VOLKSWIRTSCHAFTLICHE DISKUSSIONSBEITRÄGE

VWL

UNIVERSITÄT - GESAMTHOCHSCHULE - SIEGEN
FACHBEREICH WIRTSCHAFTSWISSENSCHAFTEN

CONSTITUTIONAL CHOICE OF RULES

Jürgen Eichberger[1] and Rüdiger Pethig[2]

Discussion Paper No. 8 – 90

March 1990

[1]Department of Economics, Faculty of Economics and Commerce, University of Melbourne, Parkville, Victoria 3052, Australia

[2]Volkswirtschaftslehre IV, Fachbereich 5, Wirtschaftswissenschaften, Universität – Gesamthochschule – Siegen, Postfach 10 12 40, D–5900 Siegen, F.R. Germany

ABSTRACT

In their **'Calculus of Consent'** Buchanan and Tullock argued that self–interested agents choose social rules at the constitutional level with unanimity provided that these agents are sufficiently **uncertain** about their precise role at the post–constitutional level at which these rules will be applied to a sequence of different situations. Unanimity follows from "high degrees of uncertainty" in which case constitutional choice is said to take place behind a veil of ignorance.

The focus of this paper is the question what formal interpretation one can give the concept of the **veil of ignorance**. A fairly general model of the decision–making process is developed encompassing both the veil of ignorance in the tradition of contractarian theories and "ordinary" uncertainty about post–constitutional situations (and preferences). It is shown that Buchanan's and Tullock's concept of minimisation of expected external and decision—making costs is in fact identical of expected utility maximisation. Moreover, two different types of uncertainty are identified that give rise to unanimous constitutional choice of rules. These two versions of the veil of ignorance are clarified by way of examples. Some results on their general relationship support the view that much of the controversy about the veil of ignorance which one finds in the literature might be caused by a lack of precision as to the meaning of it.

The final section demonstrates that the formal approach of this paper is well suited for **comparative institutional analysis**. In a simple parametric allocation model the Walrasian market procedure is juxtaposed to the principle of egalitarian distribution. It turns out that either rule may be unanimously preferred to the other depending on the specification of utility functions and uncertainty.

CONSTITUTIONAL CHOICE OF RULES

Jürgen Eichberger[1] and Rüdiger Pethig[2]

1. Introduction

Twenty five years ago Buchanan and Tullock (1965) began to investigate the question how rational individuals would choose a rule to conduct decision–making in collective choice situations. Their analysis was based on three principles,

- firstly, that agents act in their self–interest when they choose a rule,
- secondly, that a rule has to be chosen unanimously, and
- thirdly, that unanimity at the constitutional level can be expected if agents' interests are sufficiently diffuse at that stage.

While the first one of these principles is a corner stone of all economic analysis and needs no justification, the second is necessary to avoid an infinite regress of "how to choose the

[1]Department of Economics, Faculty of Economics and Commerce, University of Melbourne, Parkville, Victoria 3052, Australia

[2]Volkswirtschaftslehre IV, Fachbereich 5, Wirtschaftswissenschaften, Universität – Gesamthochschule – Siegen, Postfach 10 12 40, D–5900 Siegen, F.R. Germany

rule to choose the rule..." The third principle, however, is more open to interpretation.

Influential as the "Calculus of Consent" has been, it is surprising that until recently no attempt has been made to provide a formal model of it. Such a formalisation would be important since it requires to make precise the meaning of statements about "sufficiently diffuse preferences" as in the following quotation (Buchanan and Tullock (1965), p. 78): "Essential to the analysis is the presumption that the individual is uncertain as to what his own precise role will be in any one of the whole chain of later collective choices that will actually have to be made." The question what 'uncertainty about one's precise role' means will be the focus of this paper.

In another attempt to clarify some of the ambiguities left in the verbal argument of Buchanan and Tullock (1965), Schweizer (1989) focuses on the question of the appropriate equilibrium concept for decision making at the post–constitutional level. This aspect was completely ignored by Buchanan and Tullock (1965), largely because they emphasise the "simple majority rule" which suggests a dominant strategy equilibrium. This is a very special case, however, as Schweizer (1990) has shown convincingly. In general, one cannot hope for uniqueness of an equilibrium at the point in time when the rule under consideration is applied. If there are multiple equilibria, however, giving rise to different payoffs, then it remains unclear how individuals will evaluate the adoption of a certain rule. It remains an open question whether a general answer to this problem can be found. In any case, non–uniqueness of equilibrium at the post–constitutional level is not a remote possibility as the study of Schweizer (1990) amply demonstrates.

A further related line of work is Binmore's (1989) attempt to reconstruct Rawls' theory of the "Social Contract". Similar to Buchanan and Tullock (1965), Rawls (1971) uses the concept of a "veil of ignorance" at a pre–contractual stage. Binmore (1989) shows with a simple bargaining model that arguments concerning the idea of the veil of ignorance are usually based on some symmetry assumption and require an inter–personal comparison of utility as well as unlimited commitment possibilities.

The focus of this paper is the question what formal interpretation one can give the concept of the veil of ignorance. To investigate this point a fairly general model of the decision–making process on rules is developed (section 2). Section 3 shows that Buchanan's and Tullock's (1965) concept of minimisation of expected external and decision–making costs is in fact identical to expected utility maximisation as it is used in this paper. Our model allows to treat the examples of Binmore (1989) and Schweizer (1990) as special cases. In particular, two interpretations of the veil of ignorance can be distinguished (section 4) and related to each other (section 5). The final section 6 demonstrates the capability of this theory to compare different economic institutions.

2. The basic model

Consider a social system with a finite set of agents, A. These agents form the society under consideration and have to interact in various situations. Denote by L the **set of possible outcomes** of the social interaction. Usually, these outcomes will be commodity allocations and L will be a subset of a commodity space.[3] Let U be the **set of possible utility functions** representing preferences of the agents over possible allocations in L. Finally, Q is the **set of problems** which have to be solved. More precisely, Q is an index set for a family of subsets of allocations $(L_q)_{q\in Q}$ where each element of Q indicates the option $L_q \subset L$ open to the society if q is the problem which the society faces. A **situation** $t = (\ell_0,u,q) \in T := L_0 \times U \times Q$ consists of

(i) a **status–quo allocation** ℓ_0 from a subset L_0 of L which represents the allocation if no agreement is reached,

[3]We will use this interpretation throughout the paper, but it is easy to imagine other applications where L might be a set of candidates to be elected, etc.

(ii) a **profile of individual preferences** $u = (u_a)_{a \in A}$, where $u_a : L \rightarrow \mathbb{R}$, and

(iii) a **problem** q, i.e. a set of potential allocations $L_q \subset L$.

Typical examples of social choice problems include the allocation of public and private goods among agents, the choice whether to carry through a joint project, etc. Usually these problems involve a change in the (resource) allocation of a group of agents, but other objects of choice could be easily considered too, e.g. the choice among alternative candidates for a position. The status quo allocation ℓ_0 will be the outcome if no agreement about a change of allocation is reached among agents. Since $\ell_0 \in L$ and L is the domain of u_a, the utility level agent a achieves from the status quo, $u_a(\ell_0)$, is well defined. Notice in addition that ℓ_0 may or may not be an element of L_q, the set of choices open in situation (ℓ_0,u,q). Situations are allowed to be different in terms of status–quo allocations and preference profiles of agents as well as in regard to the choice options available. As some examples below will show it is easy to consider the case of situations characterised by either constant preferences or constant status–quo allocations by restricting L_0 and/or U to one–element sets.

