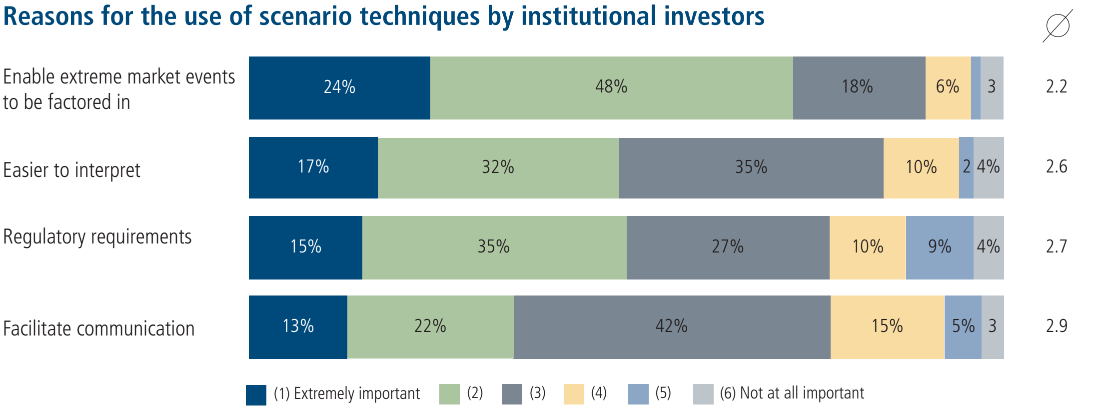
These findings demonstrate that investors already analyse a number of different expected returns even if they do not (yet) factor these into their traditional Markowitz optimisation models.

Figure 10:
Changing importance of the use of scenario techniques by institutional investors



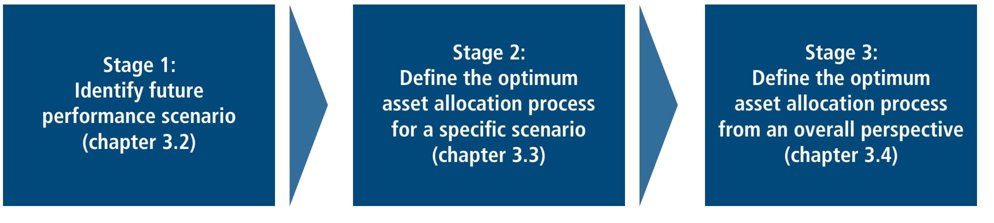
Those surveyed stated once again that the most important reason for using scenario techniques was that they enabled extreme market events to be factored in (see Figure 11).

Figure 11:
Reasons for the use of scenario techniques by institutional investors



Because investors are becoming much more interested in integrating scenario techniques into their investment decision-making processes, the question now is how this can be achieved in a structured form and how an asset allocation process that is firmly rooted in sound decision theory can be devised. To this end, the asset allocation process is divided into three stages, as shown in Figure 12.

Figure 12:
Scenario-based asset allocation process



The focus of stage 1 is to identify future performance scenarios. The aim here is firstly to specify potential performance scenarios and then to predict the returns that the analysed asset classes will achieve in these scenarios. In addition, this stage involves defining scenario-specific correlation matrices for the purpose of mapping various correlations between the asset classes in a number of different scenarios.

The next stage is to use the scenarios defined in stage 1 in order to identify the optimum asset allocation for each scenario. The third stage is to aggregate all the results and then to use the optimum scenario-specific asset allocation identified during the second stage in order to define the optimum form of asset allocation that meets all the investor's individual requirements and specifications and takes account of all scenarios.

3.2 Defining future performance scenarios

As discussed in the previous section, the first stage in the process is to identify the scenarios to be included in the asset allocation process. The first question here is what sort of scenarios are to be analysed? Figure 13 summarises the importance that institutional investors attach to the various types of scenario when making investment decisions.

Figure 13:
What importance do you think you will attach to the following types of scenario when making your investment decisions in future?

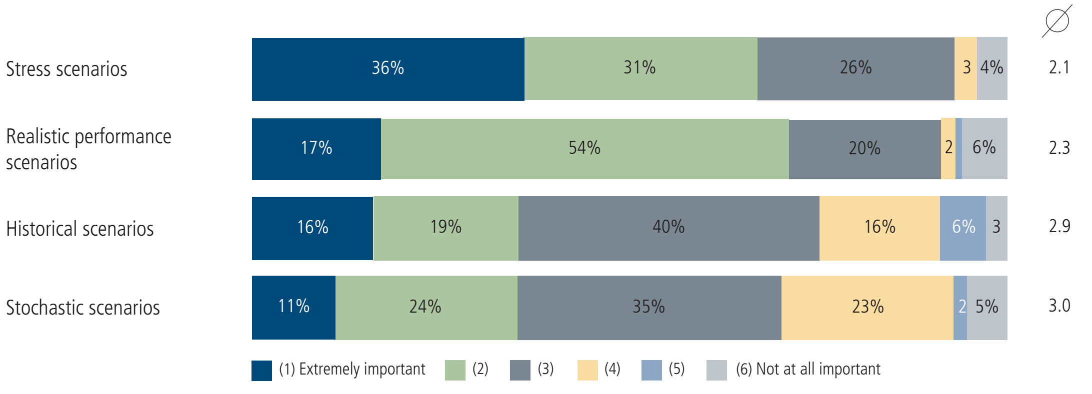
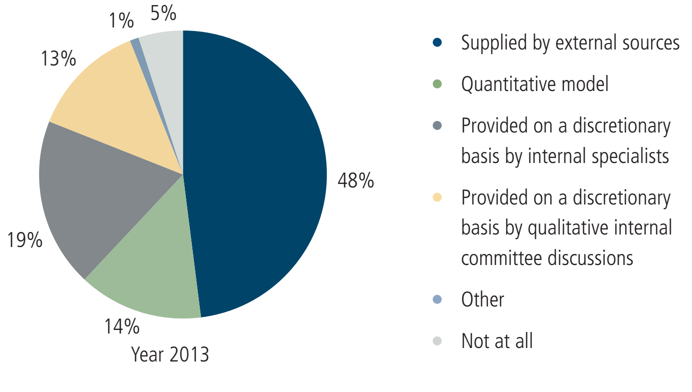


Figure 13 clearly shows that institutional investors' main focus is on stress scenarios and realistic performance scenarios. Much less importance is attached to historical and stochastic scenarios. These results once again demonstrate that scenario techniques are investors' preferred tool for analysing potential market trends in a structured way. Although stochastic scenarios offer other benefits, they are of no use here. As Figure 14 shows, almost half of all investors avail themselves of external expertise when identifying economic performance scenarios.

Figure 14:
How does your company identify economic performance scenarios?



In order to illustrate how scenario-based asset allocation works, this paper draws on the realistic performance scenarios shown in Table 6 for the seven asset classes being analysed.[20] This enables asymmetric expected returns and fat-tail events to be formulated and then addressed as part of various optimisation problems. The annual return expected to be generated by each asset class is specified for the respective performance scenario.

Table 6:
Performance scenarios used in the example

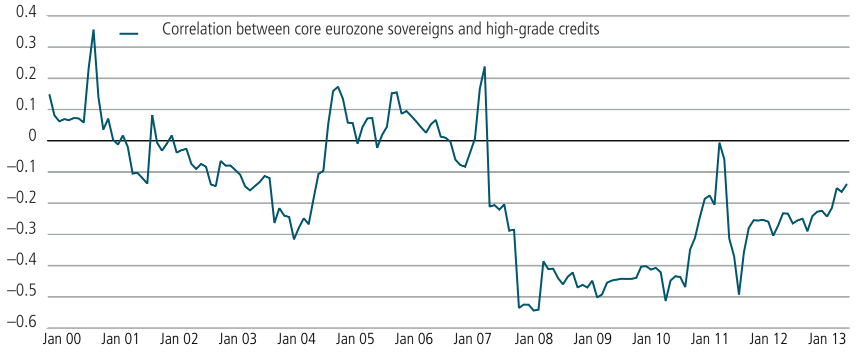
Asset classes	Main scenario	Opti-mistisc	Pessi-mistic I	Pessi-mistic II	Fat-tail
Core eurozone sovereigns	2.2%	1.5%	3.0%	0.0%	3.0%
Peripheral eurozone sovereigns	2.6%	4.0%	-2.0%	1.0%	1.0%
High-grade credits	2.8%	2.5%	1.0%	1.5%	2.0%
High-yield credits, ex financials	5.0%	6.0%	0.5%	3.0%	1.5%
Emerging market sovereigns	5.5%	7.0%	5.0%	4.0%	0.0%
Global equities	7.0%	18.0%	-10.0%	0.0%	-30.0%
Commodities	6.0%	15.0%	-7.0%	0.0%	-15.0%

The Main scenario in this illustrative example forecasts the level of investment returns in the event of moderate economic growth coupled with stable interest rates. Risk assets slightly outperform safe sovereign bonds. The Optimistic scenario is based on the assumption that there will be a sustainable global economic recovery, which will help risk assets to significantly outperform other investments in many cases. The Pessimistic I scenario predicts that the euro crisis will be reignited, causing heavy losses on peripheral eurozone bonds and in the equity market. The Pessimistic II scenario assumes that interest rates will rise sharply. This would reduce the returns on all fixed-income investments, while equities and commodities would yield zero returns. In addition to the two Pessimistic scenarios this illustrative example also includes a Fat-tail scenario, which is intended to replicate extreme market events such as the bankruptcy of Lehman Brothers. Equities would incur losses of 30 per cent in this event, whereas safe sovereign bonds could post gains.

In addition to the previously defined scenario-dependent returns expected for individual asset classes, scenario-specific variance-covariance matrices can also be integrated into the investment decision-making process. This approach takes account of the fact that the correlation behaviour between individual asset classes can vary substantially depending on the market environment.[21] This is illustrated by Figure 15 using the example of the annual moving correlation between the returns yielded by the 'core eurozone sovereigns' and 'high-grade credits' asset classes.

[20] The article by Gosling (2010) examines the definition of performance scenarios for the purposes of scenario-based asset allocation. Saffo (2007) flags up general aspects that need to be factored into forward-looking forecasts.
[21] See, for example, Loretan/Englisch (2000), p. 1; Kotkatvuori-Örnberg et al. (2013) demonstrate this by using the example of equity markets.

Figure 15:
Correlation between core eurozone sovereigns and high-grade credits



This paper bases its regime classification on the S&P 500 Volatility Index (VIX). To this end, the historical period being analysed is divided into three market phases: phases with high volatility, phases with normal volatility and phases with low volatility. This method is illustrated in Figure 16. The two 'upper VIX' and 'lower VIX' bands are used to separate these market phases.[22]

Figure 16:
Regime classification based on the VIX

The scenario-specific covariances are determined on the basis of the historical returns yielded by the seven asset classes. This method factors in a range of time periods and market volatilities that vary according to the scenario being analysed (see Table 7).[23]

[22] The lower band denotes one downward standard deviation from the VIX average, while the upper band marks 1.5 upward standard deviations from the VIX average.

[23] The scenario-specific covariance matrices used in the example are shown in Annex A.

Table 7:
Time periods used to determine scenario-specific covariance matrices

Main scenario	Total period
Optimistic scenario	100% normal-volatility phases
Pessimistic scenario I	70% high-volatility phases and 30% of total period
Pessimistic scenario II	50% low-volatility phases and 50% of total period
Fat-tail scenario	100% high-volatility phases

The following section uses the scenarios defined in this section to show how various scenario-based asset allocation methods work.

