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VWL

Computer Simulation of Reallocating Resources among Growing Regions

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I.   Growth as a Result of Regional Competition

Based on an approach presented by Buhr (2000), the computer simulation in this paper is concerned with the growth effects on the generation, distribution, and use of income. At first the private and public sectors, as well as the State, fix certain parameters which determine their behavior for the entire period under consideration. The simulation computes the results for a two-region model with respect to regional and national growth. The ensuing parameter variations reflect regional competition. The outcomes of the simulation runs are then used to determine an optimal behavior with respect to sectoral objectives. On the one hand, the private sectors try to increase an overall utility index which depends on the time path of consumption per capita; in this sense the private sectors compete for consumable and investment commodities. On the other hand, it is the task of the public sectors to supply public capital to the private sectors and to strengthen the productivity of labor by expenditure on education. Moreover, the public sectors may allocate subsidies to attract private capital from the other region. The means for raising public expenditure mainly include taxation of private income. According to this idea, the public sectors organize and determine the results of competition. Finally, the State imposes a tax on public income and reallocates these resources in order to improve the situation of one region or of both regions together. Now the State can be interpreted as an institution correcting the process of competition to some extent.

II.   Basics of the Model

Labor Force.   The model deals with two regions ($i = 1, 2$) which are embedded in a State. Figure 1 shows the structure of this approach. The quoted linkages between the regions and the State are explained in more detail by Buhr (2000). Each of the two regions includes a private and a public sector. Both regions are mainly characterized by four state variables – two types of labor and two stocks of capital. Serving as factors of production, they determine the flow of income. With regard to labor force L_i we distinguish the supply of educated labor from the supply of raw labor, $L_i^S = L_i^{edu,S} + L_i^{raw,S}$. In principle both groups of workers may grow at different natural rates, i.e. n_i^{edu} and n_i^{raw}. Moreover, both of them

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may migrate between the two regions. The net migration from region 1 (R1) to region 2 (R2) is denoted by L_{12}^{edu} or L_{12}^{raw} depending on who migrates. The boxes for both regions in Figure 1 include the corresponding differential equations $\dot{L}_i^{edu,S}$ and $\dot{L}_i^{raw,S}$, which describe the rate of change of regional supply of educated and raw labor, respectively.
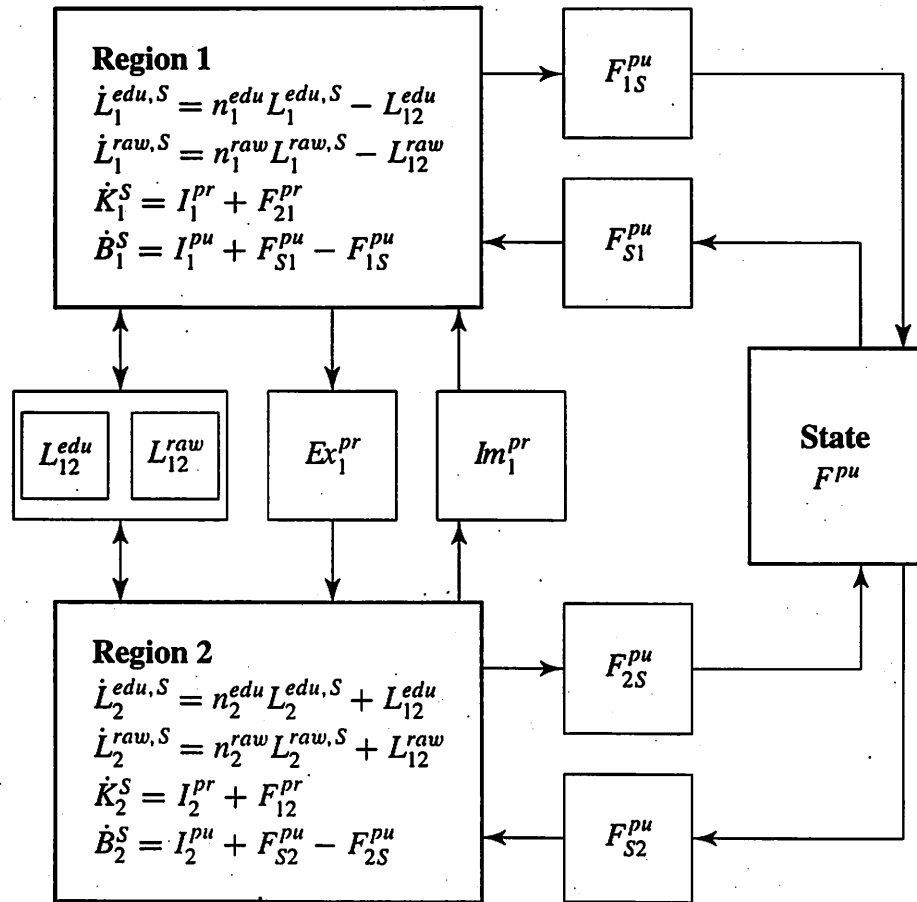

Region 1

$$\dot{L}_1^{edu,S} = n_1^{edu} L_1^{edu,S} - L_{12}^{edu}$$
$$\dot{L}_1^{raw,S} = n_1^{raw} L_1^{raw,S} - L_{12}^{raw}$$
$$\dot{K}_1^S = I_1^{pr} + F_{21}^{pr}$$
$$\dot{B}_1^S = I_1^{pu} + F_{S1}^{pu} - F_{1S}^{pu}$$

F_{1S}^{pu}

F_{S1}^{pu}

L_{12}^{edu} L_{12}^{raw} Ex_1^{pr} Im_1^{pr}

State F^{pu}

Region 2

$$\dot{L}_2^{edu,S} = n_2^{edu} L_2^{edu,S} + L_{12}^{edu}$$
$$\dot{L}_2^{raw,S} = n_2^{raw} L_2^{raw,S} + L_{12}^{raw}$$
$$\dot{K}_2^S = I_2^{pr} + F_{12}^{pr}$$
$$\dot{B}_2^S = I_2^{pu} + F_{S2}^{pu} - F_{2S}^{pu}$$

F_{2S}^{pu}

F_{S2}^{pu}

Figure 1: Dynamic aspects of the model.

Physical Capital Stocks. The third and the fourth state variables of region i describe two physical capital stocks. The private sectors supply private capital K_i^S and the public sectors supply infrastructure capital B_i^S. The accumulation of these capital stocks is known from the sectoral capital transactions accounts. It depends essentially on private and public investment which are abbreviated as I_i^{pr} and I_i^{pu}, respectively. This yields a system of differential equations regarding the supply

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of physical capital stocks, which again can be found in the corresponding boxes of Figure 1.

As far as $\dot{K}_1^S$ is concerned, the term F_{21}^{pr} indicates that residents of R2 make investments in R1. This investment is induced by subsidies G_1^{pu} payed by the public sector of R1. The opposite effect in $\dot{K}_2^S$ is caused by subsidies G_2^{pu} of R2 and is recorded by F_{12}^{pr}. The two differential equations of infrastructure capital $\dot{B}_i^S$ take into consideration that the State reallocates resources among both regions. The public sector of region i transfers resources F_{iS}^{pu} to the State and receives F_{Si}^{pu} from the State.

Expenditures Affecting Productivities. Besides private investment I_i^{pr} the private sector makes expenditures $I_i^{pr,dev}$ on research and development (R&D). Hence, total private investment in region i is $\tilde{I}_i^{pr} = I_i^{pr} + I_i^{pr,dev}$. The accumulated level K_i^{dev} of expenditures on R&D is determined by

$$\dot{K}_i^{dev} = I_i^{pr,dev} \qquad (i = 1, 2)$$

and increases the productivity parameter a_i^{edu} of educated labor. The initial values at time $t = 0$ are supposed to be a fractional part of private capital stocks: $K_1^{dev} = 0.0053 K_1^S$ and $K_2^{dev} = 0.0050 K_2^S$.

Similarly, a part of public investment is dedicated to education. However, public capital is not differentiated as to its purposes. The total level of public investment on education is merely needed, since it affects the productivities of both kinds of labor.

$$\dot{B}_i^{edu} = I_i^{pu,edu} \qquad (i = 1, 2)$$

Again the initial values are fixed at certain fractional parts of public capital: $B_1^{edu} = 0.0045 B_1^S$ and $B_2^{edu} = 0.0045 B_2^S$.

Distribution of Assets. The assets of residents of region 1 (2) held in region 2 (1) are denoted by W_{12} (W_{21}). They are treated as additional state variables and change simultaneously in accordance with the account of R2 from the point of view of R1.

$$Z_1^{pr} + F_{12}^{pr} + r_2 W_{12} + G_2^{pu} + \dot{W}_{21} = Z_2^{pr} + F_{21}^{pr} + r_1 W_{21} + G_1^{pu} + \dot{W}_{12}$$

Because the simulation depends on the isolated state variables W_{12} and W_{21}, it is supposed that this relation can be split up into two equations.[1] Notice that the term

[1] If both regions have the same interest rate $r = r_1 = r_2$, then the distinction between W_{12} and W_{21} becomes irrelevant, provided the distribution of assets is of minor importance.