Rules of decision making are the core of the analysis in this approach. Rules are viewed as governing the choice not in a once–and–for–all decision to 'solve' a problem but rather as a method to decide in a sequence of similar decision problems or, equivalently, as a choice of rule for a potentially large pool of similar problems. Thus, the focus of this paper is the choice of a rule about how to collectively decide in various situations with similar problems.

A rule therefore must determine for any situation $t \in T$:

– what strategies an agent has in the given situation,

– what outcome results from the strategic choices of the agents, and

– what each agent knows about the rule.

Hence, for a situation $t \in T$, a rule specifies a set of strategies $S_a(r,t)$ available to agent $a \in A$, a set of outcomes $\Omega(r,t) \subset L_q$, and an outcome function $o : S(r,t) \rightarrow \Omega(r,t)$ which associates with each strategy combination $s \in S(r,t) := \Pi_{a \in A} S_a(r,t)$ an outcome $o(s)$ in $\Omega(r,t)$. Thus, each rule combined with a situation $t \in T$ determines a **game form**. Since the set of outcomes $\Omega(r,t)$ is a subset of L, the utility of an agent a over possible outcomes is well–defined.

In general, a non–trivial game[4] will arise from a rule in which one has to consider strategic behaviour of agents given their information. It is worth noting that Buchanan and Tullock (1965) disregard strategic behaviour of agents completely. Since a large part of their book (Part III) considers voting rules, they seem to assume that voting according to one's preferences forms a dominant strategy for each agent. That this is not necessarily true has been shown recently by Schweizer (1989).

If the situation t and all elements of the rule r are common knowledge, then

$$\Gamma(r,t) = (A, (S_a(r,t))_{a \in A}, (u_a(o))_{a \in A})$$

defines a game in normal form. Given an appropriate equilibrium concept, one obtains the outcome of the game $o^*(r,t) := o(s^*)$ where s^* is the equilibrium strategy combination. Similarly, if there is incomplete information of the agents about the situation t or the rule r, e.g. about the characteristics of the opponents, one can specify a game of incomplete information as in Harsanyi (1967).[5]

To decide on a rule, each individual must be able to evaluate the expected consequence of her action under a certain rule. This requires a determinate outcome for all games $\Gamma(r,t)$

[4]In some applications the game resulting from the choice of a rule may be trivial in the sense that one can specify the outcome $o^*(r,t) \in \Omega(r,t)$ directly.

[5]Such an extension will be the subject of a further paper.

which may arise under rule r. Uniqueness of equilibrium in a normal form game, however, cannot be guaranteed in general, nor is there a widely accepted theory of equilibrium selection. This poses a serious problem for the present approach.[6] Nevertheless, in many applications a natural solution to this problem of multiple equilibria can be found. Since our analysis focuses on the concept of constitutional choice of rules, it seems justified to assume that there is a unique outcome[7] $o^*(r,t)$.

With r as the rule of the game, the payoff for agent a in situation t can now be written as

$$p_a(r,t) := u_a(o^*(r,t)). \tag{2.1}$$

Given a set of possible situations T and a probability distribution σ governing the random choice of a situation[8] t one obtains the expected payoff for each player from adopting rule r $\in$ R as

$$P_a(r) := \int_T p_a(r,t)d\sigma. \tag{2.2}$$

There are two phases of decision making distinguished in this approach: a **constitutional phase,** and an **operational phase.** At the constitutional stage, there is a set I of **individuals** who have not been identified with certain agents. The **agents** represent the potential roles which individuals may have to play in a society regarding the problems under consideration. The number of individuals must of course equal the number of potential roles, i.e. #I = #A must hold. In this constitutional phase, the individuals $i \in I$ have to decide which

[6]Compare Schweizer (1989) for a broader discussion of this issue.

[7]Note that requiring a unique equilibrium outcome is weaker than requiring a unique equilibrium strategy combination.

[8]We will assume throughout this paper that there is a sigma–algebra and a probability measure with T and that all functions are measurable where necessary. This is a restriction on possible spaces T which has to be kept in mind for possible applications.

rule they want to govern their behaviour at the operational level when they face a particular problem.

After the rule has been chosen at the constitutional level, each individual $i \in I$ is assigned to an agent (or rather a role) $a \in A$ according to a probability distribution π. For the case of finite sets A and I, $\pi(i,a) \in \mathbb{R}_+$ denotes the probability of individual $i \in I$ becoming agent $a \in A$. Of course, $\Sigma_{a \in A}\ \pi(i,a) = 1$ must hold for all $i \in I$, because each individual is assigned some role with certainty. After this assignment of roles, it is assumed that nature chooses randomly a situation in which the agents have to make a collective decision with the help of the rule chosen at the constitutional level. Note that there are two randomisation processes at work. The first concerns the assignment of individuals to agents (denoted **constitutional uncertainty**), the second determines the actual game to be played on the operational level by laying down the agents' preferences, the status–quo allocation, and the particular problem they face. For convenience of reference, this type of uncertainty is denoted **post–constitutional uncertainty**. Some examples may illustrate the basic model described so far.

Example 2.1: Consider the issue of accepting or rejecting an indivisible public **project** by a finite set of agents A and suppose, for a moment, that some situation $t = (\ell_o,u,q)$ is given. If the project is well defined with regard to its quantitative and qualitative dimensions and with respect to cost share assignments, its implementation clearly transforms the status quo allocation ℓ_o into some allocation $\ell_q \in L$. Hence $L_q = \{\ell_o,\ell_q\}$ describes the problem of selecting a well–defined project. If R is the set of **voting rules**, one has for each $r \in R$ the following game $\Gamma(r,t)$: Each agent has the same strategy set $S_a(r,t) = \{0,1\}$ where 0 means a vote against and 1 a vote in favour of ℓ_q. The set of outcomes is $\Omega(r,t) = \{\ell_o,\ell_q\}$, and $o(r,t) = \ell_q$ indicates that the project has been accepted, while $o(r,t) = \ell_o$ means that it has been rejected. The associated payoffs are $u_a(\ell_q)$ and $u_a(\ell_o)$, respectively. Introducing post constitutional uncertainty means that a situation t is selected randomly from T according to the probability distribution σ. If preferences and the status quo allocation are not uncertain, one sets $L_o = \{\ell_o\}$ and $U = \{u_o\}$ so that σ is concentrated on $\{\ell_o\}\times\{u_o\}\times Q$. This example is discussed at length in Schweizer (1989) for the case of $\#A = 3$.

Example 2.2: Consider an exchange economy with a finite set of agents. The set of possible (status quo) allocations is $L_o = L = \mathbb{R}_+^{\#A\cdot m}$, where m denotes the number of commodities. Finally, the set of all possible utility functions, denoted U, consists of all $u = (u_a)_{a\in A}$ with $u_a : \mathbb{R}_+^{\#A\cdot m} \to \mathbb{R}$, where u_a is selfish[9], strictly increasing, and quasi–concave for all $a \in A$. Since there is no production, one can identify the set of all possible allocation problems with L_o, the space of initial endowments. Thus, for all $q \in Q$ the set of feasible outcomes is simply $L_q = \{\ell \in L \mid \Sigma_{a\in A}\ell_a = \Sigma_{a\in A}\ell_{ao}\}$. In this example, the set of rules R is the set of all possible **allocation rules** for these economies and, therefore, the set of outcomes for any game $\Gamma(r,t)$, $r \in R$, equals $\Omega(r,t) = L_q$ for all $r \in R$. This example will be discussed further below.