3.3 Defining optimum portfolios for individual scenarios

The second stage of the scenario-based asset allocation process focuses on creating optimum portfolios for the scenarios defined in the first stage. The objective of this process is to identify portfolio structures that would be ideal for investors if the relevant scenarios materialised. The first step in this process is to select a suitable optimisation model, the form of which may be based on the variants presented in chapter 2. The choice of optimisation model must be consistent with the investor's particular risk appetite and with the internally and externally imposed restrictions. This paper discusses Variant 3 of the target functions described.[24] Consequently, the aim is to maximise the following target function in line with the vector of portfolio weights x in the individual asset classes for all analysed scenarios s = 1,…, 5:

$$\text{Utility}_s = \mu_{P,s} - \frac{\gamma}{2} \cdot \sigma^2_{P,s} \rightarrow \max!$$

The expected portfolio return $\mu_{P,S}$ is directly calculated in the example as shown below based on the returns defined in the previous stage for asset classes $\underline{R}_S^{AK}$ in the relevant performance scenarios:

$$\mu_{P,s} = \underline{x}^T \cdot \underline{R}_s^{AK}$$

We can therefore calculate the portfolio return by multiplying the transposed vector of portfolio weights $\underline{x}^T$ by the vector of the returns expected for asset classes $\underline{R}_S^{AK}$ in the various scenarios. Apart from integrating the scenario-specific expected returns directly into the optimisation model, a further option is to use the Black-Litterman model to combine the individual expected returns with the longterm equilibrium returns observable in the capital markets for the asset classes being analysed.[25]

Calculations of portfolio variance are based on scenario-specific variance-covariance matrices Σ_S, whose computation from the historical capital market data was explained in the previous section.[26] Scenario-specific portfolio variance $\sigma^2_{P,S}$ in line with the weight vector of asset classes $\underline{x}_s$ is thus determined as follows:

$$\sigma^2_{P,S} = \underline{x}_S^T \cdot \Sigma_S \cdot \underline{x}_S$$

24 It is both possible and appropriate to use alternative optimisation models.
25 See Black/Litterman (1992), p. 28 onwards, and Annex B.
26 The scenario-specific covariance matrices used in the example are shown in Annex A.

This approach ensures that the varying correlation behaviour of the returns generated by the asset classes in various market situations is taken into account when the optimum investment decision is identified.
As described in the previous chapter, the risk aversion parameter is specified as $\gamma = 6.7$. In addition to the

two constraints of full asset allocation and the non-negativity of portfolio weights in the asset classes, the maximum portfolio weight for each asset class was limited to 40 per cent.[27] Figure 17 summarises the results of the optimisation process.

Figure 17:
Results of scenario-specific optimisation on a case-by-case basis

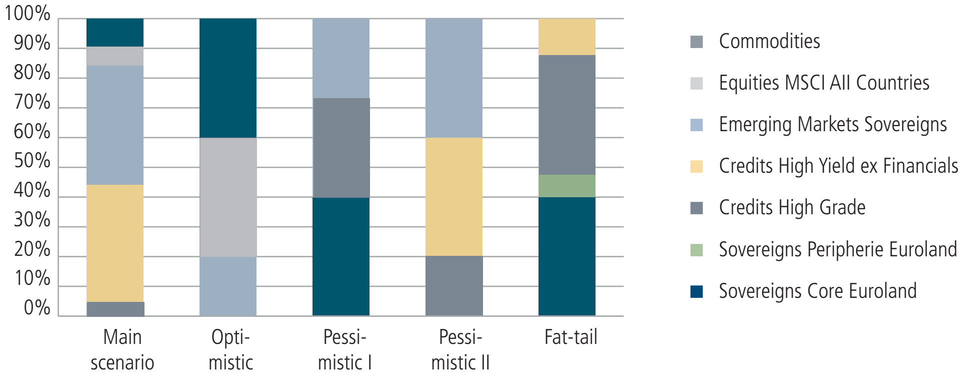
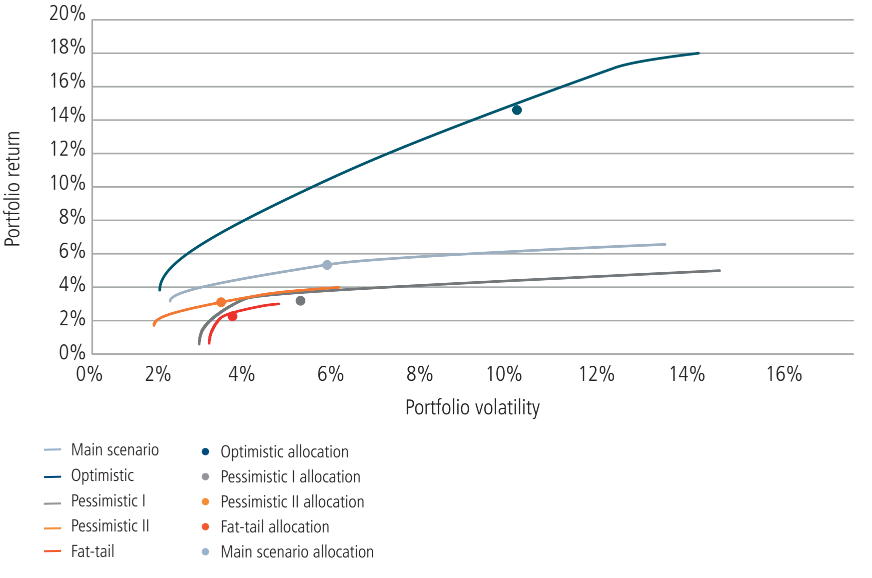


Figure 17 clearly illustrates that a large proportion of the investment under the Main and Optimistic scenarios is allocated to high-risk asset classes that offer considerable upside potential, whereas most of the investment under the Pessimistic and Fat-tail scenarios is allocated to low-risk bonds issued by core eurozone countries and to investment-grade corporate bonds. The optimum asset allocation weightings therefore vary substantially in line with the underlying scenario.

Figure 18 illustrates the scenario-specific asset allocation weightings on the risk/return chart. A separate efficient frontier can be defined for each scenario based on the various covariance matrices and on the scenario-dependent expected returns specified during stage 1 for the asset classes. By formulating and then resolving their optimisation problems, investors can identify a certain point on the efficient frontier as the optimum point for them. It should be noted here that some points will lie slightly below the efficient frontier because the maximum portfolio weight for each asset class is limited to 40 per cent. The inclusion of additional restrictions may therefore impair the efficiency of the portfolios generated, which needs to be taken into consideration when the optimisation problems are formulated.

[27] This restriction is not absolutely necessary. Nonetheless, it is included in place of a large number of internal and external restrictions that investors have to comply with when making their investment decisions (see Figure 6 and Figure 7 in chapter 2).

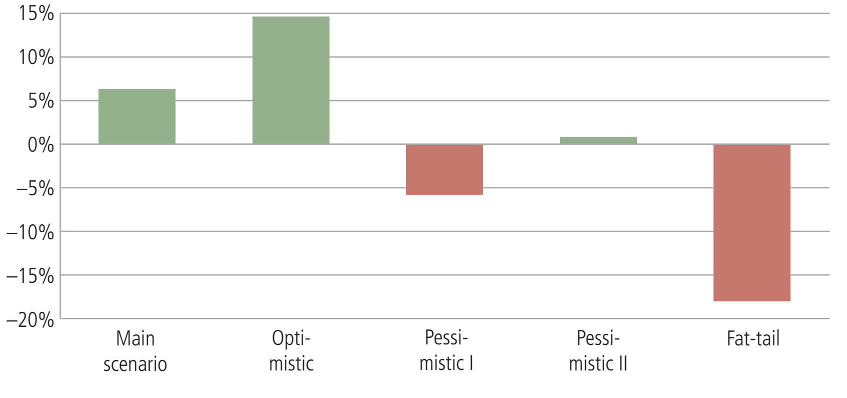
Figure 18:
Scenario-specific asset allocation weightings on the risk/return chart



Because it is uncertain whether and, if so, when the defined scenarios will materialise in future, investors are faced with the question of what decision to make, because each portfolio has been optimised to fit a specific scenario.

As Figure 19 shows, an incorrect decision can have fatal consequences. In order to illustrate the potential impact of such a decision, let's assume that the investor in our example has opted for the asset allocation weighting that has been optimised to fit the Optimistic performance scenario. Figure 19 shows the returns that this portfolio would generate if the various scenarios were to materialise.

Figure 19:
Returns generated in the various scenarios by the portfolio optimised to fit the optimistic scenario



Whereas the portfolio generates positive returns if either the Main scenario, the Optimistic scenario or the Pessimistic II scenario materialises, it yields a strongly negative return if a fat-tail event occurs.

This means that the radical approach of gearing the portfolio to just a single performance scenario only yields optimum results if the specific scenario for which the portfolio was optimised actually occurs. As soon as any scenario other than the expected one materialises, however, investors run the risk of incurring heavy losses.

This therefore poses the question of whether and, if so, how investors can identify a portfolio that is ideal for all scenarios.

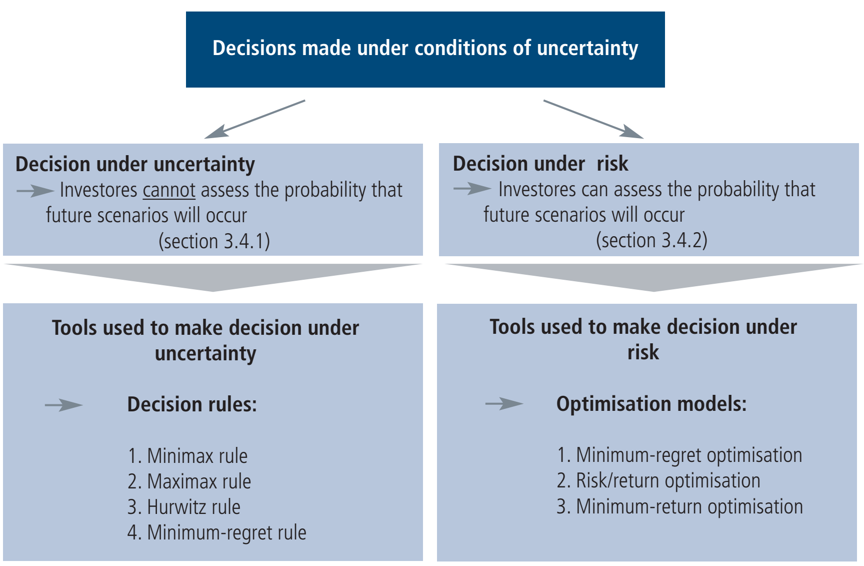
3.4 Identifying an asset allocation strategy that takes account of all scenarios

3.4.1 Decision making under uncertainty

The objective of this chapter is to identify the ideal asset allocation strategy. In contrast to the previous section, however, this optimisation strategy will take account of all scenarios rather than analysing just one scenario in isolation and then optimising the asset allocation process to fit this specific scenario.

Viewed from the perspective of decision theory, investors looking to select what for them is the ideal alternative are faced with the problem of having to make decisions under conditions of uncertainty. Decision making under conditions of uncertainty is characterised by the fact that investors cannot predict in advance what scenario will actually occur in future.[28] As Figure 20 shows, decisions made under conditions of uncertainty are described as either 'decisions under risk' or 'decisions under uncertainty'. Whereas investors making decisions under risk can assess the probability that future potential scenarios will occur, they cannot do so when making decisions under uncertainty.[29]

Figure 20:
Tools used to make decisions under uncertainty and risk[30]



[28] See Parmigiani/Inoue (2009), p. 13.
[29] See Laux et al. (2012), p. 82.
[30] There are many other decision rules. This diagram only lists the tools that are analysed in the sections below.

The following section analyses how to arrive at the optimum investment decision in both decision-making situations, discusses the pertinent decision-making processes and compares the resultant asset allocation weightings with each other. In cases where decisions are being made under uncertainty, i.e. if the investors concerned are either unable or unwilling to assess the probability that the formulated scenarios will occur, then decision rules can help them to select the ideal portfolio for their requirements. This type of situation is analysed in this section. Further portfolio optimisation options are available to investors in situations where they are making decisions under risk, i.e. if the investors concerned in each case are able to assess the probability that the formulated scenarios will occur. This type of case is discussed in the subsequent chapter (3.4.2).