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F_{12}^{pr} increases the capital stock K_2^S as it is the aim of the subsidy G_2^{pu}. Therefore, it is assigned to $\dot{W}_{12}$. Analogously, F_{21}^{pr} is a part of $\dot{W}_{21}$. It is postulated that

$$\dot{W}_{12} = G_2^{pu} + F_{12}^{pr} + Z_1^{pr} - Z_2^{pr} + r_2 W_{12} - r_1 W_{21}$$
$$= G_2^{pu} + F_{21}^{pr} + \Delta F_1^n$$
$$\dot{W}_{21} = G_1^{pu} + F_{21}^{pr}$$

Using the balance of the private sector's current account the above identity reduces to

$$\Delta F_1^n = \dot{W}_{12} - \dot{W}_{21} + G_1^{pu} - G_2^{pu},$$

where ΔF_1^n is the net investment of R1 in R2. In this case an alternative formulation could be $\dot{W}_{12} = G_2^{pu} + \Delta F_1^n$ and $\dot{W}_{21} = G_1^{pu}$.

Comments on the Runge-Kutta-Fehlberg Method. The above system of differential equations is approximated by a system of difference equations. A numerical solution to the system of difference equations is then determined.[2] While the first step introduces a truncation error, the second step involves rounding errors. In order to estimate the truncation error, the Runge-Kutta-Fehlberg 4/5 method can be thought of as having been obtained by expanding the solution function in a Taylor series at a particular point.[3] Afterwards certain terms are omitted which account for the truncation error.[4]

If the estimate for the local error between time t and $t + h$ is too big, h is reduced. The state variables are updated provided the estimated local error is acceptable. In order to compare different simulation runs h is set to 0.0002. That corresponds to a time interval of two hours. An increase of h to 0.01, i.e. an interval of 3.65 days, dramatically reduces the time needed for simulation runs. But the simulation runs are then distorted by truncation errors so that parameter variations are more difficult to compare.

Figure 2 shows a flow chart of the solution to the model at each point of time. It is to be read line by line and leads through the following steps. In accordance with the Runge-Kutta-Fehlberg method the first line mainly includes the state variables,

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[2] We make use of our own software. Nevertheless, there is more software available in order to forecast economic-demographic effects in a multi-area region context. The input-output simulation model of the Regional Economic Models, Inc. (REMI) is described in Treyz, Rickman, and Shao (1992) and the newest version of REMI's software is called REMI Policy Insight.

[3] Similarly the economic modelling software GEMPACK uses a multi-step Euler solution; cf. Harrison and Pearson (1996, p. 106 f.). Another simulation tool is SIMULINK, which requires the use of MATLAB. This software asks for the simulation method to be used; cf. Herbert and Bell (1997, p. 111).

[4] Cf. Vandergraft (1983, Chapter 8) for the numerical approximation of differential equations. The applied PASCAL algorithm has been taken from Engeln-Müllges and Reutter (1991, p. 898 ff.) and then translated to C++.

while the last line describes their change in time. For the sake of clarity the boxes of $\dot{L}_i^{edu,S}$ and $\dot{L}_i^{raw,S}$ are omitted because they are distorted by the migration terms L_{12}^{edu} and L_{12}^{raw} of line 3.
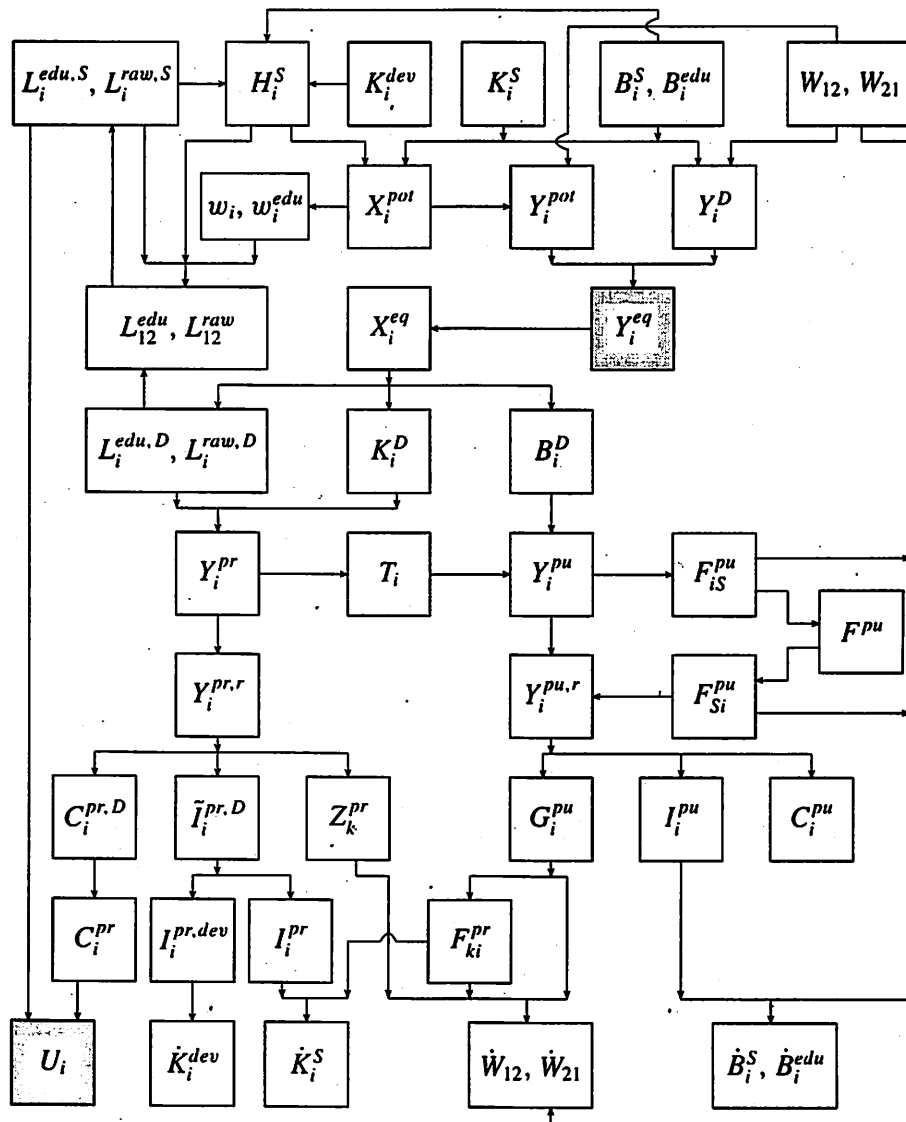

Figure 2: Sketch of the solution to the model.

III. Numerical Solution to a Standard Case

1. Aggregate Supply

The Choice of the Production Function. Starting with the state variables included in the first line of Figure 2 there is an additional variable H_i^S which denotes the supply of human capital in region i. Human capital is composed of educated labor L_i^{edu} and raw labor L_i^{raw} where $H_i = g_i(L_i^{edu}, L_i^{raw})$ is assumed to have a linear homogeneous CES structure.[5] The demand for labor is restricted to

$$L_i^{edu,D} \leq L_i^{edu,S} \qquad (i = 1, 2).$$

Hence, the maximum amount of human capital H_i^{max} is realized only if both kinds of labor are fully employed.

Line 2 of Figure 2 starts with the potential domestic production X_i^{pot} of region i. The analysis is based on two linear homogeneous CES production function $X_i^{pot} = f_i(H_i, K_i, B_i)$ where the elasticity of substitution ρ_i is the same for the two regions. Regarding this type of production function, there are two special cases of major importance. The first case concerns the elasticity of substitution $\rho_i = 0$ so that f_i takes the form of a Cobb-Douglas production function. In the second case f_i approaches a linear limitational production function as $\rho_i \to -\infty$.

Wage Rates. In accordance with the linear homogeneous production functions, it is assumed that there is no positive profit in any region and that the adding up theorem applies. Given the rental rates r_i and r_i^* of the private and public physical capital stocks, respectively, the rental rate w_i of human capital H_i is determined by the adding up theorem. Moreover, w_i is a linear homogeneous function in the wage rates w_i^{edu} and w_i^{raw} of educated and raw labor, respectively.

As long as every output X_i is produced at minimum cost the following optimum condition must be valid:

$$\frac{w_i^{raw}}{w_i^{edu}} = \frac{\partial g_i(L_i^{edu,D}, L_i^{raw,D})/\partial L_i^{raw}}{\partial g_i(L_i^{edu,D}, L_i^{raw,D})/\partial L_i^{edu}}.$$

Therefore, the demand for both kinds of labor lies on a ray through the origin whose direction is determined by (w_i^{raw}, w_i^{edu}). On a perfect labor market with full employment the two wage rates w_i^{edu} and w_i^{raw} are adjusted so that the preceding optimum condition holds good for the labor supply $L_i^{edu,S}$ and $L_i^{raw,S}$. See the ray $\overrightarrow{OA}$ in Figure 3.

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[5] In contrast to this idea of human capital, other authors define an individual's human capital to be his skill level; cf. e.g. Lucas (1988).