The model of constitutional choice as introduced in this section has been kept deliberately general to accommodate all kind of institutional choices. In particular, the question about the appropriate solution concept for the games $\Gamma(r,t)$ induced by the rules and the related question of informational constraints on agents deserves careful discussion in each application. The following sections will provide additional examples and alternative concepts of how decisions will be made in the constitutional phase.

3. External costs and decision making costs

As is well–known Buchanan and Tullock (1965) analysed the agents' choice of a rule in terms of minimisation of expected external and decision making costs. In contrast, the present paper, following Schweizer (1989), models choice as a problem of expected utility

---

[9]The utility function u_a is said to be selfish, if there is a function $v_a : \mathbb{R}_+^m \to \mathbb{R}$ such that $u_a(\ell) = v_a(\ell_a)$ for all ℓ, where ℓ_a is the projection of ℓ on agents a's allocation.

maximisation – as formalised in equation 2.2. This section will establish the equivalence of these two approaches.

According to Buchanan and Tullock (1965, p. 45), external costs of a rule are the costs an agent faces because the rule produces an outcome which is worse for her than the status quo. Hence, for each rule $r \in R$ and each situation $t \in T$, one defines

(3.1) $$k_a^e(r,t) := \max\{u_a(\ell_o) - p_a(r,t), 0\}$$

as the external cost of agent $a \in A$. As for the decision–making costs, Buchanan and Tullock (1965, p. 69) argue that not only direct bargaining costs of applying the rule should be considered but opportunity costs as well. These opportunity costs derive from outcomes which are better for an agent than the outcome actually achieved under the adopted rule. Let $\overset{*}{u}_a(t) := \max\{u_a(o^*(r,t)) \mid r \in R\}$ be the best outcome in situation t for all possible rules, and define decision–making (opportunity) costs as

(3.2) $$k_a^d(r,t) := \min\{\overset{*}{u}_a(t) - p_a(r,t), \overset{*}{u}_a(t) - u_a(\ell_o)\}.$$

Taking expectations with respect to σ, one obtains

(3.1') $$K_a^e(r) := \int_T k_a^e(r,t)d\sigma, \text{ and}$$

(3.2') $$K_a^d(r) := \int_T k_a^d(r,t)d\sigma$$

as the **expected external costs** and the **expected decision–making costs** respectively. Finally, $K_a(r) := K_a^e(r) + K_a^d(r)$ defines the total expected costs for agent a under rule r.

The following proposition shows that maximising expected payoff from a rule and minimising total expected costs lead indeed to the same decision.

Proposition 3.1: $\arg\max_{r \in R} P_a(r) = \arg\min_{r \in R} K_a(r)$ *for all* $a \in A$.

Proof: From 3.1 and 3.2, one has $k_a^e(r,t) + k_a^d(r,t) = \overset{*}{u}_a(t) - p_a(r,t)$. Taking expectations yields

$$K_a(r) := K_a^e(r) + K_a^d(r) = \int_T \overset{*}{u}_a(t)d\sigma - P_a(r).$$

Hence, a minimiser of $K_a(r)$ must be a maximiser of $P_a(r)$, too. □

It is thus demonstrated that Buchanan's and Tullock's informal analysis of expected external costs and decision making costs (the former being a decreasing and the latter an increasing function of the required quorum for acceptance) is equivalent to an agent's choice of that rule which maximises her expected utility.

Observe that the classical voting rules form a proper subset of all voting rules. Since the concepts of external costs and decision making costs have been applied to this set of rules by Buchanan and Tullock (1962), it is illuminative to apply proposition 3.1 to these rules. For that purpose consider a situation $t = (\ell_o,u,q)$, where problem q implies the set of feasible allocations $L_q = \{\ell_o,\ell_q\}$, and denote by $v_a(q) := u_a(\ell_q) - u_a(\ell_o)$ the net willingness to pay for "project q" of agent a.[10] In this case one obviously has $\overset{*}{u}_a(t) = \max[0, v_a(q)] + u_a(\ell_o)$. Moreover, it is analytically convenient to define

$$\delta(r,t) = \begin{cases} 1, \text{ if } o^*(r,t) = \ell_q, \\ 0, \text{ if } o^*(r,t) = \ell_o. \end{cases}$$

With this notation it follows that

$$k_a^e(r,t) = -\min[0, v_a(q)] \cdot \delta(r,t),$$
$$k_a^d(r,t) = \{\max[0, v_a(q)]\} \cdot [1 - \delta(r,t)],$$

[10]For more details see example 4.1 in Section 4.

$k_a^e(r,t) + k_a^d(r,t) = \max[0, v_a(q)] - v_a(q)\cdot\delta(r,t),$

and therefore $K_a(r) = \int_T \{\max[0, v_a[q(t)]] + u_a(\ell_o)\}d\sigma - P_a(r).$

Observe that the integral term is independent of r, because $\overset{*}{u}_a(r,t)$ is the same for all classical voting rules. This proves the applicability of proposition 3.1 to the constitutional choice of voting rules. The following sections will provide additional examples and alternative concepts of how decisions will be made in the constitutional phase.

4. Two versions of the veil of ignorance

This section will focus on the question how a choice among rules will be made in the constitutional phase. To avoid an infinite regress of choosing the rule for choosing the rule for choosing the rule etc., we follow Buchanan and Tullock (1965, p.77/78) and Schweizer (1989) in adopting unanimity as the rule for accepting a rule at the constitutional level. Denote by

(4.1) $$W_i(r) := \Sigma_{a\in A}\, \pi(i,a)\cdot P_a(r)$$

the expected payoff of an individual $i \in I$ from adoption of rule $r \in R$ in the constitutional phase before she is identified with a particular agent. For unanimity in the choice of rule r^* at the constitutional level, it is necessary that

$$W_i(r^*) \geq W_i(r) \quad \text{for all} \quad r \in R \text{ and all } i \in I$$

holds, i.e. that each individual weakly prefers the rule r^* which is chosen.

Clearly, there is little hope to find rules which satisfy this condition for individuals characterised by arbitrary preferences or facing arbitrary problems. The main idea of Buchanan and Tullock (1965, p. 77/78) to achieve unanimity in the constitutional phase is the assumption that individuals are sufficiently 'unspecified' during the constitutional phase that they will come to identical decisions. This ignorance of individuals in regard to the exact conditions under which they will have to make the decision at the operational stage is called the **'veil of ignorance'**.

There are, however, two distinct interpretations which one can give to the idea that agents make their choice of a rule under the veil of ignorance at the constitutional stage:

- either individuals are completely ignorant about the role they are going to play in the post–constitutional society, (perfect constitutional uncertainty);
- or they have varying chances (or even certainty) of facing certain roles at that stage but equal chances of facing situations giving them similar payoffs (perfect post–constitutional uncertainty).