We will start by analysing the process of decision making under uncertainty. In our example, the investor has a choice of five portfolios that have been optimised in isolation to fit a specific scenario. They now have to decide what form of asset allocation to choose if they are unable to assess the probability that the future scenarios will occur. In this case the various forms of asset allocation available are to be viewed as alternatives between which the investor has to decide.[31] Table 8 shows the composition of the individual portfolios, each of which has been optimised to fit a specific scenario. Alternative 1, for example, constitutes the optimum portfolio for the Main scenario ('moderate economic growth coupled with stable interest rates'). This portfolio largely comprises the 'high-yield credits, ex financials' and 'emerging market sovereigns' asset classes, which each account for 40 per cent of the total portfolio. It also contains a 10 per cent weighting of commodities, 6 per cent equities and 4 per cent high-grade credits.

Table 8:
Comparison of portfolio weightings in alternatives

Alternatives Asset classes	Alternative 1: Optimal portfolio in the Main scenario	Alternative 2: Optimal portfolio in the Optimistic scenario	Alternative 3: Optimal portfolio in the Pessimistic I scenario	Alternative 4: Optimal portfolio in the Pessimistic II scenario	Alternative 5: Optimal portfolio in the Fat-tail scenario
Core eurozone sovereigns	0%	0%	40%	0%	40%
Peripheral eurozone sovereigns	0%	0%	0%	0%	8%
High-grade credits	4%	0%	25%	20%	40%
High-yield credits, ex financials	40%	0%	0%	40%	12%
Emerging market sovereigns	40%	20%	35%	40%	0%
Global equities	6%	40%	0%	0%	0%
Commodities	10%	40%	0%	0%	0%
Total	**100%**	**100%**	**100%**	**100%**	**100%**

[31] This assumption no longer applies in section 3.4.2.

Table 9 shows the investment returns expected to be achieved by the respective alternatives if the various scenarios were to materialise. We have calculated these returns by multiplying the returns forecast during stage 1 for the respective scenarios (see Table 6 in section 3.2) by the portfolio weights specified above (see Table 8).

Table 9:
Annual returns achieved by alternatives if various scenarios were to materialise

Scenarios / Alternatives	Main scenario	Optimistic	Pessimistic I	Pessimistic II	Fat-tail
Alternative 1: Optimal portfolio in the Main scenario	5.33%	7.86%	0.95%	2.86%	–2.64%
Alternative 2: Optimal portfolio in the Optimistic scenario	6.30%	14.60%	–5.80%	0.80%	–18.00%
Alternative 3: Optimal portfolio in the Pessimistic I scenario	3.50%	3.66%	3.19%	1.77%	1.71%
Alternative 4: Optimal portfolio in the Pessimistic II scenario	4.76%	5.70%	2.40%	3.10%	1.00%
Alternative 5: Optimal portfolio in the Fat-tail scenario	2.81%	2.65%	1.51%	1.05%	2.26%

If, for example, the investor opts for Alternative 3 and the Pessimistic I scenario occurs, then a return of 3.19 per cent is expected for this portfolio. If, on the other hand, the Pessimistic II scenario occurs, then the expected portfolio return is only 1.77 per cent.

As demonstrated, decision making under uncertainty is characterised by the decision-maker (specifically the investor) finding it impossible to assign a probability to different performance scenarios. Classic decision theory has produced a variety of decision rules for specified problems of this type. The following four methods illustrated in Figure 20 are examined for the purposes of asset allocation:

1. Minimax rule
2. Maximax rule
3. Hurwitz rule
4. Minimum-regret rule

The starting point for the use of all the rules is Table 9 which was introduced in the previous section and which indicates the returns that are generated by the alternatives should different scenarios arise.

1. Minimax rule

To come to a decision using the minimax rule,[32] the minimum return generated by each alternative must first be determined as illustrated for the example in Table 10.

Table 10:
Calculation of minimum annual return for each alternative

Scenario / Alternative	Main scenario	Opti-mistic	Pessi-mistic I	Pessi-mistic II	Fat-tail	Minimum
Alternative 1: Optimal portfolio in the Main scenario	5.33%	7.86%	0.95%	2.86%	–2.64%	**–2.64%**
Alternative 2: Optimal portfolio in the Optimistic scenario	6.30%	14.60%	–5.80%	0.80%	–18.00%	**–18.00%**
Alternative 3: Optimal portfolio in the Pessimistic I scenario	3.50%	3.66%	3.19%	1.77%	1.71%	**1.71%**
Alternative 4: Optimal portfolio in the Pessimistic II scenario	4.76%	5.70%	2.40%	3.10%	1.00%	**1.00%**
Alternative 5: Optimal portfolio in the Fat-tail scenario	2.81%	2.65%	1.51%	1.05%	2.26%	**1.05%**

For example, if an investor opts for Alternative 1 this alternative would generate a return of -2.64 per cent in the worst case. This is the return generated if the 'fat-tail' scenario occurs.

According to the minimax rule, the next stage is to select the alternative for which the minimum return is greater than the minimum return generated by any of the other alternatives.[33] In the example, this is Alternative 3, for which the investor can expect a minimum return of 1.71 per cent, regardless of the probability of the scenario arising.

Obviously, investors who focus on this criterion only factor the worst possible case into their decision calculations. All other information, such as the possibility of the alternative generating high returns in different scenarios, is ignored. The minimax rule is therefore a very pessimistic decision rule, but it does enable investors to limit their losses or to generate certain minimum returns.

[32] This rule dates back to the work of Abraham Wald (1945).
[33] See Laux et al. (2012), p. 83.

2. Maximax rule

While the minimax rule represents a very pessimistic decision rule, the maximax rule is an extremely optimistic decision criterion.

Table 11:
Calculation of maximum annual return for each alternative

Scenario / Alternative	Main scenario	Opti-mistic	Pessi-mistic I	Pessi-mistic II	Fat-tail	Maximum
Alternative 1: Optimal portfolio in the Main scenario	5.33%	7.86%	0.95%	2.86%	–2.64%	7.86%
Alternative 2: Optimal portfolio in the Optimistic scenario	6.30%	14.60%	–5.80%	0.80%	–18.00%	14.60%
Alternative 3: Optimal portfolio in the Pessimistic I scenario	3.50%	3.66%	3.19%	1.77%	1.71%	3.66%
Alternative 4: Optimal portfolio in the Pessimistic II scenario	4.76%	5.70%	2.40%	3.10%	1.00%	5.70%
Alternative 5: Optimal portfolio in the Fat-tail scenario	2.81%	2.65%	1.51%	1.05%	2.26%	2.81%

As illustrated in Table 11, the maximum return for all scenarios must first be determined. The maximax rule then selects the alternative for which the maximum return is greater than the maximum return for any other alternative.

In the example, the maximum return for Alternative 2 is 14.60 per cent which exceeds the maximum return for any of the other alternatives.[34]

3. Hurwitz rule

The Hurwitz criterion represents a compromise between the pessimistic minimax rule and the extremely optimistic maximax rule. Based on the minimum or maximum achievable returns for each alternative, the Hurwitz value is calculated as follows:

[34] See Laux et al. (2012), p. 84.

Table 12:
Calculation of the Hurwitz value for the example

Scenario / Alternative	Minimum	Maximum	Hurwitz value
Alternative 1: Optimal portfolio in the Main scenario	–2.64%	7.86%	**–2.64% · (1 – 0,4) + 7.86% · 0.4 = 1.56%**
Alternative 2: Optimal portfolio in the Optimistic scenario	–18.00%	14.60%	**–18.00% · (1 – 0,4) + 14.60% · 0.4 = -4.96%**
Alternative 3: Optimal portfolio in the Pessimistic I scenario	1.71%	3.66%	**1.71% · (1 – 0,4) + 3.66% · 0.4 = 2.49%**
Alternative 4: Optimal portfolio in the Pessimistic II scenario	1.00%	5.70%	**1.00% · (1 – 0,4) + 5.70% · 0.4 = 2.88%**
Alternative 5: Optimal portfolio in the Fat-tail scenario	1.05%	2.81%	**1.05% · (1 – 0,4) + 2.81% · 0.4 = 1.75%**

The Hurwitz value is a weighted average calculated as follows:[35]

$$\text{Hurwitz value}_{\text{alternative}} = \text{Minimum return of alternative} \cdot (1 - \lambda) + \text{Maximum return of alternative} \cdot \lambda$$

Parameter λ may have a value in the range [0;1] and it is specified by the investor. It measures the degree of optimism. The greater λ, the higher the degree of optimism. According to the Hurwitz rule, the alternative with the greatest Hurwitz value should be selected. In the example with an exogenously specified optimism parameter of $\lambda = 0.4$, this is Alternative 4 (see Table 12).

Compared with the two preceding criteria, the Hurwitz criterion does not focus exclusively on one extreme (minimum or maximum). Instead, the two extremes are factored into the calculation for making the decision according to individual risk appetite. However, it is important to note that this decision rule ignores all the information that falls outside the most optimistic and most pessimistic cases.

4. Minimum-regret rule

> "I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn't in it—or if it went way down and I was completely in it. My intention was to minimize my future regret, so I split my [pension scheme] contributions 50/50 between bonds and equities."[36]
>
> Harry Markowitz

[35] See Laux et al. (2012), p. 84.
[36] Quoted in Zweig (2007), p. 4. The quotation by Harry Markowitz comes from an article in the January 1988 issue of Money magazine.

While all the decision rules presented above are based on the minimum, the maximum or a weighted average of the minimum and maximum, the minimum-regret criterion tries to make a decision based on regret. The term ‘regret’ refers to the feeling associated with having decided on one alternative when it subsequently transpires that the choice of a different alternative would have resulted in a better outcome.[37]

The concept of regret is important in the field of decision theory.[38] In the academic literature there are numerous publications investigating the suitability of this rule in decision making.[39]

To come to a decision using the minimum-regret rule, what is known as the regret matrix must first be determined. This requires the maximum achievable return to be determined for each scenario (see Table 13).

Table 13:
Calculation of maximum annual return for each scenario

Scenario / Alternative	Main scenario	Opti-mistic	Pessi-mistic I	Pessi-mistic II	Fat-tail
Alternative 1: Optimal portfolio in the Main scenario	5.33%	7.86%	0.95%	2.86%	–2.64%
Alternative 2: Optimal portfolio in Optimistic scenario	6.30%	14.60%	–5.80%	0.80%	–18.00%
Alternative 3: Optimal portfolio in the Pessimistic I Scenario	3.50%	3.66%	3.19%	1.77%	1.71%
Alternative 4: Otimal portfolio in the Pessimistic II Scenario	4.76%	5.70%	2.40%	3.10%	1.00%
Alternative 5: Optimal portfolio in the Fat-tail scenario	2.81%	2.65%	1.51%	1.05%	2.26%
Maximum return	**6.30%**	**14.60%**	**3.19%**	**3.10%**	**2.26%**

Ideally, if the Main scenario occurs, the investor should have opted for Alternative 2. If this alternative had been chosen, the investor would have received the maximum return of 6.30 per cent. If the Fat-tail scenario had materialised, Alternative 5 would have been the preferred choice in retrospect, because the highest possible return of 2.26 per cent would have been generated in this scenario.

If the investor opts for Alternative 1 and the Main scenario occurs, this alternative generates a return of 5.33 per cent but, as described above, Alternative 2 would have been the optimal choice in retrospect. If the Main scenario arises, Alternative 1 delivers a return on the portfolio that is 0.97 per cent lower than Alternative 2 (6.30 per cent minus 5.33 per cent). This reduced return compared with the best possible return in each scenario is known as ‘investor regret’. Regret can therefore be interpreted as the lost opportunity for return, and it arises because an investor does not know in advance which scenario will arise in the future.[40] Table 14 illustrates the full regret matrix for the example.