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This approach is useful to determine a plausible time path for w_i^{raw} provided w_i^{raw} is fixed outside the labor market. A time path of a minimum wage rate could be, e.g.,

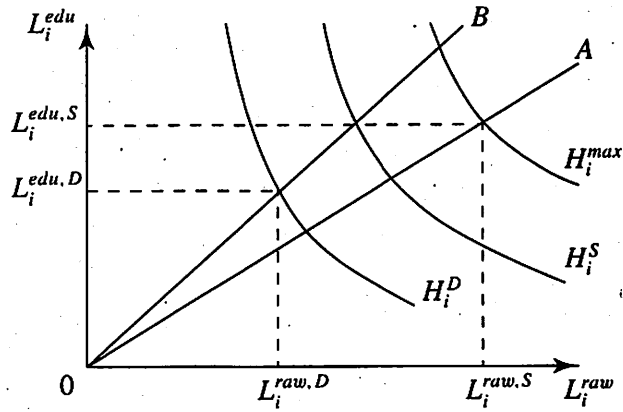$$w_i^{raw}(t) = w_i^{raw}(0) + \delta_i^{raw} t$$

$$(i = 1, 2).$$

Figure 3: Cost minimizing demand for human capital H_i of region i.

Suppose that distortions of this kind increase w_i^{raw} at each point of time. Provided w_i^{raw}/w_i^{edu} is constant, then the result is a ray depicted as $\overrightarrow{OB}$ in Figure 3.

Finally, at each point of time, the maximum supply of human capital H_i^S depends on the observed ray through the origin. According to the linear homogeneity of g_i and the restricted labor supply it follows[6]

$$H_i^S = g_i(1, \zeta_i) \cdot \min\left\{L_i^{edu,S}, L_i^{raw,S}/\zeta_i\right\} \qquad (i = 1, 2).$$

Hence, Figure 3 involves an H_i^S which corresponds to the ray $\overrightarrow{OB}$.

Time Paths of Potential Regional Products. Similarly to the two dimensional case of Figure 3, the factor demand for human and physical capital stocks determines a ray through the origin.[7] In the case of a CES production function this ray is[8]

$$\begin{pmatrix} H_i^D \\ K_i^D \\ B_i^D \end{pmatrix} = \begin{pmatrix} a_i^{-\beta_i}(w_i/\alpha_i^H)^{\beta_i-1} \\ k_i^{-\beta_i}(r_i/\alpha_i^K)^{\beta_i-1} \\ b_i^{-\beta_i}(r_i^*/\alpha_i^B)^{\beta_i-1} \end{pmatrix} \frac{X_i}{A_i} Z_i^{-1/\rho_i}.$$

Suppose that both regions maximize the potential output or domestic product X_i^{pot} with respect to the preceding ray. Then the supply of human and physical capital

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[6] The constant ζ_i is given as $\zeta_i = L_i^{raw,D}/L_i^{edu,D}$ and is a composition of parameters and rental rates. It is calculated in Buhr (2000).

[7] In accordance with Figure 3 the reader may find it useful to depict a three dimensional box where the size of the box is given by the supply of factors (H_i^S, K_i^S, B_i^S). Then each ray through the origin crossing the positive orthant intersects one face of the box at a certain point. This point yields the factor demand (H_i^D, K_i^D, B_i^D) and the corresponding potential output X_i^{pot}.

[8] Again, the constant ζ_i and the following Z_i are compositions of parameters and rental rates. They are calculated in Buhr (2000).

stocks (H_i^S, K_i^S, B_i^S) restricts this output:

$$X_i^{pot} = A_i\, Z_i^{1/\rho_i}\, \min \left\{ \frac{H_i^S\, a_i^{\beta_i}}{(w_i/\alpha_i^H)^{\beta_i-1}}, \frac{K_i^S\, k_i^{\beta_i}}{(r_i/\alpha_i^K)^{\beta_i-1}}, \frac{B_i^S\, b_i^{\beta_i}}{(r_i^*/\alpha_i^B)^{\beta_i-1}} \right\}.$$

If it is assumed that private capital K_i^S determines X_i^{pot}, then in general $H_i^D < H_i^S$ and $B_i^D < B_i^S$. The potential gross regional product of the two regions is

$$Y_1^{pot} = X_1^{pot} + r_2 W_{12} - r_1 W_{21},$$
$$Y_2^{pot} = X_2^{pot} + r_1 W_{21} - r_2 W_{12}.$$

The time path (Y_1^{pot}, Y_2^{pot}) resulting from a simulation run with two CES production functions is depicted in Figure 4, where the numbers on both axes have to be multiplied by 10^{11}.
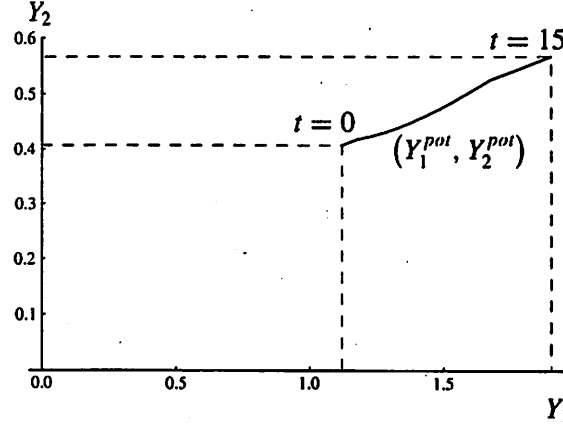
Figure 4: Time path of regional supply based on two CES production functions with $\rho_i = -2$.

2. Aggregate Demand

Because both public sectors fix their investment residually, the public incomes retained can be calculated from the income and outlay accounts of the public sectors:

$$Y_1^{pu,r} = [1 - (1 - v)\tau_1][t_1 Y_1 + (1 - t_1)r_1^* B_1^S] + v\tau_2[t_2 Y_2 + (1 - t_2)r_2^* B_2^S],$$
$$Y_2^{pu,r} = (1 - v\tau_2)[t_2 Y_2 + (1 - t_2)r_2^* B_2^S] + (1 - v)\tau_1[t_1 Y_1 + (1 - t_1)r_1^* B_1^S].$$

The corresponding equations for the incomes retained of the private sectors are

$$Y_i^{pr,r} = (1 - t_i)(Y_i - r_i^* B_i^S) \qquad (i = 1, 2).$$

In accordance with these four relationships *MATHEMATICA* analytically solves the following system of equations for the regional products Y_1 and Y_2 demanded, so that all the variables of line 2 in Figure 2 are known.

$$I_1^{pr} = S_1^{pr} - Z_1^{pr} - F_{12}^{pr} + Z_2^{pr} + r_1 W_{21} - r_2 W_{12} - G_2^{pu}$$
$$I_2^{pr} = S_2^{pr} - Z_2^{pr} - F_{21}^{pr} + Z_1^{pr} + r_2 W_{12} - r_1 W_{21} - G_1^{pu}$$

The resulting linear equations

$$Y_1 = f_1(Y_2)\ [\Longleftrightarrow\ Y_2 = f_1^{-1}(Y_1)], \quad \text{and} \quad Y_2 = f_2(Y_1)$$

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involve complex coefficients which consist of the parameters of the problem at hand. After these parameters have been fixed numerically,[9] that is

$$Y_1 \approx 0.226 K_1^S + 0.021 B_1^S - 0.006 B_2^S + 1.016 W_{12} - 1.350 W_{21} + 0.429 Y_2,$$

$$Y_2 \approx 0.113 K_2^S - 0.002 B_1^S + 0.018 B_2^S - 0.565 W_{12} + 0.753 W_{21} + 0.106 Y_1,$$

MATHEMATICA sends the values of the coefficients to the external simulation program written in C++.[10]

When the simulation run has finished, the two functions f_1^{-1} and f_2 can be depicted graphically as shown by Figure 5.

As the factor stocks grow over time both graphs of the respective linear equations are shifted away from the origin, where their slopes remain constant. The resulting time path (Y_1^D, Y_2^D) indicates all points of intersection. In order to calculate a point of intersection the explicit solution of the preceding system of equations is needed.
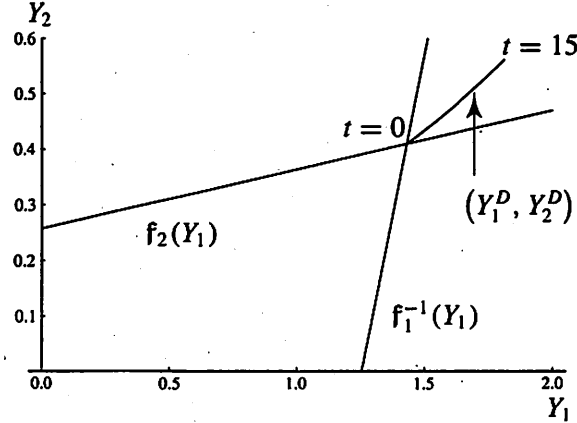
Figure 5: Time path of regional demand based on two CES production functions with $\rho_i = -2$.

$$Y_1^D \approx 0.237 K_1^S + 0.051 K_2^S + 0.021 B_1^S + 0.002 B_2^S + 0.811 W_{12} - 1.081 W_{21}$$

$$Y_2^D \approx 0.025 K_1^S + 0.119 K_2^S - 0.0001 B_1^S + 0.018 B_2^S - 0.479 W_{12} + 0.638 W_{21}$$

3. Adjustment of Supply and Demand

After Y_i^{pot} and Y_i^D have been computed as shown above, the next step is to go over from line 2 to line 3 of Figure 2. At each point of time aggregate supply and aggregate demand have to be adjusted so that the realized values of Y_i fulfill the constraints

$$Y_i^{eq} \leqq Y_i^{pot} \quad \text{and} \quad Y_i^{eq} \leqq Y_i^D \qquad (i = 1, 2).$$

The adjustment procedure applied has been presented in Buhr (2000).