The following proposition shows that both concepts of the veil of ignorance provide a sufficient condition for unanimous evaluation of the rules at the constitutional level.

Proposition 4.1:

(i) *If* $\pi(i,a) = \pi(a)$ *for all* $i \in I$ *holds, then* $W_i(r) = W(r) \equiv \Sigma_{a \in A}\ \pi(a) \cdot P_a(r)$ *for all* $i \in I$.

(ii) *If* $P_a(r) = P(r)$ *for all* $a \in A$ *holds, then* $W_i(r) = W(r) \equiv P(r)$ *for all* $i \in I$.

Proof: Obvious from equation 4.1. □

Proposition 4.1 captures the spirit of the two concepts of the 'veil of ignorance' and shows that both versions are sufficient conditions for unanimous acceptance of a rule. The first concept is best represented by the description in Rawls (1977, chapter 24). It obviously puts no constraints on the form of the problem which has to be solved by the rule considered but requires each individual to have equal probability of becoming a specific type $a \in A$.

On the other hand, the second interpretation requires all agents in A to evaluate rules identically. This requires, of course, a sufficient diversity of problems which each agent may face and an equal probability of being confronted with a specific problem.

Alternatively, if the set of problems is not large enough to satisfy this condition, one could allow for sufficient diversity of preferences and/or allocations such that each agent could expect the same payoff given the distribution σ. In any case, the latter interpretation of the veil of ignorance implies substantial constraints on the problems and/or the characteristics of agents. It does not appear to be entirely clear what Buchanan's and Tullock's (1965) own interpretation is. There are indications (compare Buchanan and Tullock (1965, chapter 7)) that they favor the second version which is also Schweizer's (1990) interpretation of 'The Calculus of Consent'.

As proposition 4.1 shows, under certain conditions both concepts lead to the necessary unanimity in the choice of rules and avoid the infinite regress mentioned above. The first version of the veil of ignorance has the advantage of leaving the set of situations without any constraints, but may be subject to a criticism of too much arbitrariness. The second version may have the undesirable consequence of not allowing to discriminate between rules if the set of problems in which they are to be applied fails to exhibit a special structure. The following example which is a special case of the example by Schweizer (1990, section 2) will illustrate the difference between the two concepts.

Example 4.1: Consider three agents, A = {1,2,3}, who want to decide which voting rule should be applied to a finite set of projects Q in a situation where U = {u} and $L_0 = \{\ell_0\}$. The voting rules considered will be:

- r_1 : unanimity for acceptance,
- r_2 : simple majority voting,
- r_3 : unanimity for rejection.

Formally, the strategy sets implied by all these rules are $S_a = \{0,1\}$ (for all $a \in A$) where 0 means rejection and 1 acceptance of the project under consideration. If the voting rule r_i is applied to the situation $t = (\ell_o,u,q)$, where q defines $L_q = \{\ell_o,\ell_q\}$, the outcome of the game $\Gamma(r,t)$ is either $o = \ell_q$ or $o = \ell_o$, i.e. acceptance or rejection of project q. Hence one can describe each rule by the strategy sets $(S_a)_{a \in A}$ and the outcome function

$$o(s_1,s_2,s_3;r_j,t) = \begin{cases} \ell_q & \text{if } \Sigma_{a \in A}\, s_a \geq n(r_j), \\ \ell_o & \text{otherwise,} \end{cases}$$

where $r_j \in \{r_1,r_2,r_3\}$, $t = (\ell_o,u,q)$ and where $n(r_j)$ denotes the necessary quorum for acceptance of the project, i.e. $n(r_1) = 3$, $n(r_2) = 2$ and $n(r_3) = 1$. In summary, the induced game is $\Gamma(r,t) = (A, (S_a)_{a \in A}, (u_a(o))_{a \in A})$. The simple structure of this example allows us to replace $u_a(\ell_q)$ by $v_a(q) = u_a(\ell_q) - u_a(\ell_o)$, which is agent a's net willingness to pay for project q. Using this notation, the game $\Gamma(r,t)$ can be represented by the following matrices where player 1 chooses rows, player 2 chooses columns, and player 3 chooses the matrix: For r_2 and $q \in Q$, one obtains:

s_1/s_2	1	0
1	$v_a(q)$	$v_a(q)$
0	$v_a(q)$	0

$s_3 = 1$

s_1/s_2	1	0
1	$v_a(q)$	0
0	0	0

$s_3 = 0$

It is easy to check from these matrices that in this game each agent has the same dominant strategy namely

$$\overset{*}{s}_a(r_2,t) = \begin{cases} 1 & \text{for } v_a(q) > 0, \\ 0 & \text{for } v_a(q) \leq 0. \end{cases}$$

Similarly, one can check that the same dominant strategy holds for the games $\Gamma(r_1,t)$ and $\Gamma(r_2,t)$. Hence, there is a unique dominant strategy equilibrium $(\overset{*}{s}_a(r_j,t))_{a \in A}$ in each of these three games leading to the outcome

$$o^*(r_j,t) = \begin{cases} \ell_q & \text{for } \Sigma_{a \in A}\, s_a^*(r_j,t) \geq n(r_j), \\ \ell_o & \text{otherwise.} \end{cases}$$

The case for using a dominant strategy equilibrium in this example is particularly strong, since this concept does not require that each agent knows the net willingness to pay of her opponents.

If an agent $a \in A$ plays the game $\Gamma(r_j,t)$ in situation $t = (\ell_o, (u_a)_{a \in A}, q)$ her payoff is $p_a(r_j,t) = u_a[o^*(r_j,t)]$. It is obvious that the preference for one or the other of these rules depends now entirely both (i) on the distribution σ on $T = L \times U \times Q$, i.e. on the probability of an agent receiving a specific allocation, having specified preferences and facing a certain problem (post constitutional uncertainty), and (ii) on the probability π of an individual to become one specific agent (constitutional uncertainty). The following two cases will illustrate the difference between the two concepts of the veil of ignorance. For both cases $U = \{u\}$ and $L_o = \{\ell_o\}$ is assumed to hold, i.e. there is just one profile of preferences and one status–quo allocation.

Case i: Suppose that $Q = \{q\}$ holds. Hence $T = \{t\}$ with $t = (\ell_o,u,q)$ and $\sigma(t) = 1$. Let $v(q) = [v_1(q), v_2(q), v_3(q)] = (1,-1,1)$ be the preference profile for this unique project. Obviously, the outcome for both the unanimous rejection rule r_2 and the majority rule r_3 equals ℓ_q, and ℓ_o is the outcome for the unanimity rule r_1. This leads to the expected payoff profile

$$[P_1(r_j), P_2(r_j), P_3(r_j)] = \begin{cases} (0,0,0) & \text{for } j = 1, \\ (1,-1,1) & \text{for } j = 2,3. \end{cases}$$

Let $I = \{1,2,3\}$ and assume that $\pi(a,i) = 1$ for $a = i$ and zero otherwise. Then unanimous choice of a rule is impossible. Clearly, individuals 1 and 3 will strictly prefer rule r_2 and rule r_3 over rule r_1, but individual 2 will prefer rule r_1 to all other rules. Hence, unanimous agreement on a rule fails in this case.