[37] See Coricelli et al (2007), p. 259.
[38] See Connolly/Zeelenberg (2002), p. 212.
[39] See also Kahneman/Tversky (1982) and Gilovich/Medvec (1995) etc.
[40] See King (1993), p. 174.

Table 14:
Regret matrix for the example

Scenario / Alternative	Main scenario	Opti-mistic	Pessi-mistic I	Pessi-mistic II	Fat-tail
Alternative 1: Optimal portfolio in the Main scenario	0.97%	6.74%	2.24%	0.24%	4.90%
Alternative 2: Optimal portfolio in the Optimistic scenario	0.00%	0.00%	8.99%	2.30%	20.26%
Alternative 3: Optimal portfolio in the Pessimistic I scenario	2.80%	10.94%	0.00%	1.33%	0.55%
Alternative 4: Optimal portfolio in the Pessimistic II scenario	1.54%	8.90%	0.79%	0.00%	1.26%
Alternative 5: Optimal portfolio in the Fat-tail scenario	3.49%	11.95%	1.68%	2.05%	0.00%

The second step towards selecting the optimal alternative using the minimum-regret criterion is to calculate the maximum regret for each alternative, which Table 15 shows for the example.

Table 15:
Calculation of the maximum regret for each alternative

Scenario / Alternative	Main scenario	Opti-mistic	Pessi-mistic I	Pessi-mistic II	Fat-tail	Maximum regret
Alternative 1: Optimal portfolio in the Main scenario	0.97%	6.74%	2.24%	0.24%	4.90%	**6.74%**
Alternative 2: Optimal portfolio in the Optimistic scenario	0.00%	0.00%	8.99%	2.30%	20.26%	**20.26%**
Alternative 3: Optimal portfolio in the Pessimistic I scenario	2.80%	10.94%	0.00%	1.33%	0.55%	**10.94%**
Alternative 4: Optimal portfolio in thePpessimistic II scenario	1.54%	8.90%	0.79%	0.00%	1.26%	**8.90%**
Alternative 5: Optimal portfolio in the Fat-tail scenario	3.49%	11.95%	1.68%	2.05%	0.00%	**11.95%**

The maximum regret that could arise if Alternative 1 is chosen is 6.74 per cent, and for Alternative 2 it is 20.26 per cent. According to the minimum-regret criterion, the alternative that should be selected is the one for which the maximum regret is lower than the maximum regret value of any other alternative.[41] In the example, Alternative 1 minimises the maximum possible potential investor regret.

[41] See Laux et al. (2012), p. 85.

Table 16 summarises the decisions reached on the basis of the decision rules that have been presented.

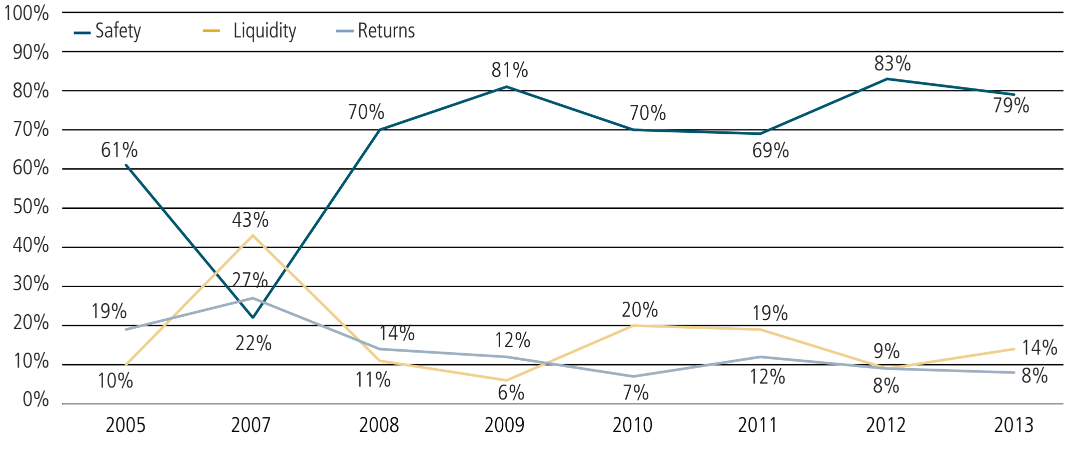
Table 16:
Comparison of decisions based on rules

Decision rule	Decision
Minimax rule	Alternative 3
Maximax rule	Alternative 2
Hurwitz rule	Alternative 4
Minimum-regret rule	Alternative 1

It is clear that the use of different decision rules results in different outcomes. The criterion to be applied depends on the investor's personal attitude to risk.

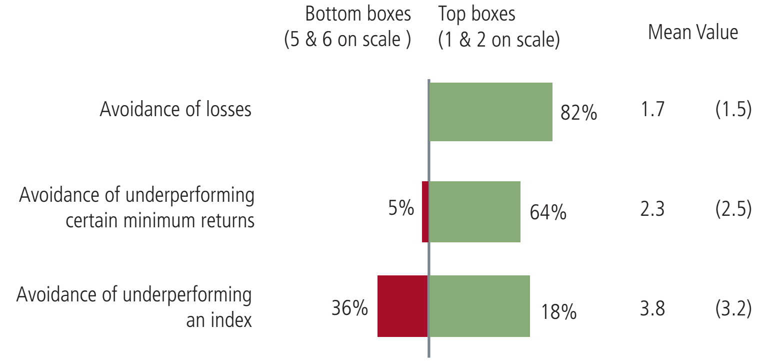
During the 2013 risk inventory, 79 per cent of all institutional investors surveyed stated that safety was the primary objective of their investment decisions, followed at some distance by liquidity and a minimum required rate of return (see Figure 21). Although this percentage is slightly lower than in 2012, it remains very high.

Figure 21:
Which of the following aspects is generally the most important for your company when making investment decisions now?



The survey also showed that the most important aspects taken into account when making investment decisions are the avoidance of losses and the achievement of a certain minimum rate of return. This is illustrated in Figure 22.

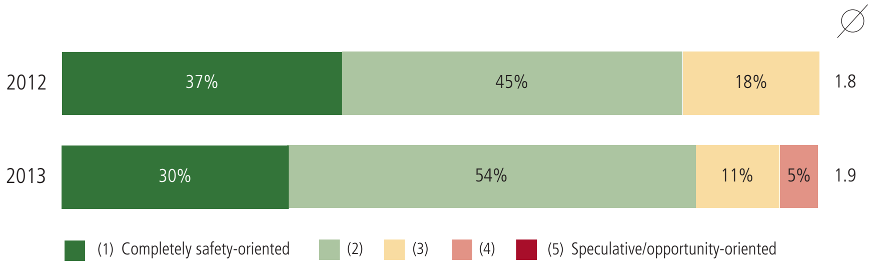
Figure 22:
Aspects taken into account during asset allocation, and their importance



Question 5; all respondents (n = 104); scale of 1 = 'extremely important' to 6 = ' not at all important'; 2012 values in brackets

The 2013 risk inventory showed that a preference for safety remains the dominant aspect when making investment decisions. However, the percentage of investors who are absolutely safety-oriented was seven percentage points lower (a fall from 37 per cent to 30 per cent, see Figure 23). In addition, 5 per cent of investors described their risk appetite as speculative/opportunity-oriented in 2013. This is remarkable because none of the respondents had stated that they adopted a strategy of this kind the previous year. The conclusion that can be drawn is that a strong preference for safety still predominates, but it has declined slightly in comparison with the previous year.

Figure 23:
Risk appetite of institutional investors

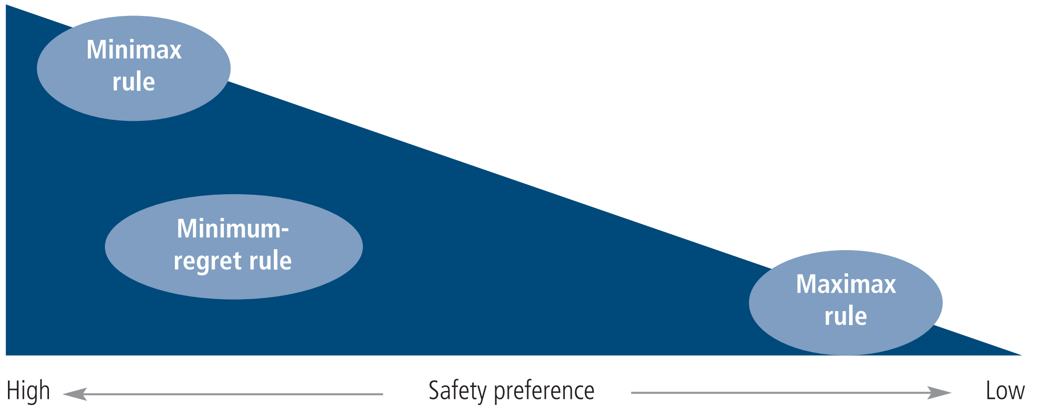


The results of the survey indicate that optimisation methods which prioritise the safety criterion are of great interest for investors. When examining decision making under uncertainty, the minimax criterion enables a high level of risk aversion to be incorporated into the investment decision process.

Figure 24 shows how the decision rules being analysed can be assigned on the basis of an investor's personal risk appetite.[42]

[42] Because allocation of the Hurwitz criterion depends on the level of optimism parameter λ, this process can occupy any of the positions in the diagram corresponding to the definition of λ, so it is not shown explicitly.

Figure 24:
Systematisation of decision rules according to risk attitude



The minimax rule ensures that the highest of the minimum returns is achieved. However, this extremely strong focus on safety also means that investors receive the bulk of their potential returns in all the other scenarios. This trade-off is illustrated in Figure 25 which shows the performance of portfolios selected according to the decision criteria in the different scenarios.

Figure 25:
Comparison of portfolio performances

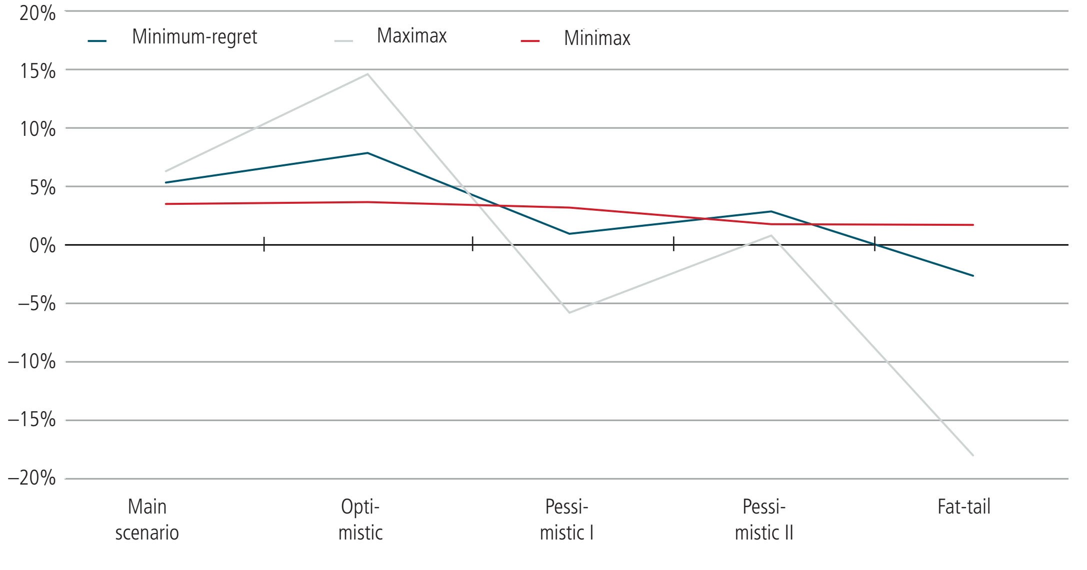


Figure 25 also shows that the alternative determined using the minimax principle always generates positive returns, although the investor forgoes the high potential returns that would arise if the Optimistic scenario or the Main scenario were to materialise. The alternative selected using the maximax principle has the opposite effect, in that investors benefit from a positive return if the Optimistic or Main scenarios arise, but incur losses, some heavy, if these scenarios do not arise. To some extent, the decision reached on the basis of the minimum-regret criterion is a compromise, because this rule tends to be more safety-oriented because it minimises the maximum possible regret. In this case, investors largely benefit from positive returns in the Main and Optimistic scenarios, while only incurring moderate losses should the scenarios that are advantageous in light of their position fail to arise.