[9] An external program is used to fix the parameters. It immediately shows the graphical representation of f_1^{-1} and f_2 with respect to the changed parameter. This is helpful to get an idea about the starting point of (Y_1^D, Y_2^D) in relation to (Y_1^{pot}, Y_2^{pot}).

[10] The communication protocol for *MATHEMATICA* is called MathLink and comes with the distribution of *MATHEMATICA*.
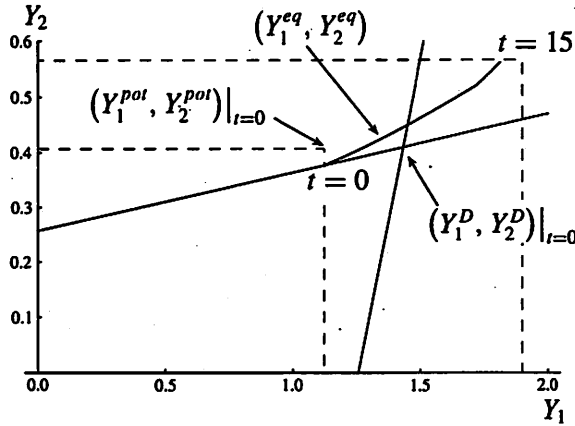
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Figure 6: Time path of realized incomes based on two CES production functions with $\rho_i = -2$.

The outcome with respect to the simulation at hand is shown in Figure 6. In particular, at time $t = 0$ it can be verified that $Y_i^{eq} \leqq Y_i^{pot}$ and $Y_i^{eq} \leqq Y_i^D$ hold good for the two regions.

Notice that Y_i^{eq} determines the realized regional outputs X_i^{pot} such that $X_i^{eq} \leqq X_i^{pot}$. These values in turn yield the factor demand (H_i^D, K_i^D, B_i^D) and, therefore, $(L_i^{edu.D}, L_i^{raw.D})$. The described approach ensures that the restrictions given by the factor supply are fulfilled. That is, the factor demand does not exceed the factor supply.

With these results, the values of factor demand in line 4 of Figure 2 are known. The rest of this flow chart is straightforward. Line 5 describes private and public
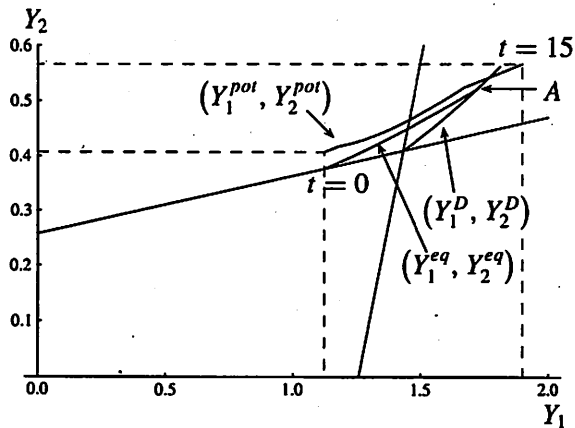
Figure 7: The outcome of the simulation run based on two CES production functions with $\rho_i = -2$.

incomes corresponding to factor payments, where T_i is the tax payed by the private sector to its public sector. In accordance with the reallocation by the state, i.e. $F_{iS}^{pu} - F_{Si}^{pu}$, line 6 shows the private and public incomes retained. These incomes determine the private and public demand terms of line 7 via the assumed behavioral relationships. The next line reflects the realized demand terms of the private sector. It includes the case in which there is still an excess demand for private consumption C_i^{pr} and private total investment $\tilde{I}_i^{pr.D}$. Again the procedure for getting rid of this excess demand is described in Buhr (2000). The last line includes an additional variable U_i, which describes an overall utility index. This index will be introduced at a later stage of the analysis.

Figure 7 is obtained by laying the preceding figures on top of each other.

The characteristics of these results may be summarized as follows. Both regions start in a situation with an excess demand, $Y_i^D > Y_i^{pot}$. However, $Y_2^D > Y_2^{pot} \approx$

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1.007, see Figure 8, part (1). After 6.3 years Y_2^{pot} passes Y_2^D, see Figure 8, part (2). The next event happens at $t = 11.5$. Since Y_1^{pot} passes Y_1^D both regions have a lack of demand, see Figure 8, part (3). From now on the time paths (Y_1^D, Y_2^D) and (Y_1^{eq}, Y_2^{eq}) are equal as it is shown by Figure 7, point A.
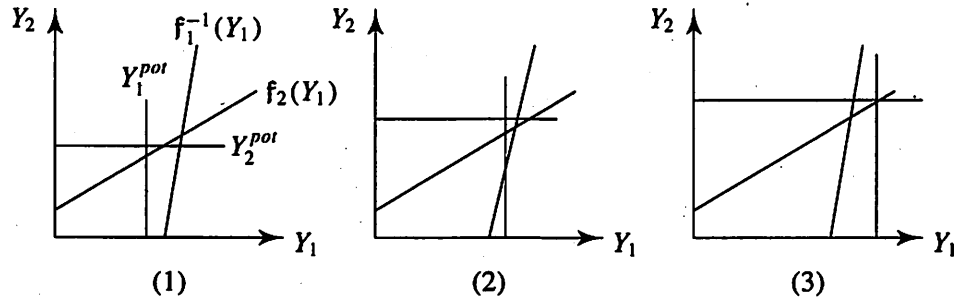

Figure 8: Sketch of the characteristic development of supply and demand.

Figure 9 shows the effects on the rates of unemployment. The kinks at $t = 11.5$ are caused by the lack of demand in both regions.
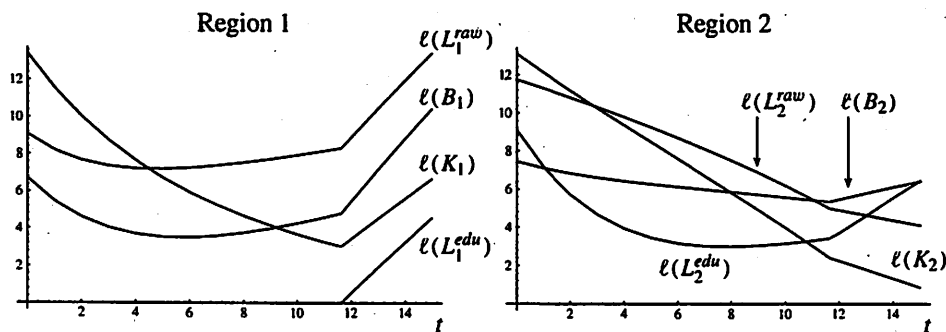

Figure 9: Implied rates of factor stock idleness.

IV. Regional Competition Expressed by Parameter Variations

1. Basic Assumptions for Both Regions

In the general setting R1 is supposed to be larger and more productive.[11] On the one hand, this assumption is reflected by the initial supply of factor stocks. On the other hand, the parameters of the production functions are relevant.[12] A complete list of initial values and parameters can be found at the end of the paper.

[11] This assumption is in line with Garcia-Milà and McGuire (1992, p. 236). They find that states with larger populations have a productive comparative advantage.

[12] Cf. Buhr (1995, pp. 258–260) for the empirical evidence of the data.

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The ratio of labor forces is $L_1^S : L_2^S \approx 2 : 1$ where the labor force of both regions is split up into $L_i^{edu,S} : L_i^{raw,S} \approx 1 : 3$. Private capital is supplied with a ratio of $K_1^S : K_2^S \approx 2.5 : 1$. Finally, the ratio of infrastructure capital is chosen to be $B_1^S : B_2^S \approx 2.9 : 1$.

With regard to the production function the setting of parameters shows higher productivities in R1. Bear in mind that the coefficients and the factor supply change over time. The coefficients of human capital are $a_1 : a_2 \approx 19920 : 14600$. The analogous ratio with respect to private capital is $k_1 : k_2 \approx 0.22 : 0.20$. Eventually, public capital shows a ratio $b_1 : b_2 \approx 1.38 : 1.35$. Recall that human capital consists of educated and raw labor. Again R1 shows higher coefficients. First, the ratio with respect to educated labor is $a_1^{edu} : a_2^{edu} \approx 1.23 : 0.98$. Second, raw labor is marked by the ratio $a_1^{raw} : a_2^{raw} \approx 0.91 : 0.87$. However, the coefficients of human capital are $A_1^H : A_2^H \approx 1$.

Because of the fixed price levels ($P_1 = P_2 = 1$) and the higher productivities in R1, the factor prices of R1 exceed those of R2. This hypothesis is confirmed by a simulation run which endogenously adjusts factor prices to the factor supply given at each point of time. Recall also that the factor price of human capital is determined by the adding up theorem. The fixed rental rates of private capital are $r_1 : r_2 = 0.06 : 0.045$. Similarly, the rental rates of public capital are set to $r_1^* : r_2^* = 0.021 : 0.018$. The implied rental rates for human capital increase over time, however the initial ratio $w_1 : w_2 \approx 14100 : 11100$ does not change very much. The wage rates for educated and raw labor are determined by the assumed distortions of the labor markets, i.e. $w_1^{raw} : w_2^{raw} = 11500 + 300t : 7000 + 100t$. This assumption implies $w_1^{edu} : w_2^{edu} \approx 22100 : 18500$ for the initial wage rates of educated labor.