If, however, the veil of ignorance is interpreted as an equal chance for each individual $i \in I$ to become each agent $a \in A$ after the constitutional phase, then unanimity will arise. In this version of the veil $\pi(a,i) = 1/3$ holds for all $i \in I$ and all $a \in A$. Therefore,

$$[W_1(r_j), W_2(r_j), W_3(r_j)] = \begin{cases} (0,0,0) & \text{for } j = 1 \\ (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) & \text{for } j = 2,3 \end{cases}$$

follows, and either rule r_2 or r_3 is accepted unanimously.

Case ii: There is a second way to obtain unanimity at the constitutional level. Assume that there is a problem $q_o \in Q$ which leads to the net willingness to pay $v(q_o) = (1,-1,1)$ and assume that there are seven other projects q_1 to q_7 yielding all possible permutations of payoffs of $v(q_o)$, namely

	q_o	q_1	q_2	q_3	q_4	q_5	q_6	q_7
v_1 (q)	1	1	1	1	- 1	- 1	- 1	- 1
v_2 (q)	- 1	1	1	- 1	- 1	1	1	- 1
v_3 (q)	1	- 1	1	- 1	1	- 1	1	- 1

It is easy to check that the following outcomes result:

$o^*(r_1,q_j) = \ell_{qj}$ for $j = 2$;

$o^*(r_2,q_j) = \ell_{qj}$ for $j = 0,1,2,6$;

$o^*(r_3,q_j) = \ell_{qj}$ for $j = 0,1,2,3,4,5,6$.

Suppose now that Q consists of these eight problems only and that the probability of each problem is the same, namely 1/8 for all j = 0,1,...,7. One checks easily that in this case the expected return from each rule is identical for each agent $a \in A$ and therefore for each individual:

$$[P_1(r_j), P_2(r_j), P_3(r_j)] = \begin{cases} (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}) & \text{for } j = 1, \\ (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) & \text{for } j = 2, \\ (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}) & \text{for } j = 3. \end{cases}$$

Note that in this case one has unanimity as well, but rule r_2 is now strictly preferred to all other rules. In particular, rule r_3 is now unacceptable even though it was acceptable to all individuals in the first case.

We conclude this extensive example with the observation that an identical chance for each individual to become each agent as well as identically independently distributed problems are both sufficient conditions for unanimity at the constitutional stage. The choice of the rule, however, is affected by the version of the veil of ignorance applied. This difference is a consequence of the fact that the assumption of identically and independently distributed net willingness to pay for the agents amounts to a selection of certain situations and certain probability distributions over the set of situations implying a severe restriction on the structure of the problems of constitutional choice to be investigated.

5. On the relationship between the two veils of ignorance

To avoid an infinite regress of "how to choose a constitution of 'how to choose...'", all agents must agree unanimously on what constitutes the "best" rule. This unanimity is guaranteed if all agents' ranking of rules is identical. As argued above such a coincidence is, however, rather unlikely unless the constitutional choice problem implies a degree of uncertainty about roles and situations great enough to turn at least one of the "veils of uncertainty" into a veil of ignorance. Clearly, differences in the probability density functions π

an σ along with changes in T, the support space of σ, simply mean that one deals with different problems of constitutional choice. It is not surprising, therefore, that solutions, if they exist, differ from each other in general.

In section 4, two versions of the veil of ignorance have been suggested, one relying on sufficient diversity of situations, the other using the device of "mixing agents" to obtain a unanimous choice of rules. Both methods achieve a unanimous preference ordering over rules by taking away the self–interest of a particular individual. Certainty about the post–constitutional settings into which an individual might be put creates particular preferences over rules. If, on the other hand, the degree of uncertainty about these post–constitutional situations is sufficiently high a mutually consistent assessment of rules emerges.

The choice of a specific version of the veil of ignorance seems to depend on the applications which one has in mind. If one is willing to take a broad view of a "situation" allowing for different preferences and status–quo allocations, then uncertainty about an individual's role assignment may become unimportant. On the other hand, if one has fairly specific post–constitutional situations in mind, with little or no variety in agents' characteristics, then uncertainty about the future role becomes essential. The set–up of the model in this paper is sufficiently general to encompass both interpretations.

In the tradition of contractarian theories the constitutional veil of ignorance is considered a mental experiment with a normative motivation: people should make constitutional choices as if they were completely uncertain about their future roles and as if they would face each role with the same probability, i.e., in the case of a finite set of agents A, $\pi(i,a) = \#A^{-1}$ for all $a \in A$ and all $i \in I$. But since the constitutional veil of ignorance is a fiction, there appears to be no basis to compare it with real–world post–constitutional uncertainty. The following analysis will show, however, that under some additional assumptions useful comparisons between the two versions of the veil of ignorance can be provided.

For that purpose, let us completely eliminate the contractarian fiction of ignorance, i.e., for all $i \in I$, put $\pi(i,a) = 1$ for some $a \in A$ and $\pi(i,a') = 0$ for all other $a' \in A$. Hence, every individual $i \in I$ is certain about her post–constitutional role when society has to decide on what rule to adopt. We know from proposition 4.1ii that even in this case unanimous agreement is a possibility, if all agents are sufficiently uncertain about their own future and their own future preferences. According to proposition 4.1ii, $P_a(r) = P(r)$ for all $a \in A$ is a sufficient condition for identical individual rankings of rules (with or without additional "contractarian uncertainty"). It remains unclear, however, what kind of post–constitutional or 'real–world' uncertainty yields this equality.

Under a few additional assumptions at least a partial answer can be given. Note that each situation $t = (\ell_o,u,q)$ can be decomposed into those components which are agent specific, t_a, and those which are not, t_A. Components of t_a comprise preferences of agent a, u_a, depending on the situation, agent–specific status–quo allocations, ℓ_{oa}, or, eventually, agent–specific outcome allocation sets L_{qa}. The vector t_A, on the other hand, contains all those elements of a situation which are common to all agents, e.g. public good allocations. Hence, one can write $t = ((t_a)_{a \in A}, t_A)$ for each situation $t \in T$.

Let $\Psi \equiv \{\psi \mid \psi\colon A \to A$, bijective) be the set of permutations of A and denote by Ψ_o the maximal subset of permutations such that, for any $\psi,\psi' \in \Psi_o$, $\psi(a) \neq \psi'(a)$ for all $a \in A$ holds. Finally, denote by $t_\psi = ((t_{\psi(a)})_{a \in A}, t_a)$ the vector of characteristics where agent a has characteristics of agent $\psi(a)$, $t_{\psi(a)}$, and by $\Theta(t) \doteq \{t_\psi \mid \psi \in \Psi_o\}$ the set of situations derived from t by permuting agent characteristics with permutations in Ψ_o.

Assumption 5.1:

(i) The set of agents A and the set of individuals I are finite sets.

(ii) The set of rules R contains anonymous rules only, i.e. rules satisfying $u_a(o*(r,t)) = u_{\psi(a)}(o*(r,t_\psi))$ *for all* $a \in A$ *and all permutations* $\psi \in \Psi$.