The common feature of all the decision rules presented so far is that they assume it is not necessary to assess the probability of the performance scenarios arising. In the following section, this assumption is set aside and we demonstrate how optimal portfolios can be generated for all scenarios while taking probability into account.

3.4.2 Decision making under risk

While the premise of the section above was that investors are unable to allocate a probability to the occurrence of future scenarios, the purpose of the following section is to show how probability can be factored into scenario-based asset allocation.

Neither the minimax nor the maximax principle is particularly suitable as a rule for decision making under risk. This is because the minimax criterion usually identifies portfolios that focus on worst-case scenarios (while the maximax criterion focuses on best-case scenarios) and all other scenarios are ignored. Consequently, not all the available information is used for the investment decision.[43] Because the Hurwitz criterion is a combination of the minimax and the maximax rules, the same criticism applies to it.

By contrast, decision making under risk focuses on identifying optimal portfolios by factoring in all the available information (i.e. all scenarios). For this purpose, the following three optimisation models are presented below:

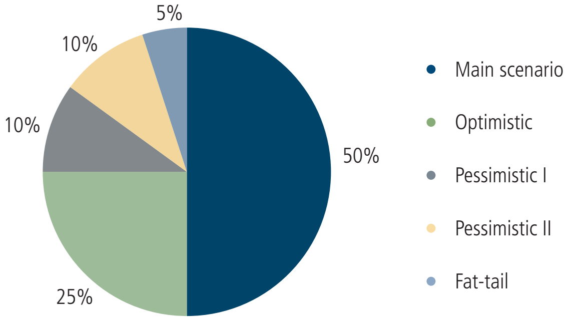
1. Minimum-regret optimisation
2. Risk/return optimisation
3. Minimum-return optimisation

In the section above, the results of the 2013 risk study identified that institutional investors have a strong preference for safety when making investment decisions. As we show below, minimum-return optimisation can be used to factor this preference into asset allocation. Classic risk/return optimisation, which was introduced in chapter 2, is suitable for less risk-averse investors, although the concept needs to be extended to include decision making under risk. The same applies to minimum-regret optimisation which is based on the concept of minimising expected regret and provides additional options. The degree of risk appetite can be adapted in line with an investor's personal risk attitude by incorporating restraints into the optimisation model, such as a minimum required rate of return.

As explained above, decision making under risk presupposes that investors are able to assess the probability of the defined scenarios arising, e.g. by means of appropriate research. The probabilities for the example are set out in Figure 26.

[43] This drawback of the two methods can also be seen as an advantage, because the future performance of the portfolios is completely independent of actual probabilities and it is therefore immune to forecasting errors.

Figure 26:
Imputed probabilities for scenarios in the example



The methodology of the three optimisation models mentioned above is introduced first below and the way they operate is illustrated using the example with the addition of probabilities. We begin with the two opportunity-oriented methods of minimum-regret optimisation and risk/return optimisation, before introducing the minimum-return optimisation model which focuses strongly on safety.

In addition to the way in which the models operate, we show how the basic models can be modified by introducing additional constraints which makes it possible to factor the individual degrees of risk aversion among investors into investment decisions.

Even at this early stage, we stress that it is not possible to say that one of the optimisation models being analysed is generally superior. The decisive factors for selecting a model are the risk attitude specific to an investor and the investment objectives that are being pursued, so the choice of a particular optimisation model always depends on the individual case.

1. Minimum-regret optimisation

In the section above, it was demonstrated that the minimum-regret criterion is a useful decision rule for investors because this method enables portfolios to be generated that benefit from future positive performance while limiting the risk of losses.

As we described, the regret criterion is important in the field of decision theory and regret is increasingly gaining in importance as a risk-management criterion.[44] Because the minimum-regret criterion was originally designed for decision making under uncertainty, the methodology first has to be adapted to the characteristics of decision making under risk. The optimisation model used for this purpose is illustrated in Table 17.[45]

[44] Michenaud/Solnik (2008) show how currency risk can be hedged using this concept. Braun/Muermann (2004) model demand for insurance on the basis of the minimum-regret criterion. Muermann et al. (2006) transfer the regret concept to asset investment decisions.
[45] Based on Dembo/King (1992), p. 153.

Table 17:
Minimum-regret optimisation

Target function	$\sum_{s \in S} p_s \cdot (R_{opt.,s} - \underline{R}_S^T \cdot \underline{w})^2 \rightarrow \min!$
Constraints	1. $\sum_{i=1}^{A} w_i = 1$ (Full allocation) 2. $w_i \geq 0$ (Positive weighting)

As figure 27 shows, $\underline{R}_S^T$ denotes the scenario-specific return vectors, while $\underline{w}^T = \{w_1; w_2; \ldots; w_A\}^T$ represents the weight vectors for the alternatives and p_s designates the probabilities of the performance scenarios S.

Figure 27:
Scenario-specific return vectors

Scenarios / Alternatives	Main scenario	Optimistic	Pessimistic I	Pessimistic II	Fat-tail
Alternative 1	0.97%	6.74%	2.24%	0.24%	4.90%
Alternative 2	0.00%	0.00%	8.99%	2.30%	20.26%
Alternative 3	2.80%	10.94%	0.00%	1.33%	0.55%
Alternative 4	1.54%	8.90%	0.79%	0.00%	1.26%
Alternative 5	3.49%	11.95%	1.68%	2.05%	0.00%
	$\underline{R}_1$	$\underline{R}_2$	$\underline{R}_3$	$\underline{R}_4$	$\underline{R}_5$

Constraint 1 requires the total of all weightings to be 1, or 100 per cent, while Constraint 2 ensures that the weightings are always positive. Table 18 shows the operation of the minimum-regret optimisation problem.

Table 18:
Operation of minimum-regret optimisation factoring in probability

	Return in relation to weight vector $\underline{w}$	Regret in relation to weight vector $\underline{w}$	Weighted regret in relation to weight vector $\underline{w}$
Main scenario s = 1	$\underline{R}_1^T \cdot \underline{w}$	$R_{opt.,1} - \underline{R}_1^T \cdot \underline{w}$	$P_1 \cdot (R_{opt.,1} - \underline{R}_1^T \cdot \underline{w})^2$
Optimistic s = 2	$\underline{R}_2^T \cdot \underline{w}$	$R_{opt.,2} - \underline{R}_2^T \cdot \underline{w}$	$P_2 \cdot (R_{opt.,2} - \underline{R}_2^T \cdot \underline{w})^2$
Pessimistic I s = 3	$\underline{R}_3^T \cdot \underline{w}$	$R_{opt.,3} - \underline{R}_3^T \cdot \underline{w}$	$P_3 \cdot (R_{opt.,3} - \underline{R}_3^T \cdot \underline{w})^2$
Pessimistic II s = 4	$\underline{R}_4^T \cdot \underline{w}$	$R_{opt.,4} - \underline{R}_4^T \cdot \underline{w}$	$P_4 \cdot (R_{opt.,4} - \underline{R}_4^T \cdot \underline{w})^2$
Fat-tail s = 5	$\underline{R}_5^T \cdot \underline{w}$	$R_{opt.,5} - \underline{R}_5^T \cdot \underline{w}$	$P_5 \cdot (R_{opt.,5} - \underline{R}_5^T \cdot \underline{w})^2$

Dembo/King propose the use of quadratic distances but point out that any other distance measures can be used instead of quadratic distances.[46] By selecting quadratic distance measures, large deviations from the scenario-specific optimal return $R_{opt.,S}$ are penalised more than smaller deviations. As a result, this target function is able to comply with the safety-oriented attitudes of investors identified in the preceding section.

The target function proposed by Dembo/King represents the total number of probability weighted, quadratic deviations from the optimal scenario outcome. If the scenario-specific return vectors $\underline{R}_S^T$ weighted by vector w for the alternatives are replaced by the expected portfolio return, the target function would represent the variance of the portfolio returns across the scenarios being analysed. Minimising them results in the weighted portfolio returns being smoothed around the expected value. This method enables the correlation structure within the defined scenario returns to be incorporated into the investment decision. Quadratic deviations are also minimised during minimum-regret optimisation, although the deviations relate to the optimal scenario outcomes rather than to the expected value, which means that the correlations are also implicitly incorporated.

In the example, the minimisation of the probability-weighted total regret produces the outcome shown in Table 19.

Table 19:
Value of weight vector in the example

Alternatives	Alternative 1	Alternative 2	Alternative 3	Alternative 4	Alternative 5
Weight vector $\underline{w}$	77%	23%	0%	0%	0%

[46] See Dembo/King (1992), p. 153.

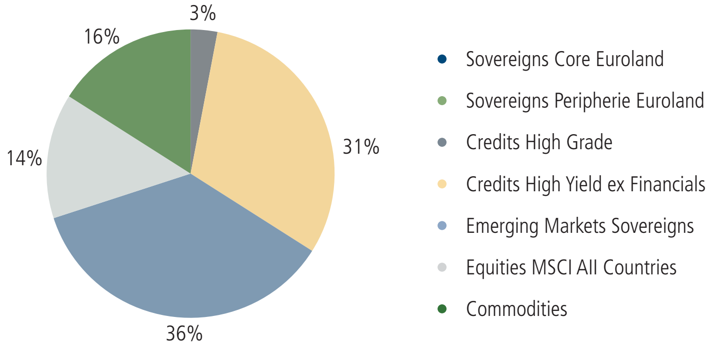
Consequently, 77 per cent of the optimal portfolio based on the minimum-regret criterion is invested in Alternative 1 and 23 per cent in Alternative 2. Nothing is invested in the remaining three alternatives which were optimised for the Pessimistic I, Pessimistic II and Fat-tail scenarios. This means that the whole investment volume available is allocated to the portfolios for the two scenarios that are estimated to be the most probable.

The percentage of each asset class in the portfolio is the weighted total for the portfolios optimised for the individual scenarios in line with section 3.3. The total is weighted using weight vector w as described above, which produces the following portfolio structure in this case:[47]

$$\underbrace{0.77}_{\substack{\text{Proportion of the total}\\ \text{portfolio represented}\\ \text{by the portfolio optimi-}\\ \text{sed for the Main}\\ \text{scenario}}} \cdot \underbrace{\begin{vmatrix} 0\% \\ 0\% \\ 4\% \\ 40\% \\ 40\% \\ 6\% \\ 10\% \end{vmatrix}}_{\substack{\text{Optimal 'Main}\\ \text{scenario' portfolio}\\ \text{structure}}} + \underbrace{0.23}_{\substack{\text{Proportion of the total}\\ \text{portfolio represented}\\ \text{by the portfolio}\\ \text{optimised for the}\\ \text{Optimistic scenario}}} \cdot \underbrace{\begin{vmatrix} 0\% \\ 0\% \\ 0\% \\ 0\% \\ 20\% \\ 40\% \\ 40\% \end{vmatrix}}_{\substack{\text{Optimal}\\ \text{'Optimistic' scenario}\\ \text{portfolio structure}}} \approx \underbrace{\begin{vmatrix} 0\% \\ 0\% \\ 3\% \\ 31\% \\ 36\% \\ 14\% \\ 16\% \end{vmatrix}}_{\substack{\text{Optimal portfolio}\\ \text{structure for all}\\ \text{scenarios based on}\\ \text{minimum-regret}\\ \text{optimisation}}}$$

This combination of alternatives produces a portfolio with the percentage of each asset class shown in Figure 28. Nothing would be invested in relatively safe sovereign bonds.