The demand side is mainly represented by consumption and investment. While the propensities to consume of the public sectors are set to $c_1^{pu} : c_2^{pu} = 0.88 : 0.91$, the analogous ratio for the private sectors is $c_1^{pr} : c_2^{pr} = 0.905 : 0.901$. Since public investment is determined residually, only private investment needs to be characterized. The investment parameters referring to disposable income are fixed at $u_1 : u_2 = 0.003 : 0.001$. The parameters referring to autonomous private investment equal $u_1^{aut} : u_2^{aut} = 0.01 : 0.009$. The two regions do not differ with respect to private investment on research and development ($\varepsilon_1^{dev} = \varepsilon_2^{dev}$) or public expenditures on education ($\varepsilon_1^{edu} = \varepsilon_2^{edu}$).

2. Robustness

The model has been tested with respect to variations of all the included parameters. As no parameter variation, within a plausible range, causes drastic effects on the behavior of the simulation, this tedious part of testing the model is omitted.

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Instead, the attention is drawn to those cases where some structural parts of the model change.

Comparison of the outcomes based on different production functions. In order to check the plausibility of the proposed parameters the results based on different production functions are firstly compared to each other. Recall that the indicator of the type of production function is the parameter ρ_i of the production function f_i. It can be shown that this parameter determines the elasticity of substitution between two factors of production, i.e. the curvature of the isoquants at hand. The first task is to find plausible values for the parameters $(\alpha_i^H, \alpha_i^K, \alpha_i^B)$ such that the production functions are comparable. (The initial coefficients (a_i, k_i, b_i) are always the same.) In the case of a Cobb-Douglas function the production coefficients $(\alpha_i^H, \alpha_i^K, \alpha_i^B)$ correspond to the factor shares. But for a linear limitational or Leontief production function these parameters become superfluous. Hence, the factor shares are calculated on the basis of a simulation run with a linear limitational production function and then used to fix the parameters considered.[13]

$$(\alpha_1^H, \alpha_1^K, \alpha_1^B) = (0.75, 0.2349, 0.151)$$

$$(\alpha_2^H, \alpha_2^K, \alpha_2^B) = (0.77, 0.2168, 0.132)$$

Using these parameters the simulation results vary with regard to qualitative and quantitative aspects, but only slightly. (The initial factor supply is always the same.) Both regions start in a situation as indicated by the following table and by

time	region i Y_i^D / Y_i^{pot}	Leontief $\rho_i \to -\infty$	CES $\rho_i = -2$	CES $\rho_i = -0.5$	Cobb-Douglas $\rho_i = 0$
0	1	1.257	1.276	1.295	1.312
	2	1.011	1.007	1.002	0.997
15	1	0.959	0.954	0.948	0.956
	2	0.965	0.992	1.019	1.047

Table 1: Initial and final relationship of income demanded Y_i^D to potential income Y_i^{pot}.

Figure 7. They also end up with situations which are quantitatively quite similar to each other.

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[13] Note that the factor shares sum up to one. At least on a metropolitan area level the coefficients α_i^B seem to be too high. For this case Eberts (1986) found a statistically significant output elasticity of 0.03. Based on a linear homogeneous Cobb-Douglas function for almost all states of the U.S. Munnell (1990) estimates the exponents to be $(\alpha^H, \alpha^K, \alpha^B) = (0.65, 0.27, 0.08)$. Moreover, she finds that dropping the assumption of linear homogeneity reduces the standard error. Then $(\alpha^H, \alpha^K, \alpha^B) = (0.59, 0.31, 0.15)$. Other authors like Aschauer (1989), Duffy-Deno and Eberts (1991) and Garcia-Milà and McGuire (1992) obtain different estimates for α^B from 0.03 up to 0.39.

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The corresponding rates of unemployment show for all cases that educated labor is the growth barrier of the first region up to year 11.5.

The main quantitative results can be summarized as follows, where a leading γ indicates the average growth rate (in percent) of the indicated state variable. While

average growth rate per year	Leontief $\rho_i \to -\infty$	CES $\rho_i = -2$	CES $\rho_i = -0.5$	Cobb-Douglas $\rho_i = 0$
$\gamma(Y_1^{eq})$	3.099	3.201	3.279	3.341
$\gamma(Y_1^{eq}/L_1^S)$	1.427	1.521	1.586	1.629
$\gamma(C_1^{pr}/L_1^S)$	1.513	1.615	1.689	1.739
$\gamma(Y_2^{eq})$	2.641	2.666	2.567	2.407
$\gamma(Y_2^{eq}/L_2^S)$	2.323	2,369	2.299	2.185
$\gamma(C_2^{pr}/L_2^S)$	2.325	2.372	2.288	2.154
$\gamma(Y_1^{eq} + Y_2^{eq})$	2.988	3.071	3.106	3.115

Table 2: Average annual growth rates of selected variables *in percent.*

the first region grows faster with an increasing ρ_1, the second region seems to have an "optimal" ρ_2 which ensures the highest growth rates. Both regions together would prefer the highest possible ρ_i.

Losses induced by distortions of the labor markets. Each simulation run can be compared to another simulation run based on a perfect labor market. That is, the labor market adjusts the two wage rates of educated and raw labor such that the condition of cost minimization

$$\frac{w_i^{raw}}{w_i^{edu}} = \frac{\partial g_i(L_i^{edu,S}, L_i^{raw,S})/\partial L_i^{raw}}{\partial g_i(L_i^{edu,S}, L_i^{raw,S})/\partial L_i^{edu}}$$

holds good[14] (see Figure 3).

In all of the above cited cases the values of $Y_i^{pr,r}$, $Y_i^{pu,r}$, Y_i^{eq} are increased when w_i^{raw} is not fixed at each point of time, but the numerical effect is small (about 0.2% at $t = 15$). A remarkable effect can be seen when drawing attention to incomes or consumption per capita. When Y_2^{eq}/L_2^S and C_2^{pr}/L_2^S are raised, the corresponding values of R1 slow down. This result is caused by a significant higher net migration to R1, which in turn is implied by the new wage rate differentials and the lower rates of unemployment in R1.

Perfect adjustment of all rental rates. As the rental rates of private and public capital are some of the most important parameters, it is useful to test what values these rates will take if they are adjusted in accordance with the capital supply. The

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[14] This approach implies that both kinds of labor have the same rate of unemployment.

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appropriate simulation run has been done for the Cobb-Douglas case. Using the regional outputs

$$X_i^{max} = A_i(a_i H_i^S)^{\alpha_i^H} (k_i K_i^S)^{\alpha_i^K} (b_i B_i^S)^{\alpha_i^B}$$

and the adding up theorem $c_i(w_i, r_i, r_i^*, X_i^{max}) = X_i^{max}$, the rental rates can be calculated from the factor shares.

$$w_i = \alpha_i^H X_i^{max}/H_i^S, \quad r_i = \alpha_i^K X_i^{max}/K_i^S, \quad r_i^* = \alpha_i^B X_i^{max}/B_i^S$$

The corresponding simulation run now ensues for the entire period

$r_1 \in [0.0508, 0.0634]$,

$r_2 \in [0.0421, 0.0479]$,

$r_1^* \in [0.0215, 0.0207]$,

$r_2^* \in [0.0172, 0.0186]$,

so that the given rental rates $r_1 = 0.06, r_1^* = 0.021, r_2 = 0.045$, and $r_2^* = 0.018$ seem to be in a plausible range. Nevertheless, the behavior of the system changes

Figure 10: The outcome of the simulation run based on two Cobb-Douglas functions with adjusted rental rates.

significantly. Compare Figure 7 to Figure 10. The reason is that Y_1^D starts at a remarkable higher level because of the lower interest rate ($r_1 = 0.0508$ at $t = 0$). As r_1 increases during the simulation run, Y_1^D diminishes and comes closer to the standard case. Eventually after 15 years, the relative deviation of all income terms is smaller than 0.05%. In particular the realized time paths of (Y_1^{eq}, Y_2^{eq}) do not differ to the same extent as the paths of (Y_1^D, Y_2^D).
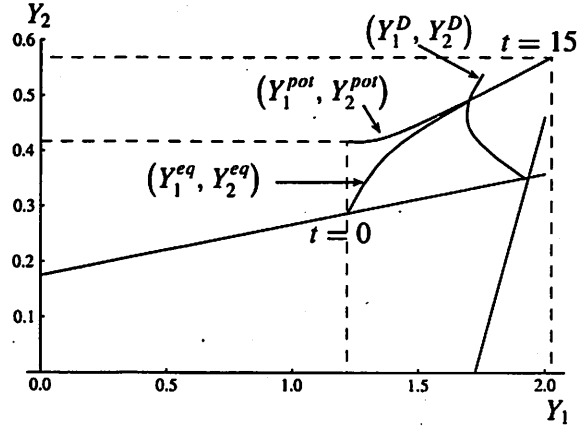
3. Competition

Overall Utility. Because of the linear homogeneous production functions no positive profits will occur. Therefore, the private sectors are assumed to assess the value of the resulting time paths with respect to an overall utility U_i as given by

$$U_i = \int_{t_0}^{t_1} L_i^S(t) \, u_i(\tilde{c}_i^{pr}(t)) \, e^{-\varrho_i t} \, dt$$

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with $t_0 = 0$ and $t_1 = 15$.[15] The included felicity function[16]

$$u_i\big(\tilde{c}_i^{pr}(t)\big) = \frac{\big(\tilde{c}_i^{pr}(t)\big)^{1-\vartheta_i} - 1}{1 - \vartheta_i} \xrightarrow{\vartheta_i \to 1} log\big(\tilde{c}_i^{pr}(t)\big)$$

depends on private consumption per capita at time t.

$$\tilde{c}_i^{pr}(t) := C_i^{pr}(t)/L_i^S(t)$$

The felicity function is weighted by $e^{-\varrho_i t}$ to get the present value with respect to the rate of time preference ϱ_i. The higher ϱ_i, the more the private sector prefers an early consumption.