Consider now the set of situations $T' = \bigcup_{t \in T} \phi(t)$ derived from T. Without loss of generality assume that T is finite and let $\sigma(t)$ be the probability of $t \in T$. The probability of $t' \in T'$ is then easily derived as

$$\sigma'(t') = \Sigma_{t \in T} \text{ prob } \{t' \in \Theta(t)\} \cdot \sigma(t).$$

Assumption 5.2: *The probability of all permutations in Ψ_0 is the same.*

It is easy to check that $\#\Psi_0 = \#A$ holds. Hence, by assumption 5.2, prob $\{t' \in \Theta(t)\} = \chi_t(t')/\#A$ is the indicator function of the set $\Theta(t)$, i.e. $\chi_t(t') = 1$ for $t' \in \Theta(t)$ and $\chi_t(t') = 0$ for $t' \notin \Theta(t)$. Therefore, one has

$$\sigma'(t') = \Sigma_{t \in T}\, \chi_t(t') \cdot \sigma(t)/\#A.$$

Clearly, (T', σ') is an expanded set of situations with a derived probability distribution σ'. Given a rule $r \in R$, one has a related constitutional choice problem where a game $\Gamma(r,t')$ is defined for each $t' \in T'$ which leads to the (by assumption) unique outcome $o^*(r,t')$ and to a payoff of agent $a \in A$ ($p_a(r,t') = u_a(o^*(r,t'))$. This induces an expected payoff $P_a'(r) = \Sigma_{t' \in T'} p_a(r,t') \cdot \sigma^*(t')$. We are now in a position to state and prove the main result of this section.

Proposition 5.1: *Given assumption 5.1 and 5.2. For any finite set of situations T with associated probability distribution σ, the derived constitutional choice with set of situations T' and probability distribution σ' satisfies*

$$P_a'(r) = P(r) \equiv \#A^{-1} \cdot \Sigma_{a \in A} P_a(r) \quad \textit{for all } a \in A.$$

Proof: By construction of $\Theta(t)$, all games $\Gamma(r,t')$ with $t' \in \Theta(t)$ are identical except for the order of the players. By assumption 5.1ii only anonymous rules are considered. Hence the outcome of a game $\Gamma(r,t_\psi)$, $o^*(r,t_\psi)$, with $t_\psi \in \Theta(t)$ must satisfy

$$u_a(o^*(r,t_\psi)) = u_{\psi(a)}(o^*(r,t)).$$

Consequently, it is true that

$$P_a'(r) = \Sigma_{t'\in T'} u_a(o^*(r,t'))\cdot\sigma'(t') = \Sigma_{t'\in T'} u_a(o^*(r,t')\cdot[\Sigma_{t\in T}\chi_t(t')\cdot\sigma(t)/\#A]$$

$$= \#A^{-1}\cdot\Sigma_{t\in T}\Sigma_{t'\in T'} u_a(o^*(r,t'))\cdot\chi_t(t')\cdot\sigma(t),$$

$$= \#A^{-1}\cdot\Sigma_{t\in T}\Sigma_{t_\psi\in\Theta(t)} u_a(o^*(r,t_\psi))\cdot\sigma(t),$$

$$= \#A^{-1}\cdot\Sigma_{t\in T}\Sigma_{\psi\in\Psi_o} u_{\psi(a)}(o^*(r,t))\cdot\sigma(t),$$

$$= \#A^{-1}\cdot\Sigma_{t\in T}\Sigma_{a\in A} u_a(o^*(r,t))\cdot\sigma(t) = \#A^{-1}\cdot\Sigma_{a\in A} P_a(r) \equiv P(r) \;\square$$

Proposition 5.1 says that one can obtain unanimity in the choice of a rule if one has sufficient diversity in the set of situations and sufficient uncertainty about the realisation of these situations. Thus, with sufficient post–constitutional uncertainty the contractarian fiction is no longer necessary for a unanimous decision.

Remark: Most of the assumptions made for this proposition can be weakened. In particular assumption 5.2 and finiteness of T could be generalised. To cover the case of general distributions on T is straightforward but requires some measure theoretic complications which would have distracted from the main line of argument. Assumption 5.1, on the other hand, is essential. This is pretty obvious for assumption 5.1i, since the decomposition of a situation t in the way described above depends on the finiteness of A. To see that assumption 5.2ii is necessary, reconsider the case i of example 4.1. If the rule r were that agent 1 is a dictator deciding on implementation of ℓ_q, then the outcome of the game would be affected by the permutation of characteristics. In this case, the only individual characteristics are preferences. For net–willingness to pay (1, –1,1) ℓ_q would be the outcome. In the permutation where agent 1 gets the net–willingness to pay of agent 2 the outcome would change to

ℓ_0. But $v_1(q) = 1 \neq v_{\psi(1)}$ (*status quo*) $= 0$. It is clear that a permutation of preferences cannot create unanimity in the choice of this rule.

A related question can now be answered easily. Suppose that one starts with a model in which constitutional uncertainty is necessary to achieve a unanimous choice of a rule, i.e. where $P_a(r) \neq P_b(r)$ for some $a,b \in A$ holds. Then one can ask the question whether it is possible to abandon the fictitious constitutional uncertainty and substitute an increased post–constitutional uncertainty for it such that the same choice of rule results as under the constitutional veil of ignorance.

Corollary 5.2: *Given assumptions 5.1 and 5.2, a rule r^* which is chosen in the model with situations (T,σ) and a constitutional veil $\pi(i,a) = \#A^{-1}$ for all $i \in I$ and all $a \in A$ will be chosen in the model with situations (T',σ') and arbitrary $\pi(i,a)$ as well.*

Proof: Suppose that $W_i(r^*) \equiv \Sigma_{a\in A}\ \#A^{-1}\cdot P_a(r^*) \geq W_i(r) \equiv \Sigma_{a\in A}\ \#A^{-1}\cdot P_a(r)$ holds for all $r \in R$, i.e. r^* is prefered by all agents in the model (T,σ) given the constitutional veil of ignorance. By proposition 5.1,

$$P_a'(r) = P(r) = \#A^{-1}\cdot\Sigma_{a\in A}\ P_a(r) \quad \text{for all } a \in A$$

is true. Hence, for any $\pi(i,a)$ one has

$$W_i'(r^*) \equiv \Sigma_{a\in A}\ \pi(i,a)\cdot P_a'(r^*) = \Sigma_{a\in A}\ \pi(i,a)\cdot P(r^*) = P(r^*) \equiv \#A^{-1}\cdot\Sigma_{a\in A}\ P_a(r^*)$$

$$\geq \#A^{-1}\cdot\Sigma_{a\in A}\ P_a(r) = W_i'(r) \quad \text{for all } r \in R. \ \square$$

Proposition 5.1 provides a constructive method to expand the set of possible situations such that the choice of a rule is unanimous without the contractarian fiction of ignorance about an individual's identity. On the other hand, this proposition raises the question of what makes the "identity" of an individual. In the interpretation of this paper identity means an association of an individual with a specific role name but not with specific characteristics. What really matters for an individual's choice among rules, however, are their prospective post–constitutional characteristics in the widest sense, i.e. including preferences and allocational options. It seems as if some of the controversy about the veil of ignorance which one finds in the literature might be caused by a lack of precision as to the meaning of it.