Figure 28:
Minimum-regret portfolio factoring in scenario probability



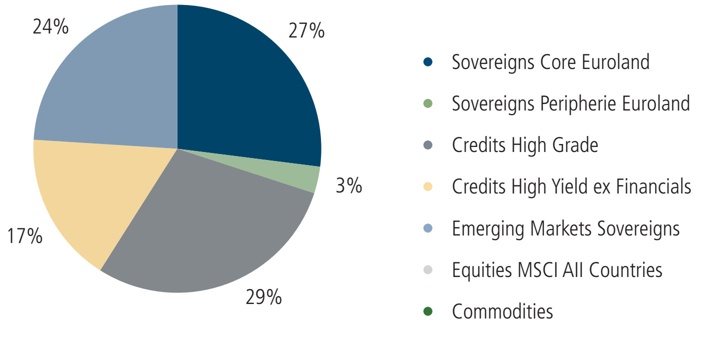
Incorporating the following constraint requiring a minimum return to be achieved in each scenario into the optimisation problem described above enables investors to take greater account of their safety requirements.[48]

$\underline{R}_S^T \cdot \underline{w} \;\underline{\geq}$ minimum return $\forall s \in S$

[47] The calculation was carried out using exact values. Only rounded results are shown.
[48] Alternatively, the level of the minimum required rate of return can be set for each scenario.

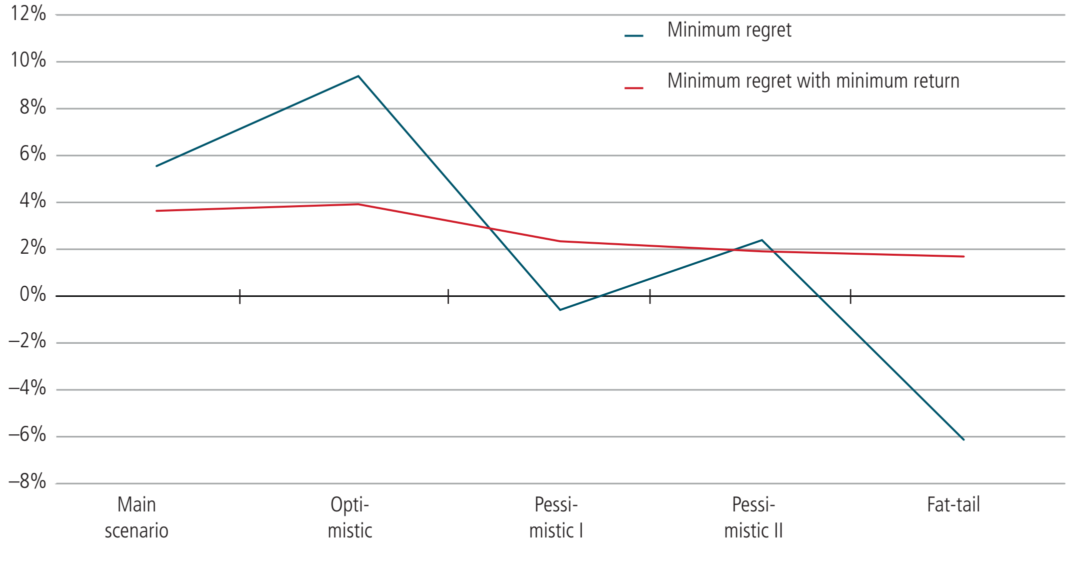
If a minimum return of 1.7 per cent is required for each scenario in the example, weight vector $\underline{w}^T = \{0\%; 0\%; 0\%; 60\%; 40\%\}$ minimises the minimum-regret optimisation problem to which a minimum-return requirement has been added.

Figure 29:
Minimum-regret portfolio with a minimum return of 1.7 per cent



If we compare the portfolio structures with a minimum required rate of return (Figure 29) and without a minimum required rate of return (see Figure 28), it is clear that the high proportions of equities and commodities in the portfolio generated without a minimum required rate of return are largely reallocated to low-risk, core eurozone sovereign bonds. Adding the minimum-return requirement limits risk primarily because the portfolio structure is more closely aligned to the Fat-tail scenario. Of course, the 'price paid' for the safety gained in this way is that the investor forgoes the upside potential in the Optimistic and Main scenarios (see Figure 30).

Figure 30:
Performance of the minimum-regret portfolio with and without a minimum return



When interpreting Figure 30, please note that, although investors who secure a minimum required rate of return forgo potentially higher returns, these scenarios are those regarded as most likely to arise, with probabilities of 50 per cent and 25 per cent.

2. Risk/return optimisation

Classic risk/return optimisation, as described in chapter 2, can also be transferred to this decision-making situation. The optimisation problem shown in Table 20 is examined for this purpose.

Table 20:
Risk/return optimisation

Target function	Constraint
$\text{Expected utility} = \sum_{s=1}^{S} p_s \cdot \text{Utility}_s = \sum_{s=1}^{S} p_s \cdot (\mu_{p,s} - \frac{\gamma}{2} \cdot \sigma^2_{p,s}) \rightarrow \max!$ with: $\sigma^2_{P,s} = \underline{x}^T \cdot \sum_s \cdot \underline{x}$ and $\mu_{P,s} = \underline{x}^T \cdot \underline{R}_S^{AK}$	1. $\sum_{i=1}^{N} x_i = 1$ 2. $x_i \geq 0$

As illustrated in Table 20, the Von Neumann-Morgenstern expected utility function is used to determine expected utility. It determines expected utility by weighting the scenario-specific utility values utility$_S$ by the probability of each scenario arising p_S.

The utility function required to determine scenario-specific utility values utilitys is the utility function explained in chapter 2.[50]

Full allocation (Constraint 1) and non-negativity of the elements used for the weight vector (Constraint 2) are again required as constraints. Please note that optimisation at this point is not based on optimised individual strategies from stage 2 as is the case for minimum-regret optimisation. Instead, optimisation is based directly on the expected returns for the individual asset classes $\underline{R}_S^{AC}$ specified within the scenario definition and on the scenario-specific variance-covariance matrices $\sum$. Therefore, optimisation does not relate to weight vector $\underline{w}$, which denotes the weightings of the individual strategies derived in stage 2 of minimum-regret optimisation, instead it relates to weight vector x, which directly indicates the portfolio weighting of the individual asset classes. This is illustrated by the following detailed formulation of the target function in Table 20:

$$\text{Expected utility} = \sum_{s=1}^{S} p_s \cdot (\mu_P - \frac{\gamma}{2} \cdot \sigma_P^2) = \sum_{s=1}^{S} p_s \cdot (\underline{x}^T \cdot \underline{R}_S^{AK} - \frac{\gamma}{2} \cdot \underline{x}^T \cdot \underline{\Sigma}_s \cdot \underline{x}) \rightarrow \max!$$

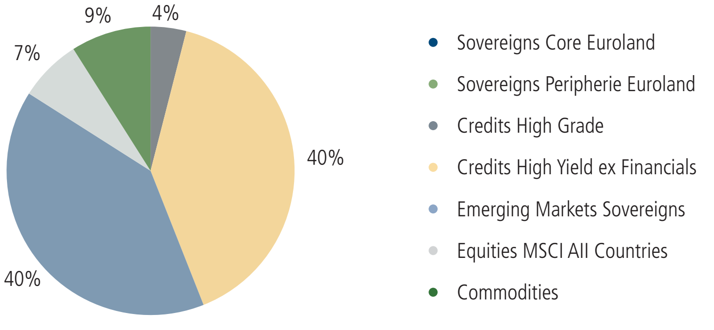
[49] See Grinold (1999), p. 13.
[50] Other utility functions can also be used.

A minimum required rate of return can also be secured in the context of risk/return optimisation by incorporating a further constraint into the optimisation model for each scenario:[51]

$$\underline{x}^T \cdot \underline{R}_s^{AK} \geq \text{minimum return} \; \forall s \in S$$

The portfolio structure shown in Figure 31 (no minimum return specified) reflects the result of risk/return optimisation for the example.

Figure 31:
Portfolio structure based on risk/return optimisation



While portfolio volatility is used to measure risk during risk/return optimisation, risk is measured on the basis of expected regret during minimum-regret optimisation. Table 21 compares the structure of the minimum-regret optimisation portfolio with the risk/return optimisation portfolio.

Table 21:
Comparison of portfolio structures for minimum-regret and risk/reward optimisation

	Minimum-regret optimisation	Risk/return optimisation
Core eurozone sovereigns	0%	0%
Peripheral eurozone sovereigns	0%	0%
High-grade credits	3%	4%
High-yield credits, ex financials	31%	40%
Emerging market sovereigns	36%	40%
Global equities	14%	7%
Commodities	16%	9%

The basic difference between the two portfolio structures is that a larger proportion of the minimum-regret optimisation portfolio is invested in equities and commodities and it is therefore slightly more opportunity-oriented than the risk/return optimisation portfolio. The consequences of these strategies are shown in Figure 32 which compares the future upside potential of both portfolios in the scenarios being analysed.

[51] Scenario-dependent minimum required returns can also be incorporated.

Figure 32:
Performance of the minimum-regret portfolio compared with the risk/return portfolio

The minimum-regret portfolio generates higher returns in both the Main and Optimistic scenarios—due to the higher proportion of equity and commodity asset classes in the portfolio. Conversely however, lower returns are generated in the Pessimistic and Fat-tail scenarios. This outcome can be explained by the structure of the target function in which three-quarters of the probability mass is concentrated on the Main and Optimistic scenarios. Because the weighted regret in these scenarios can be particularly high, minimum-regret optimisation tries to generate returns that are close to the optimal at these particular points. Deviations in the other scenarios are less relevant because the scenarios are much less likely to arise.

3. Minimum-return optimisation

The two preceding sections showed how asset allocation can be undertaken using minimum-regret optimisation and risk/return optimisation. Using the example, we explained that both methods focus closely on the two most probable scenarios in which high returns are generated if the scenarios arise. Consequently, both methods tend to be opportunity-oriented, albeit to different extents. We also demonstrated that the inclusion of a minimum-return constraint can take more account of the Fat-tail and Pessimistic scenarios. Minimum-return optimisation is the final method to be examined. Using this method, it is possible to undertake asset allocation for investors with a strong safety focus. The structure of the basic risk/return optimisation model used in this paper is shown in Table 22.

Table 22:
Minimum-return optimisation

Target function	$\sum_{s \in S} p_s \cdot (\underline{R}_S^T \cdot \underline{w} - \text{minimum return}^2 \rightarrow \text{min!}$
Constraints	1. $\sum_{i=1}^{N} w_i = 1$ (Full allocation) 2. $w_i \geq 0$ (Positive weighting)

Please note that optimisation at this point is another two-step process, so the results are based on the second, separate optimisation step. Optimisation relates to weight vector $\underline{w}$, which indicates the proportions of the optimal individual strategies (stage 2) in the total portfolio.

The target function is very similar to that used for minimum-regret optimisation. While quadratic deviations from scenario-specific maximum returns were calculated in minimum-regret-optimisation, the total probability-weighted quadratic deviation from an exogenously specified minimum return is minimised for minimum-return optimisation. The minimum required rate of return remains the same for all scenarios. Full allocation (Constraint 1) and a positive weighting (Constraint 2) are again required as constraints.