Suppose the private sector of R1 seeks to maximize its overall utility with respect to the flow budget constraint

$$\begin{aligned}
\dot{W}_1 &= (1 - t_1)\big(w_1 H_1^D + r_1 K_1^D + r_2 W_{12} - r_1 W_{21}\big) \\
&\quad - r_1^*(B_1^S - B_1^D) - C_1^{pr} - I_1^{pr,dev} \\
&= Y_1^{pr,r} - C_1^{pr} - I_1^{pr,dev} = S_1^{pr}
\end{aligned}$$

where $W_1 := K_1^S + W_{12} - W_{21}$ is the wealth of R1. An approximation[17] of the optimality condition of the Ramsey model is then

$$\varrho_i \approx (1 - t_i)r_i - \frac{d\tilde{c}_i^{pr}/dt}{\tilde{c}_i^{pr}}.$$

That is, in the case of $\varrho_i \approx (1 - t_i)r_i$ the consumption per capita should be approximately constant. The standard simulation yields the regional growth rate of consumption per capita as 1.513% and 2.325%. Hence, it seems to be plausible to choose $\varrho_1 = 0.032$ and $\varrho_2 = 0.013$.

Notation. The following tables show the compressed results for a lot of simulation runs. Each table refers to one parameter, which is varied over a certain range. In order to save space there is almost always a threefold set of information. (a) The first line includes the object variables (obj.), which are of most importance. (b) The second line shows the maximum average deviation *in percent* (dev.) from a simulation run with standard parameters after 15 years.[18] This

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[15] Cf. Barro and Sala-i-Martin (1995, pp. 59–65). It should be kept in mind that the households have no perfect foresight as it is typically assumed in dynamic Computable General Equilibrium (CGE) models. In this sense the overall utility has to be interpreted as an ex post criterion.

[16] The instantaneous elasticity of intertemporal substitution between consumption at times t and $t + dt$ for u_i is the constant $1/\vartheta_i$.

[17] The approximation assumes that $r_1^*(B_1^D - B_1^S)$ is relatively small, that $r_2 W_{12} \approx r_1 W_{12}$, and that the growth rates of labor n_1^{edu} and n_1^{raw} corrected by migration can be approximated by a constant.

[18] As in Harrison and Pearson (1996) the simulations are the answer to "What if" questions such as "If the taxes are reduced by 10 percent, how much different would the economy be in 15 years time from what it would otherwise have been".

reference simulation is based on two linear limitational production functions. An arrow indicates that the variable either increases or decreases monotonically. (c) The third line gives the value of the respective parameter at which the maximum average deviation is attained.

For the sake of brevity, national income terms are defined by summing up over regional incomes.

$$Y^{eq} := Y_1^{eq} + Y_2^{eq} \quad \text{and} \quad Y^{pr,r} := Y_1^{pr,r} + Y_2^{pr,r}$$

Moreover, a small letter y denotes an income term per capita, e.g. $y_i^{eq} := Y_i^{eq}/L_i^S$. The analysis starts with the private sectors, goes over to the public sectors and ends up with the State.

Private Sectors. The first two tables serve for deriving an "optimal" propensity to consume c_i^{pr} and, therefore, an optimal tuning of consumption given the investment demand function. In Table 3, the utility index U_1 rises with increasing values

obj.			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
dev.			4.52%	1.64%	3.84%	4.60%	1.66%	3.89%
c_1^{pr}			0.911	0.912	0.911	0.909	0.913	0.912
obj.	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
dev.	↗	↘	4.40%	1.91%	3.84%	4.48%	1.93%	3.90%
c_1^{pr}			0.911	0.911	0.911	0.909	0.912	0.909

Table 3: Variation of the marginal propensity to consume c_1^{pr} over the range $[0.89, 0.94]$. For the first region $c_1^{pr} = 0.905$ is given by default.

of c_1^{pr}. This is even true for very high values of c_1^{pr}. At least two reasons can be indicated for this result. Firstly, consumption is valued less the later it is received. Secondly, although private and public investment go down, R1 is able to live on the excess supply of its private (and public) capital stocks: $\ell(K_1)|_{t=0} = 7.5\%$ and $\ell(K_1)|_{t=15} = 2.8\%$. Since educated labor is always the scarce input, the outcomes change only slightly.

Observation of the outcomes for R2 yields quite similar results, where $c_2^{pr} = 0.901$ is the default value. But here very high values of c_2^{pr} (e.g. 95%) reduce U_2. The excess capacity $\ell(K_2)|_{t=0} = 5.8\%$ gets used up after a period of about 8 years, since investment is not high enough. If the private sectors are interpreted as competitors, an adverse effect of competition on both regions can be seen in the percentage deviations of U_1 and U_2 from the standard simulation.

Next the parameter of exports i_{12} (or analogously i_{21}) is investigated. Notice again the adverse effects on both regions with respect to all variables. Regarding the

obj. dev. c_2^{pr}			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
			1.16%	3.89%	1.80%	1.18%	3.93%	1.83%
			0.909	0.908	0.909	0.911	0.909	0.910
obj. dev. c_2^{pr}	U_1^*	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
	-1.20%	2.99%	2.46%	1.71%	1.75%	2.49%	1.75%	1.79%
	0.912	0.9145	0.910	0.905	0.909	0.910	0.905	0.909

Table 4: The same variation as in Table 3 but with respect to R2. The reference value of c_2^{pr} is 0.901.

* U_1 attaines a minimum at $c_2^{pr} = 0.912$.

common income terms, the "optimal" values $i_{12} = 0.052$ (or $i_{21} = 0.005$) seem to yield the smallest values with respect to the balance of trade $Ex_1^{pr} - Im_1^{pr}$. In this case the balance turns into a surplus after about 4 years. In the standard case with $i_{12} = 0.03$ and $i_{21} = 0.008$ the balance shows always a deficit.

obj. dev. i_{12}			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
			↗	↘	2.92%	↗	↘	2.90%
					0.051			0.051
obj. dev. i_{12}	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
	↗	↘	4.14%	↘	3.05%	4.27%	↘	3.03%
			0.051		0.051	0.051		0.051

Table 5: Variation of the export parameter i_{21} of R1 over the range $[0.04, 0.08]$.

The parameters of investment u_i and u_i^{aut} have two effects. First of all they determine one part of aggregate demand. For R1 this income effect can be described as follows. Given the reference values $u_2 = 0.001$, $u_1^{aut} = 0.01$ and $u_2^{aut} = 0.009$, an unknown parameter u_1 yields

$$Y_1^D \approx \frac{6.4 \cdot 10^{-6}}{0.006 - u_1}[117.7K_1^S + 25.2K_2^S + (20.0 - 3263.8u_1)B_1^S + B_2^S$$
$$+ 403.3W_{12} - 537.7W_{21}].$$

Similarly with the default value $u_1 = 0.003$, an unknown parameter u_1^{aut} results in

$$Y_1^D \approx 23.65u_1^{aut}K_1^S + 0.05K_2^S + 0.02B_1^S + 0.002B_2^S + 0.81W_{12} - 1.08W_{21}.$$

Secondly, the parameters of investment influence the growth rate of K_i^S and, therefore, of Y_i^{pot}, provided that private capital is the relevant growth barrier. This is the capacity effect.

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Although increases or decreases of u_1 or u_1^{aut} by the same ratio yield very similar results, u_1^{aut} is more important with respect to the growth rates. The reason can be found in the private investment function.

$$I_1^{pr} = \frac{u_1}{r_1} Y_1^{pr,r} + u_1^{aut} K_1^S = 0.05 Y_1^{pr,r} + 0.01 K_1^S \quad \text{where} \quad K_1^S / Y_1^{pr,r} \approx 5.8$$

The variation of the autonomous investment parameter u_1^{aut} over the range $[0.008, 0.015]$ shows that all of the income terms in the preceding tables grow monotonically with an increase of u_1^{aut}. However, the utility index of R2 diminishes. Merely U_1 attains a maximum at $u_1^{aut} = 0.105$, where the relative deviation from the standard simulation is 3.0%. The most striking facts are that smaller values of u_1^{aut} make K_1^S the scarce input so that $\ell(L_1^{edu})$ jumps up to 22% or even 30%. In the opposite case with an increase of u_1^{aut}, educated labor slows down the growth in general, $\ell(L_1^{edu}) = 0$.