6. Comparison of institutions: some examples

Consider a pure exchange economy in which each agent $a \in A = \{1,2\}$ has an endowment $\omega_a > 0$ and $\nu_a > 0$ of the commodities X and Y, respectively. Total endowments are $\omega := \omega_1 + \omega_2 > 0$ and $\nu := \nu_1 + \nu_2 > 0$. Agent a's utility from the consumption bundle (x_a,y_a) is

(6.1) $$u_a = x_a^{\alpha} \cdot y_a^{\alpha} \quad \text{with} \quad \alpha \in \mathbb{R}_{++}.$$

Let $Q \equiv \mathbb{R}^4_{++}$ be the set of initial endowments. Each element q indexes a set of potential allocations $L_q = \{(x_1,x_2,y_1,y_2) \in \mathbb{R}^4_+ \mid x_1 + x_2 \leq \omega,\ y_1 + y_2 \leq \nu\}$, the set of feasible allocations. A problem q corresponds therefore to the search of a feasible allocation in L_q. The set of potential status–quo allocations $L_o = \mathbb{R}^4_{++}$ can here be identified with the index set of problems Q. Among the large number of rules which have been suggested for "solving this problem" is the **Walrasian market procedure** r_w as well as the **principle of egalitarian distribution** r_e. In what follows, we wish to compare these two rules at the constitutional level.

The Walrasian market rule. Rather than describing the market game completely with its players and strategies (for such an exposition compare Arrow and Debreu (1954)) it suffices to simply indicate how to calculate the (unique) Walrasian equilibrium and the associated payoffs. With p_x and p_y as prices for the goods X and Y, respectively, individual utility maximisation yields with the help of equation 6.1

$$x_a = \tfrac{1}{2}\cdot(\omega_a + \frac{p_y}{p_x}\nu_a) \text{ and } y_a = \tfrac{1}{2}\cdot(\nu_a + \frac{p_y}{p_x}\omega_a) \text{ for } a = 1,2.$$

Hence the market for good X clears (implying the simultaneous clearance of the market for good Y) if

$$0 = x_1 + x_2 - \omega = \tfrac{1}{2}\cdot(\omega_1 + \frac{p_y}{p_x}\nu_1) + \tfrac{1}{2}\cdot(\omega_2 + \frac{p_y}{p_x}\nu_2) - \omega \text{ or } \frac{p_y}{p_x} = \frac{\omega}{\nu}.$$

It follows that the Walrasian equilibrium allocation, hence the outcome of the market game is

(6.2) $$o(r_w,t) = (x_{1w}, x_{2w}, y_{1w}, y_{2w})$$

with $x_{aw} = \dfrac{\omega_a\nu + \omega\nu_a}{2\nu}$ and $y_{aw} = \dfrac{\omega\nu_a + \nu\omega_a}{2\omega}$. In view of 6.2 and 6.1 the payoffs are for a $\in$ A

(6.3) $$p_a(r_w,t) = u_a[o(r_w,t)] = \left[\frac{(\nu\omega_a + \omega\nu_a)^2}{4\nu\omega}\right]^{\alpha}.$$

The egalitarian distribution rule. For any initial distribution of goods, the outcome of this rule is $o(r_e,t) = (\frac{\omega}{2}, \frac{\omega}{2}, \frac{\nu}{2}, \frac{\nu}{2})$ so that one uses eq. 6.1 to calculate the individual payoffs as

(6.4) $$p_a(r_e,t) = u_a[o(r_e,t)] = \left[\frac{\omega\nu}{4}\right]^{\alpha}.$$

Note that in the present example the egalitarian allocation happens to be Pareto efficient because both agents were assumed to have the same preferences.

The next step is to clarify and numerically specify the set of situations T. A situation t is completely described by an endowment vector in L_0 and a profile of utility functions which are parameterised by α in this example. To simplify our exposition even further, let the total endowments for both goods be 4. Any initial endowment for agent 1 now specifies (ω_2

$= 4 - \omega_1$, $\nu_2 = 4 - \nu_1$) as the endowment vector of agent 2. If we consider only $\omega_1 = 1,2,3$ and $\nu_1 = 1,2,3$ as possible endowments for agent 1, then the set of initial allocations consists of nine elements only and can conveniently be written as

$$L_o = \{(\omega_1,\nu_1,\omega_2,\nu_2) \mid \omega_2 = 4 - \omega_1,\ \nu_2 = 4 - \nu_1;\ \omega_1,\nu_1 \in \{1,2,3\}\ \}.$$

Similarly, one has $U = \{[(x_1 \cdot y_1)^{\alpha},(x_2 \cdot y_2)^{\alpha}] \mid \alpha \in \mathbb{R}_{++}\}$ as the set of preferences. Recalling that $L_q = L_o$ and $Q = \{q\}$ in this example, the set of situations T is $L_o \times U \times Q$, and any element $t \in T$ can conveniently be described by the three numbers (ω_1,ν_1,α).

For any situation $t \in T$ the associated payoffs are, respectively,

$$p_a(r_w,t) = \left[\frac{(\omega_a + \nu_a)^2}{4}\right]^{\alpha} \text{ and } p_a(r_e,t) = (4^2)^{\alpha} = 16^{\alpha}.$$

Case 1: Suppose first that $\alpha = 1$ and the probability distribution σ over T is flat. Then each endowment prevails with probability 1/9, and the Walrasian market rule yields the following payoffs where each cell of the table corresponds to a different situation (characterised by ω_1, ν_1, and $\alpha = 1$):

ν_a \ ω_a	1	2	3
1	1	$\frac{9}{4}$	4
2	$\frac{9}{4}$	4	$\frac{25}{4}$
3	4	$\frac{25}{4}$	9

Moreover, the agent's expected payoff is $P_1(r_w) = P_2(r_w) = \frac{1}{9}\cdot\frac{156}{4} = 4.\overline{3}$. On the other hand, if the rule of egalitarian distribution is used, one obtains $P_1(r_e) = P_2(r_e) = \frac{1}{9}\cdot 9\cdot 4 = 4$. It follows that r_w is unanimously preferred to r_e independent of the specification of $\pi(i,a)$. Hence part ii of proposition 3.1 applies.

Case 2: To see that the equality of $P_1(r)$ and $P_2(r)$ depends crucially on the probability distribution, suppose now that the endowments $(\omega_1,\nu_1) = (1,1)$, $(1,2)$, $(2,1)$, and $(2,2)$ with their complementary endowments $(\omega_2,\nu_2) = (3,3)$, $(3,2)$, $(2,3)$, and $(2,2)$ occur each with probability 1/4 whereas the other $\ell \in L$ occur with zero probability. Then it turns out that $P(r_w) = 2.375 \neq P_2(r_w) = 6.375$, whereas $P_1(r_e) = P_2(r_e) = 4$ still holds. In this case the constitutional choice among r_w and r_e fails to be conclusive unless the constitutional uncertainty is perfect in the sense that $\pi(i,a) = \frac{1}{2}$ for all $i \in I$ and $a \in A$. Under this condition it is easy to see that $W_i(r_w) = 4.375 > W_i(r_e) = 4$ for all $i \in I$.