The target function characterised in Table 22 minimises the weighted quadratic deviations from a given minimum required rate of return. Squaring the deviations ensures that small deviations from the required minimum return are penalised less than larger deviations. Investors who base their investment decisions on this target function are trying to minimise the regret caused by the portfolio return deviating from the specified minimum required return. Positive and negative deviations from the minimum required return are penalised equally, but because only negative deviations are undesirable to investors, the target function shown above is modified by the target function shown in Table 23.[52]

Table 23:
Minimum-return optimisation with modified target function

Modified Target function	$\sum_{s \in S} p_s \cdot d_s \cdot (\underline{R}_S^T \cdot \underline{w} - \text{minimum return}^2 \rightarrow \text{min!}$
	with: $d_s = \begin{cases} 1, \text{ if } (\underline{R}_S^T \cdot \underline{w} - \text{minimum return}) < 0 \\ 0, \text{ if } (\underline{R}_S^T \cdot \underline{w} - \text{minimum return}) \geq 0 \end{cases}$

Variable d_s is added to the target function. If the portfolio return weighted by the weight vector is lower than the minimum required return in the scenario being analysed, it is assigned a value of one, which means that negative deviations are still penalised quadratically. However, if the portfolio return weighted by the weight vector is higher than the minimum required return, d_s has a value of zero so that positive deviations are no longer penalised.[53]

[52] Based on Rudolf et al. (1999), p. 89.

[53] It is also possible to provide a special reward for positive deviations from the minimum return by defining d_s appropriately, i.e. adjusting the relative level of the reward compared with that of the penalty.

Figure 33:
Portfolio structures optimised for a minimum return of 4 per cent

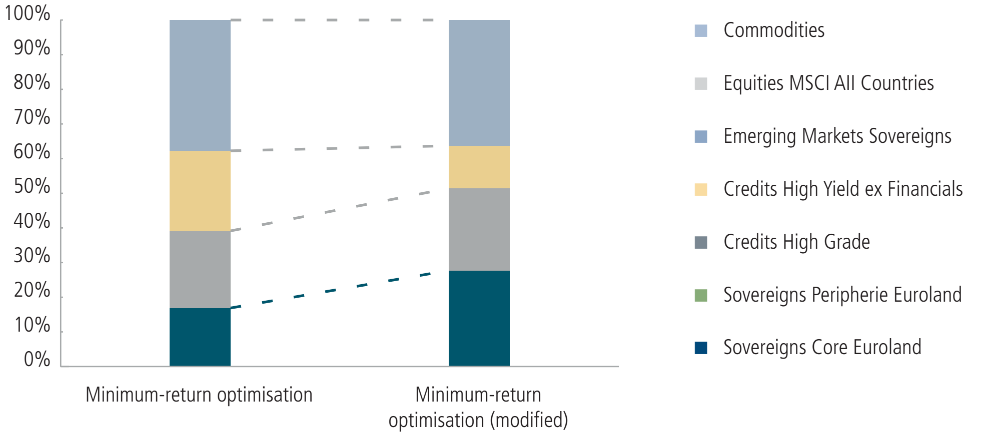
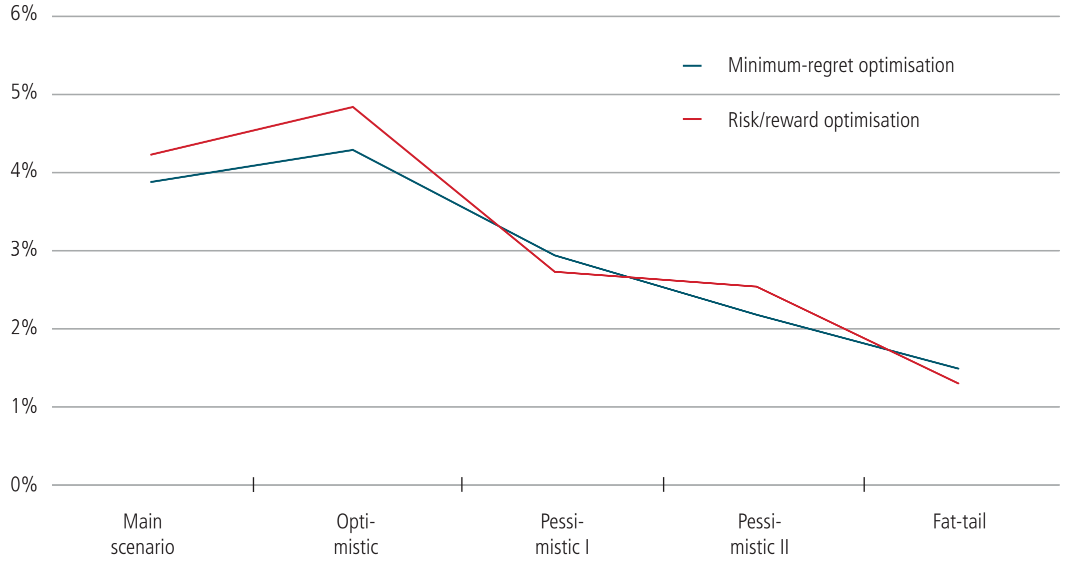


Figure 33 compares the portfolio structures for the two target functions with a minimum required return of 4 per cent. It is clear that both target functions tend to produce investments in lower-risk asset classes, i.e. there are no equities or commodities in the portfolios. This ensures that deviations from the minimum required return are not too pronounced, even in the Pessimistic and Fat-tail scenarios. Because the modified target function does not penalise positive deviations, the proportions of high-yield bonds in the associated portfolio structure are higher and the proportions of low-risk, core eurozone sovereign bonds are lower. Figure 34 compares the performance of both portfolios across all scenarios.

Figure 34:
Comparison of the performance of minimum-return optimisation portfolios



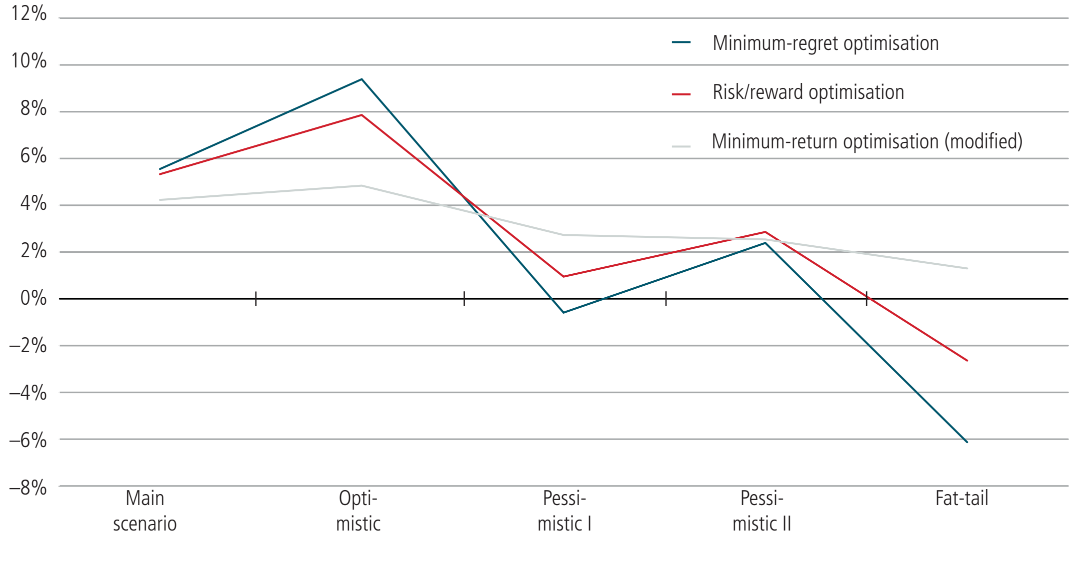
Because they are of a similar composition, both portfolios result in a similar performance in the individual scenarios. If the Main scenario arises, the modified minimum-return optimisation variant generates a return of 4.23 per cent and if the Optimistic scenario arises it generates a return of 4.84 per cent. By contrast, the portfolio based on the 'simple' target function generates lower returns of 3.88 per cent (Main scenario) and

4.29 per cent (Optimistic scenario). These differences may appear small, but if we consider that the Main and the Optimistic scenarios are those most likely to occur—with probabilities of 50 per cent and 25 per cent—they become much more relevant.

It is important to stress that the return generated in the context of minimum-return optimisation can easily fall short of the minimum rate specified but (quadratic) deviations from it are minimised. By contrast, the return constraint introduced in the context of minimum-regret optimisation excludes returns that fall short of the minimum required rate.

We pointed out earlier that it is not possible to say that one of the optimisation models being analysed is generally superior, because investors' specific attitudes to risk or the investment objectives they are pursuing are always the deciding factors when selecting a target function. Investors who use the minimum-value optimisation model for asset allocation are adopting a safety-oriented strategy, in return for which they are consciously forgoing any upside potential in advance. Figure 35 compares the performance of the portfolio based on the modified variant of minimum-value optimisation with the performance of the portfolios generated using minimum-regret optimisation and risk/return optimisation.

Figure 35:
Performance comparison of optimisation models



The performance of the modified, minimum-return optimisation portfolio is much less volatile than that of the portfolios generated using minimum-regret optimisation and risk/return optimisation. No losses are generated in any scenario but in return, the investor has to forgo the high potential returns offered in the Main and Optimistic scenarios. As mentioned above, when interpreting the differences it should be borne in mind at all times that the overall probability of the Main and Optimistic scenarios arising in the example is 75 per cent.

It has been shown that a constraint that requires a mandatory minimum rate of return can be added to both minimum-regret optimisation and risk/return optimisation. As also demonstrated, the addition of this constraint to the models eliminates some of the volatility range, depending on the level set for the minimum required return. Of course, some of the upside potential is also forgone in return.

Finally, we reiterate that the specific optimisation model selected always depends on the individual case and, specifically, on the level of investor risk aversion. It should also be stressed that all the optimisation models presented in this paper represent 'basic versions' to which any additional, investor-specified restrictions can be added.

4 Conclusion

The results of the 2013 risk study have shown that institutional investors are increasingly turning their attention to the inclusion of economic performance and stress scenarios in the investment decision process. This was the impetus for this research paper to provide a systematic demonstration of how performance scenarios that factor in investor-specific risk appetite can be incorporated into the portfolio optimisation calculation.

Firstly, the underlying concept of classic Markowitz portfolio optimisation was explained, and the weaknesses of this approach were highlighted. This was followed by the systematic development of a method of integrating performance scenarios into the decision-making process. In particular, classic Markowitz-style risk/return optimisation was transferred to the scenario case and compared with an alternative optimisation model (minimum-regret optimisation). The basics of this model were first illustrated for decision making under uncertainty, before the model was transferred to decision making under risk. Unlike classic risk/return optimisation, minimum-regret optimisation is based on the intuitive risk measure known as regret. The minimum-return optimisation model was also introduced and compared with the two models that had previously been explained.

We found that the portfolio structures based on the underlying models differ widely in some respects. This is not a cause for regret, in fact it highlights the importance of a structured allocation process for overall investment performance. The use of decision theory and scenario techniques compels investors to pay close attention to their attitude to risk and their investment objectives.[55] The choice of an optimisation model is then 'merely' a formal means of incorporating personal risk attitudes and the investment objectives pursued by investors into the decision process and of translating them into a specific allocation proposal. Only when this has been clearly formulated is it possible to determine the optimal asset allocation. This applies equally to subsequent performance and risk reports because any assessment of the performance of individual investments must be based on predetermined targets.

55 Banks in particular also have to integrate their management of deposits into their overall management strategy and the objectives it pursues. See Wiedemann/Wiechers (2013) for more details of the concept of overall return/risk management for banks.