Investment on R&D is represented by the parameter ε_i^{dev}. An increase of ε_i^{dev} raises the productivity coefficient a_i^{edu} (but not a_i^{raw}). Therefore, the isoquants – as H_i^S shown by Figure 3 – are shifted further away from the origin. Given a constant demand for human capital H_i^D, this tends to reduce $L_i^{edu,D}$ and $L_i^{raw,D}$. As long as $\ell(L_i^{edu}) = 0$ and $\ell(L_i^{raw}) > 0$ hold true, only $L_i^{raw,D}$ diminishes. As soon as the ray $\overrightarrow{OB}$ is reached, $\ell(L_i^{edu})$ increases faster than $\ell(L_i^{raw})$. These effects can be shown by appropriate parameter variations. They come along with a rising rental rate w_i and a rising wage rate w_i^{edu}. Similarly one can show for $(K_i^S = K_i^D, H_i^S > H_i^D)$

$$(a_i \uparrow \rightarrow w_i \uparrow, H_i^D \downarrow) \implies (L_i^{edu,D} \downarrow, L_i^{raw,D} \downarrow)$$

with respect to the productivity coefficient a_i.

All of the parameter variations affect the endogenous wage rates of educated labor and the rates of unemployment. Hence, the implied migration distorts the supply of labor and, therefore, the supply of human capital. For instance, the case of perfect adjusted wage rates does not increase U_1, as one would expect. The reason is a high immigration such that consumption per head $\bar{c}_1^{pr}$ falls.

Public Sectors. The public sector of region i fixes the tax rate t_i. As the public has the main task to offer public capital B_i^S and to strengthen the productivity of labor,[19] it is relevant to ask for the optimal tax rates while holding the rest of the parameters fixed, cf. Table 6.

The "optimal values" of t_1 and t_2 are influenced by the reallocation of the State. Among other things, R2 prefers higher tax rates than R1 since each $ of public

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[19] As pointed out by Aschauer (1989), these aspects should be attributed to public investment decisions when assessing the role the government plays in the course of economic growth and productivity improvement.

obj.			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
dev.			8.31%	0.98%	5.35%	0.59%	1.01%	0.69%
t_1			0.33	0.11	0.30	0.11	0.11	0.11
obj.	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
dev.	0.01%	↘	6.34%	0.96%	5.39%	2.49%	0.95%	0.65%
t_1	0.20		0.30	0.25	0.30	0.11	0.25	0.11
obj.			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
dev.			1.8%	↗	1.13%	0.18%	0.71%	0.07%
t_2			0.23		0.24	0.23	0.05	0.23
obj.	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
dev.	↘	2.39%	1.37%	1.79%	1.09%	1.37%	↘	0.04%
t_2		0.27	0.26	0.23	0.24	0.26		0.23

Table 6: Variation of the tax rates t_1 and t_2 over the range $[0.0, 0.45]$ and $[0.0, 0.3]$, respectively. The standard tax rates of the first and the second region are $t_1 = 0.2075$ and $t_2 = 0.1975$.

income payed to the State comes back with a profit, $F_{2S}^{pu} < F_{S2}^{pu}$. This effect vanishes if the parameter of reallocation holds $v > 0.78$.

According to the public sectors there are four tasks to look at.

(1) Up until now, the main aspect of the public sector was the supply of infrastructure capital B_i^S.

(2) To what extent is the public sector able to repair the problem of a lack of private capital, i.e. $K_i^S < K_i^D$? The public sectors can pay subsidies $G_i^{pu} = h_i I_i^{pu}$ with $h_i > 0$ in order to attract private capital from the other region. But this takes resources away from B_i^S. Hence, it depends on the situation whether or not it is useful to pay subsidies. Moreover, the outcome of paying subsidies depends on the multipliers v_{12} and v_{21} which determine the private transfers F_{12}^{pr} and F_{21}^{pr}, respectively.

$$\dot{K}_1^S \approx 0.01 K_1^S - (0.001 - 0.002 h_1 v_{21}) B_1^S - (0.00004 + 0.001 h_1 v_{21}) B_2^S$$
$$+ (0.040 + 0.031 h_1 v_{21}) Y_1^D - (0.001 + 0.015 h_1 v_{21}) Y_2^D$$
$$\dot{K}_2^S \approx 0.009 K_2^S - (9.1\,10^{-6} + 0.002 h_1 v_{21}) B_1^S - (0.0003 - 0.001 h_1 v_{21}) B_2^S$$
$$- (0.0001 + 0.031 h_1 v_{21}) Y_1^D + (0.018 + 0.015 h_1 v_{21}) Y_2^D$$

Eventually, a change of the parameter h_i referring to subsidies mainly affects the demand side of the opposite region (see below).

(3) To what extent is the public sector able to correct the problem of a lack of human capital, i.e. $H_i^S < H_i^D$? The relevant parameter is now ε_i^{edu} which determines the coefficients a_i^{edu} and a_i^{raw}. Each increase of these parameters raises H_i^S.

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Again, if $H_i^S > H_i^D$, there arises the problem that each increase of ε_i^{edu} decreases H_i^D and this leads to unemployment of labor.

(4) To what extent is the public sector able to remove the problem of a lack of aggregate demand, i.e. $Y_i^D < Y_i^{pot}$? Regarding the parameters t_i (see above) and c_i^{pu} the most important equations of R1 concerning dynamics are

$$I_1^{pr,D} \approx 0.010 K_1^S + 0.050(1 - t_1)(Y_1^D - 0.021 B_1^S),$$

$$I_1^{pu} \approx (0.021 - 0.020 c_1^{pu})(1 - t_1) B_1^S - 0.002 c_1^{pu}(1 - t_2) B_2^S$$
$$+ (1 - 0.966 c_1^{pu}) t_1 Y_1^D - 0.086 c_1^{pu} t_2 Y_2^D.$$

In this sense the public propensity to consume c_i^{pu} is a parameter of minor importance. "Each increase in c_i^{pu} worsens everything" because the residually determined public investment I_i^{pu} is reduced and, therefore, B_i^S grows at a slower rate.[20] In the model at hand, even large excess capacities of B_i are more useful than public consumption.

If the result $Y_1^D < Y_1^{pot}$ is caused by a fall in c_1^{pr}, then the public sector of R1 is recommended to compensate for this lack of demand by an increase of its tax rate t_1. Table 7 includes the optimal tax rates with respect to a maximum U_1, Y_1^{eq} and $Y_1^{pr,r}$. From a static point of view, this result can be explained as follows for time $t = 0$. While a decrease of c_1^{pr} causes Y_1^D to fall, an increase of t_1 reverses this effect. In this sense, public demand covered by taxes substitutes for private demand. Be aware that this explanation might not be true under dynamic change. At year 15 the relation Y_1^D / Y_1^{pot} may have a minimum with respect to t_1.

c_1^{pr}	U_1	Y_1^{eq}	$Y_1^{pr,r}$
0.890	37.5%	45.0%	45.0%
0.895	32.5%	40.0%	40.0%
0.900	27.5%	35.0%	35.0%
0.905	19.5%	32.5%	30.5%
0.910	17.5%	32.5%	30.0%

Table 7: Optimal tax rates with respect to selected variables

In some cases, a coordinate strategy of paying subsidies may overcome the lack of demand. An isolated strategy of R1 – i.e. h_1 and v_{21} – yields at once the above cited equations for K_1^S and K_2^S. However, this policy mainly affects the demand side of R2. For unknown h_1 and v_{12} it follows

$$f_1(Y_2) \approx 0.226 K_1^S + 0.021 B_1^S - 0.006 B_2^S + 1.016 W_{12} - 1.355 W_{21} + 0.429 Y_2$$

$$f_2(Y_1) \approx \frac{0.0731392}{5.306 + h_1 + h_1 v_{21}} [8.195 K_2^S$$
$$- (0.162 - 2.266 h_1(1 + v_{21})) B_1^S + (1.306 - h_1(1 + v_{21})) B_2^S$$
$$- 40.973 W_{12} + 54.631 W_{21} + (7.697 + 28.258 h_1(1 + v_{21})) Y_1]$$

In this sense an isolated strategy of R1 is not useful to remove the problem $Y_1^D < Y_1^{pot}$.

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[20] This outcome is in line with the empirical result in Munnell (1990, p. 205), i.e., an investment in public capital has a positive impact on that region's employment growth.

State. The State fixes the public tax rates τ_1 and τ_2 and, therefore, the volume of resources to be reallocated. Then it assigns the received revenue F^{pu} in accordance with the parameter v to the public sectors, that is $F_{S1}^{pu} = vF^{pu}$ and $F_{S2}^{pu} = (1 - v)F^{pu}$. The objectives involved may be the maximization of the regional or national incomes.[21] As long as v corresponds to the part of F^{pu} payed by R1, i.e. $F_{1S}^{pu} = F_{S1}^{pu}$, there will be no net effect. The same outcome would result from $\tau_1 = \tau_2 = 0$.