Case 3: Whenever the constitutional choice was conclusive in the preceding examples, the Walrasian market rule dominated the egalitarian distribution rule. In order to show that this is not generally true, reconsider the case where σ is a flat distribution but leave α unspecified. This yields for each $a \in A$

$$P_a(r_w) - P_a(r_e) = \tfrac{1}{9}\cdot[1 + 9^{\alpha} + 2(\tfrac{9}{4})^{\alpha} + 2(\tfrac{25}{4})^{\alpha} - 6\cdot 4^{\alpha}].$$

For $\alpha = 1/2$ this difference is zero and for $\alpha = 1/4$ it attains the value -0.0158 implying that the egalitarian distribution rule dominates the Walrasian market rule if α is small enough. Since an increase in α corresponds to a monotonic transformation of the utilities, case 3 shows that such transformations change the preference for one or the other rule at the constitutional level. Consequently, utility functions are not just representations of ordinal preferences, but rather cardinal measures of intensity of preferences. Indeed, this is a direct consequence of treating our utility functions as von–Neumann–Morgenstern utilities.

The simplicity of our example makes transparent the reason for this varying preference for egalitarian versus Walrasian allocations. Since a smaller parameter α means a utility function which is more concave along the line $\{(x,y) \mid x = y\}$ in the commodity space, a higher α means a less risk–averse agent. Given the uniform distribution σ, in case 3 agents have equal chances to end up with a high or with a low endowment in the post–constitutional phase. From equation 6.3 it is clear that the Walrasian allocation rule favours the agent with the higher endowment, while the egalitarian rule neutralises any effect of the individual distribution of the initial endowment on the final allocation.

If only initial endowment distributions were allowed which give each agent the same quantity of each commodity, i.e. if with probability 1/3 either $(\omega_1,\nu_1) = (1,1)$ or $(\omega_1,\nu_1) = (2,2)$ or $(\omega_1,\nu_1) = (3,3)$ would hold, then for $\alpha < 1$ the egalitarian rule would be chosen, while for $\alpha > 1$ the Walrasian rule would dominate, as one easily checks from the equations 6.3 and 6.4. Note firstly that in this case the Walrasian equilibrium would imply no trade, and secondly that for $\alpha < 1$ the utility function is strictly concave along the locus of equal quantities, while it is strictly convex along this locus for $\alpha > 1$. Thus, risk considerations would decide the choice in this case alone.

On the other hand, if as in case 3 off–diagonal initial endowment distributions have a positive probability, then these risk related considerations have to be balanced against the advantage of mutually beneficial trade away from relative asymmetric endowment positions. This accounts for the fact that risk–aversion has to be rather strong ($\alpha < 1/2$) before the egalitarian allocation rule dominates the Walrasian rule. It is worth noting that Buchanan and Tullock (1965, pp. 195–199) were well aware of this trade–off between insurance and allocative efficiency advantages.

Finally, the conflict of interest between agents in case 2 is a straightforward consequence of the asymmetry of possible situations in favouring agent 2. The difference between case 1 and case 2 can be viewed as an expansion of situations which are considered as possible applications of the rule at the constitutional stage. Given that both agents have identical preferences, one can interprete the randomisation with $\pi(i,a) = 1/2$ which was necessary to achieve unanimity as giving positive probability to endowment situations (3,3), (3,2), and (2,3) for agent 1 which were possible for agent 2 only previously.

7. Concluding remarks

Welfare economics has been plagued by the problem that economic states could not be compared if they lay in the set of pareto–optimal allocations. In very many cases it was even impossible to argue for a move from a pareto–inferior to a pareto–optimal situation without prior specification of compensatory payments, since a move to a pareto–preferred state might involve a loss for some agents. It is one of the very appealing aspects of the theory of constitutional choice as developed by Buchanan and Tullock (1965) that it provides a concept to overcome this problem, at least in a number of applications.

Though there are a few problems to be solved, e.g. uniqueness of the outcome from the social interaction resulting from a rule in a given situation, this approach seems to be promising not only for "constitutional" choice problems but for a broad range of social choice problems in general. This is important since very many social choice problems do involve a once–and–for–all choice of an allocation rather than instituting a process of decision making for many different cases.

Consider for example the case of public decision–making on an environmental issue. The decision to declare a forest area a National Park usually meets opposition from logging industries and local workers using the forest as a resource for their business. People favouring the instalation of a National Park are often city dwellers using the forest for recreational purposes. The implied conflict of interest results from the specific situation in which agents find themselves when the decision has to be made.

While in the constitutional choice problem a rule together with a situation determine a game and (ideally a unique) outcome in the environmental problem, the decision to create a National Park (or not to create it) directly produces an outcome. With such a modification, one could ask the question what an agent would vote for if she were uncertain as to her role after the decision, e.g. whether she could want to use the forest for recreational purposes or as input into a production process. Framing the question in this way may yield a unanimous choice for or against the project under consideration.

This example suggests a much broader range of applications for the theory presented in this paper than Buchanan and Tullock had in mind in their book "The Calculus of Consent" (1965). At the same time, however, it indicates some of the problems involved in this approach. Firstly, the determinacy of the social interaction, following the implementation of a rule or public decision. As mentioned before, with multiple outcomes from such an interaction the assessment of the rule is impossible. This problem of equilibrium selection, however, is pervasive to all economic theory. Secondly, since one cannot take the constitutional phase as a literal description of the situation in which a decision on a rule takes place, one has to view it as a hypothetical situation. To ask an individual, however, how she would decide under some hypothetical situation, raises the familiar problem of truthful revelation of preferences. Again, this is a fundamental problem of all economic analysis. But in spite of these open problems, the approach deserves further investigation.

References

Arrow, K.J., and Debreu, G. (1954), "Existence of an equilibrium for a competitive economy", **Econometrica** 22, 265–290

Binmore, K. (1989), "Social contract I: Harsanyi and Rawls", **Economic Journal** (Supplement 1989) 99, 84–102

Buchanan, J.M., and Tullock, G. (1965), **The Calculus of Consent. Logical Foundations of Constitutional Democracy**

Harsanyi, J.C. (1967/68), "Games with incomplete information played by 'Bayesian' players; Part I: The basic model; Part II: Bayesian equilibrium points; Part III: The basic probability distribution of the game", **Management Science** 14, 159–182, 320––334, 486–502

Pethig, R. (1990), "Calculus of consent: A game–theoretic perspective. Comment", **Journal of Institutional and Theoretical Economics** (JITE) 146, 55 – 60

Rawls, J. (1971), **The Theory of Justice**, Harvard University Press

Schweizer, U. (1990), "Calculus of consent: A game theoretic perspective", **Journal of Institutional and Theoretical Economics** (JITE) 146, 28 – 54

Schweizer, U. (1988). "Unanimous choice of rules", Discussion Paper No. A–172, Universität Bonn

Seit 1989 erschienene Diskussionsbeiträge:
Discussion papers released as of 1989/1990:

1-89: **Klaus Schöler**, Zollwirkungen in einem räumlichen Oligopol

2-89 **Rüdiger Pethig**, Trinkwasser und Gewässergüte. Ein Plädoyer für das Nutzerprinzip in der Wasserwirtschaft

3-89 **Rüdiger Pethig**, Calculus of Consent: A Game-theoretic Perspective. Comment

4-89 **Rüdiger Pethig**, Problems of Irreversibility in the Control of Persistent Pollutants

5-90 **Klaus Schöler**, On Credit Supply of PLS-Banks

6-90 **Rüdiger Pethig**, Optimal Pollution Control, Irreversibilities, and the Value of Future Information

7-90 **Klaus Schöler**, A Note on "Price Variation in Spatial Markets: The Case of Perfectly Inelastic Demand"