Annex A

Scenario-specific variance-covariance matrices

Main scenario covariance matrix	Core eurozone sover-eigns	Pheripheral eurozone sover-eigns	High gradae credit	High yield credits, ex financials	Emerging market sover-eigns	Global equities	Commod-ities
Core eurozone sovereigns	0.14%	0.09%	0.08%	-0.01%	0.00%	–0.18%	–0.09%
Peripheral eurozone sovereigns	0.09%	0.26%	0.06%	0.02%	0.04%	0.02%	0.00%
High-grade credits	0.08%	0.06%	0.08%	0.02%	0.03%	-0.08%	–0.03%
High-yield credits, ex financials	–0.01%	0.02%	0.02%	0.47%	0.15%	0.32%	0.11%
Emerging market sovereigns	0.00%	0.04%	0.03%	0.15%	0.65%	0.55%	0.26%
Global equities	–0.18%	0.02%	–0.08%	0.32%	0.55%	2.92%	1.06%
Commodities	–0.09%	0.00%	–0.03%	0.11%	0.26%	1.06%	2.90%

Global economic recovery covariance matrix	Core eurozone sover-eigns	Pheripheral eurozone sover-eigns	High gradae credit	High yield credits, ex financials	Emerging market sover-eigns	Global equities	Commod-ities
Core eurozone sovereigns	0.13%	0.08%	0.08%	-0.01%	0.03%	–0.12%	–0.06%
Peripheral eurozone sovereigns	0.08%	0.26%	0.05%	0.02%	0.06%	0.08%	0.04%
High-grade credits	0.08%	0.05%	0.08%	0.01%	0.03%	–0.05%	–0.02%
High-yield credits, ex financials	–0.01%	0.02%	0.01%	0.30%	0.06%	0.19%	0.07%
Emerging market sovereigns	0.03%	0.06%	0.03%	0.06%	0.31%	0.28%	0.13%
Global equities	–0.12%	0.08%	–0.05%	0.19%	0.28%	1.89%	0.68%
Commodities	–0.06%	0.04%	–0.02%	0.07%	0.13%	0.68%	2.43%

Resurgence of eurozone crisis covariance matrix	Core eurozone sover-eigns	Pheripheral eurozone sover-eigns	High gradae credit	High yield credits, ex financials	Emerging market sover-eigns	Global equities	Commod-ities
Core eurozone sovereigns	0.21%	0.15%	0.12%	–0.02%	–0.08%	–0.46%	–0.21%
Peripheral eurozone sovereigns	0.15%	0.31%	0.10%	0.03%	–0.02%	–0.18%	–0.10%
High-grade credits	0.12%	0.10%	0.13%	0.07%	0.01%	–0.21%	–0.08%
High-yield credits, ex financials	–0.02%	0.03%	0.07%	1.24%	0.50%	0.84%	0.28%
Emerging market sovereigns	–0.08%	–0.02%	0.01%	0.50%	2.03%	1.63%	0.78%
Global equities	–0.46%	–0.18%	–0.21%	0.84%	1.63%	7.27%	2.61%
Commodities	–0.21%	–0.10%	–0.08%	0.28%	0.78%	2.61%	4.45%

Sharp rise in interest rates covariance matrix	Core eurozone sover-eigns	Pheripheral eurozone sover-eigns	High gradae credit	High yield credits, ex financials	Emerging market sover-reigns	Global equities	Commod-ities
Core eurozone sovereigns	0.11%	0.09%	0.07%	–0.01%	0.02%	–0.09%	–0.04%
Peripheral eurozone sovereigns	0.09%	0.18%	0.06%	0.01%	0.04%	0.01%	0.01%
High-grade credits	0.07%	0.06%	0.06%	0.01%	0.03%	–0.04%	–0.01%
High-yield credits, ex financials	–0.01%	0.01%	0.01%	0.26%	0.08%	0.17%	0.05%
Emerging Markets sovereigns	0.02%	0.04%	0.03%	0.08%	0.39%	0.32%	0.13%
Global equities	–0.09%	0.01%	–0.04%	0.17%	0.32%	1.72%	0.59%
Commodities	–0.04%	0.01%	–0.01%	0.05%	0.13%	0.59%	2.82%

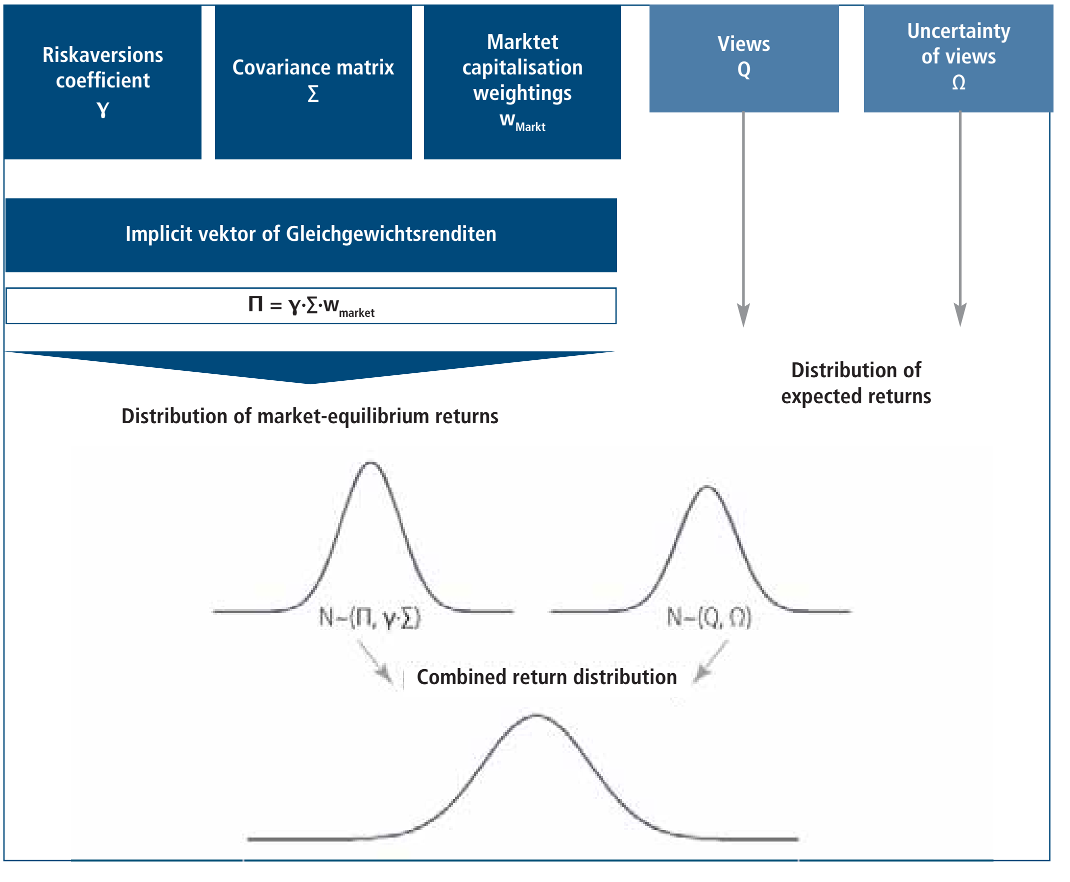
Extreme scenario covariance matrix	Core eurozone sover-eigns	Pheripheral eurozone sover-eigns	High gradae credit	High yield credits, ex financials	Emerging market sover-eigns	Global equities	Commod-ities
Core eurozone sovereigns	0.24%	0.18%	0.14%	–0.03%	–0.12%	–0.58%	–0.26%
Peripheral eurozone sovereigns	0.18%	0.33%	0.11%	0.03%	–0.04%	–0.26%	–0.14%
High-grade credits	0.14%	0.11%	0.14%	0.09%	0.00%	–0.26%	–0.10%
High-yield credits, ex financials	–0,.03%	0.03%	0.09%	1.57%	0.65%	1.06%	0.35%
Emerging market sovereigns	–0.12%	–0.04%	0.00%	0.65%	2.62%	2.09%	1.01%
Global equities	–0.58%	–0.26%	–0.26%	1.06%	2.09%	9.13%	3.28%
Commodities	–0.26%	–0.14%	–0.10%	0.35%	1.01%	3.28%	5.11%

Annex B

Basic concept of the Black-Litterman model

Figure 36 illustrates the basic operation of the Black-Litterman model.[56] The distribution of market-equilibrium returns is determined on the basis of risk aversion coefficient γ estimated using capital-market data, covariance matrix of returns Σ and market capitalisation weightings of the asset classes w_{market}. In the model, this represents a normal distribution with mean Π and variance $\gamma \cdot \Sigma$. The investor-specified expected returns in the scenarios are factored into the model using the Views Q vector. Investors can record the uncertainty of their views using matrix Ω.[57] The distribution of the investor-specified expected returns also represents a normal distribution with mean Q and variance Ω.

Figure 36:
Operation of the Black-Litterman model



Finally, both distributions are merged into a combined return distribution which also represents a normal distribution in the Black-Litterman model.[58] Interposing the Black-Litterman filter provides investors with the opportunity to combine their own expected returns in each scenario with market-equilibrium returns.[59]

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[56] Based on Idzorek (2004), p. 16.
[57] Here and below, see Idzorek (2004), p. 3 onwards.
[58] See Idzorek (2004), p. 16.
[59] Important note: use of the Black-Litterman filter is optional. Please also note that the Black-Litterman model has recently been criticised; see Michaud et al. (2013), p. 16.

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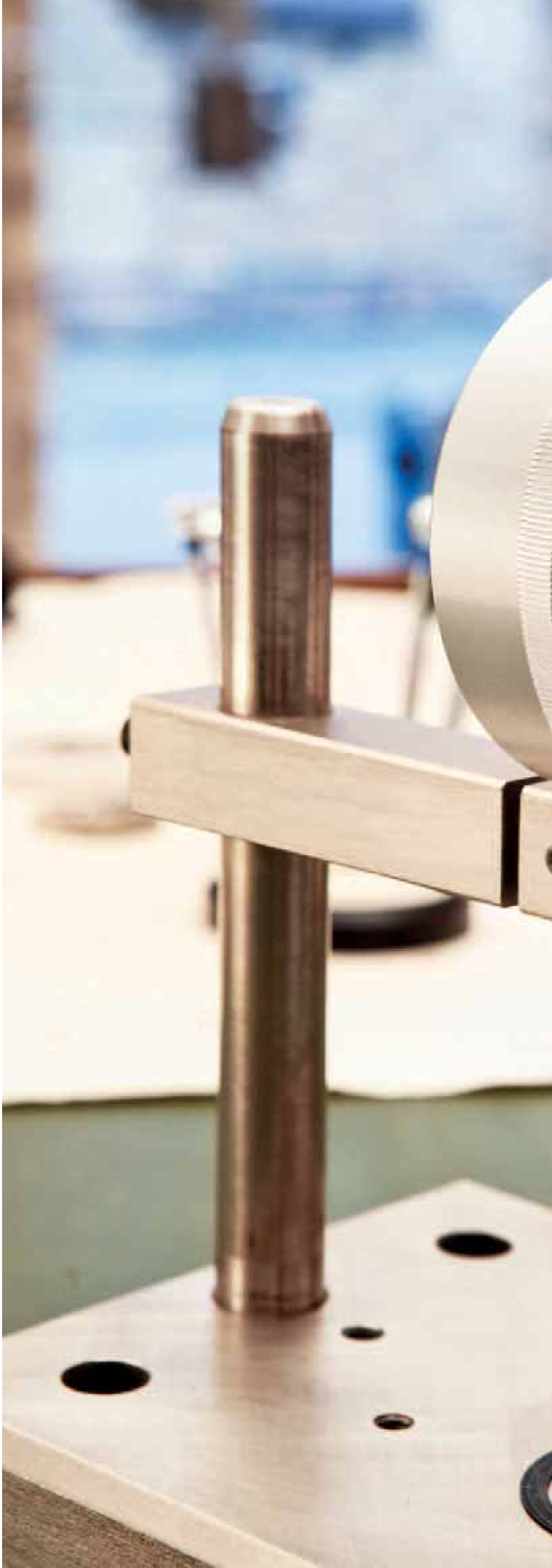


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Dated: November 2013
005614 11.13