Using the default values $\tau_1 = \tau_2 = 0.12$ and $F_{S1}^{pu}/F^{pu} = v = 0.72$, then $F_{1S}^{pu}/F^{pu} \approx 0.77$, that is $F_{1S}^{pu} > F_{S1}^{pu}$. R1 pays more than it receives. An increase of v beyond 0.77, as shown by Table 8, reverses the net effect of reallocation, i.e.

obj.			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
dev.			↗	↘	0.61%	↗	↘	0.61%
obj.	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
dev.	↗	↘	↗	↘	0.61%	↗	↘	0.61%

Table 8: Variation of the State's parameter of reallocation v over the range [0.7, 1.0]. The standard value of v is 0.72. The maximum relative deviations of the aggregated income terms refer to $v = 1$.

$F_{1S}^{pu} < F_{S1}^{pu}$. The maximum effect on the growth of both regions together is attained at $v = 1$. In this case, the State transfers all of its resources to R1. The induced losses of R2 are compensated for by the gains of R1 in terms of aggregate incomes.

The higher τ_1 and τ_2 are chosen, the higher will be F^{pu} and the more sensitive is the model with respect to variations of v. Be aware that v is fixed for a period of 15 years. In the worst case, $\tau_1 = \tau_2 = 1$, even small differences in the regional growth rates of Y_i^{pu} will have a great net effect on reallocation.

obj.			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
dev.			↗	↘	0.85%	↗	↘	0.85%
v				0.95				0.95
obj.	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
dev.	↗	↘	1.18%	↘	0.86%	1.15%	↘	0.85%
v			0.950		0.925	0.950		0.925

Table 9: Analogous variation as in Table 8 but using $\tau_1 = \tau_2 = 0.27$.

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[21] An alternative aim is suggested by the Art. 130b of the Rome Treaty. This objective to even out incomes per head has not been investigated.

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The next table is based on $\tau_1 = \tau_2 = 0.27$. Notice that it is no longer optimal to choose $\nu = 1$, provided the aim is to maximize Y^{eq} or $Y^{pr,r}$. Although it is a realistic approach to set $\tau_1 = \tau_2 = 12\%$ and $\nu = 0.72$, the best fine tuning of transferring resources from R1 to R2 would be achieved by $\tau_1 = 0$, $\nu = 0$ and an appropriate small $\tau_2 > 0$. In this case, R1 pays nothing and receives all State payments.

obj.			Y_1^{eq}	Y_2^{eq}	Y^{eq}	$Y_1^{pr,r}$	$Y_2^{pr,r}$	$Y^{pr,r}$
dev.			1.52%	↘	0.91%	1.46%	↘	0.91%
τ_2			0.425		0.195	0.375		0.195
obj.	U_1	U_2	y_1^{eq}	y_2^{eq}	y^{eq}	$y_1^{pr,r}$	$y_2^{pr,r}$	$y^{pr,r}$
dev.	↗	↘	1.12%	↘	0.91%	1.17%	↘	0.91%
τ_2			0.250		0.195	0.1975		0.195

Table 10: Results for the case where R1 pays nothing and receives all.

For $\tau_2 = 0.195$ the net effect $F_{S1}^{pu} - F_{1S}^{pu} > 0$ is very similar to the case with $\tau_1 = \tau_2 = 0.27$ and $\nu = 0.950$ in the preceding Table 9.

V. List of Parameters and Initial Values

Initial values of state variables:

$$L_1^{edu,S} = 1\,500\,000 \qquad L_2^{edu,S} = 800\,000$$

$$L_1^{raw,S} = 4\,593\,700 \qquad L_2^{raw,S} = 2\,472\,000$$

$$K_1^S \; = 561\,100\,000\,000 \qquad K_2^S \; = 211\,400\,000\,000$$

$$B_1^S \; = 87\,000\,000\,000 \qquad B_2^S \; = 30\,000\,000\,000$$

$$W_{12} \; = 120\,000\,000\,000 \qquad W_{21} \; = 92\,000\,000\,000$$

Private expenditures on R&D and public expenditures on education:

$$K_1^{dev} = 0.0053 K_1^S \qquad I_1^{dev} = 0.001 \tilde{I}_1^{pr} \qquad (\eta_1^{dev}, \varepsilon_1^{dev})$$

$$K_2^{dev} = 0.0050 K_2^S \qquad I_2^{dev} = 0.001 \tilde{I}_2^{pr} \qquad (\eta_2^{dev}, \varepsilon_2^{dev})$$

$$B_1^{edu} = 0.0045 B_1^S \qquad I_1^{edu} = 0.001 I_1^{pu} \qquad (\eta_1^{edu}, \varepsilon_1^{edu})$$

$$B_2^{edu} = 0.0045 B_2^S \qquad I_2^{edu} = 0.001 I_2^{pu} \qquad (\eta_2^{edu}, \varepsilon_2^{edu})$$

Coefficients of factors of production:

$$a_1 = 19910 + 350t + 5.00\,10^{-11} B_1^S + 1.6\,10^{-11} K_1^S \qquad (a_{10}, a_{11}, a_{12}, a_{13})$$

$$a_2 = 14600 + 340t + 1.28\,10^{-10} B_2^S + 6.7\,10^{-11} K_2^S \qquad (a_{20}, a_{21}, a_{22}, a_{23})$$

$$k_1 = 0.23 + 0.0030t + 1.09\,10^{-13} B_1^S - 3.50\,10^{-14} K_1^S \qquad (k_{10}, k_{11}, k_{12}, k_{13})$$

$$k_2 = 0.22 + 0.0015t + 5.82\,10^{-13} B_2^S - 1.65\,10^{-13} K_2^S \qquad (k_{20}, k_{21}, k_{22}, k_{23})$$

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$$b_1 = 1.380 + 0.0008t \qquad\qquad (b_{10}, b_{11})$$
$$b_2 = 1.345 + 0.0013t \qquad\qquad (b_{20}, b_{21})$$

Regional production functions:

$$X_1 = [0.75(a_1 H_1^S)^{\rho_1} + 0.2349(k_1 K_1^S)^{\rho_1} + 0.0151(b_1 B_1^S)^{\rho_1}]^{1/\rho_1} \qquad (\alpha_1^H, \alpha_1^K, \alpha_1^B)$$
$$X_2 = [0.77(a_2 H_2^S)^{\rho_2} + 0.2168(k_2 K_2^S)^{\rho_1} + 0.0132(b_2 B_2^S)^{\rho_2}]^{1/\rho_2} \qquad (\alpha_2^H, \alpha_2^K, \alpha_2^B)$$

Coefficients of human capital referring to educated and raw labor:

$$a_1^{edu} = 0.85 + 2.0\,10^{-10} B_1^{edu} + 1.0\,10^{-10} K_1^{dev} \qquad (a_{10}^{edu}, a_{11}^{edu}, a_{12}^{edu})$$
$$a_2^{edu} = 0.85 + 2.0\,10^{-10} B_2^{edu} + 1.0\,10^{-10} K_2^{dev} \qquad (a_{20}^{edu}, a_{21}^{edu}, a_{22}^{edu})$$
$$a_1^{raw} = 0.84 + 2.0\,10^{-10} B_1^{edu} \qquad\qquad (a_{10}^{raw}, a_{11}^{raw})$$
$$a_2^{raw} = 0.84 + 2.0\,10^{-10} B_2^{edu} \qquad\qquad (a_{20}^{raw}, a_{21}^{raw})$$

Human capital:

$$H_1 = 1.90[0.25(a_1^{edu} L_1^{edu})^{-1} + 0.75(a_1^{raw} L_1^{raw})^{-1}]^{-1} \qquad (A_1^H, \varphi_1, \omega_1)$$
$$H_2 = 1.92[0.25(a_2^{edu} L_2^{edu})^{-1} + 0.75(a_2^{raw} L_2^{raw})^{-1}]^{-1} \qquad (A_2^H, \varphi_2, \omega_2)$$

Natural growth rates of labor:

$$n_1^{edu} = 0.011, \quad n_2^{edu} = 0.015, \quad n_1^{raw} = 0.011, \quad n_2^{raw} = 0.015$$

Parameters of migration:

$$e_{A1}^{edu} = 0.000001 \qquad\qquad e_{A2}^{edu} = 0.000002$$
$$e_{B1}^{edu} = 100000 \qquad\qquad e_{B2}^{edu} = 180000$$
$$e_{C1}^{edu} = 0.025 \qquad\qquad e_{C2}^{edu} = 0.05$$
$$e_{A1}^{raw} = 0.000001 \qquad\qquad e_{A2}^{raw} = 0.000002$$
$$e_{B1}^{raw} = 100000 \qquad\qquad e_{B2}^{raw} = 180000$$

$r_1 = 6.0\%$	$r_1^* = 2.1\%$	$r_2 = 4.5\%$	$r_2^* = 1.8\%$
$w_1^{raw} = 11500$ $+300t$		$w_2^{raw} = 7000$ $+100t$	
$c_1^{pr} = 90.5\%$	$c_1^{pu} = 88.0\%$	$c_2^{pr} = 90.1\%$	$c_2^{pu} = 91.0\%$
$t_1 = 20.75\%$	$\tau_1 = 12.0\%$	$t_2 = 19.75\%$	$\tau_2 = 12.0\%$
$i_{12} = 0.03$		$i_{21} = 0.008$	
$u_1 = 0.003$	$h_1 = 0$	$u_2 = 0.001$	$h_2 = 0.01$
$u_1^{aut} = 0.010$	$v_{21} = 0$	$u_2^{aut} = 0.009$	$v_{12} = 1.8$

Reallocation by the state: $v = 0.72$

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