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A Note on "Price Variation in Spatial Markets: The Case of Perfectly Inelastic Demand"

by

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Abstract:

This note on the paper of Mulligan and Fik (Annals of Regional Science 23: 187–201) is based on the need for consistent models. If we look at the spatial oligopoly approach of Mulligan and Fik from this point of view, we can draw two conclusions: First, their model does not indicate any reasons for different conjectural price variations. Second, exactly one conjectural reaction coefficient is consistent with the profit maximizing behavior of the firms.

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I. Introduction

In the Annals of Regional Science (1989a) and in other publications (1989b, 1989c), Mulligan and Fik presented a rather interesting and important model of spatial competition. It is largely due to them that for spatial chain oligopolies the dependency of prices in a short run equilibrium is derived from the spatial distribution of the firms, the production cost, and the conjectural reactions of the firms. The model can be applied equally to perfectly inelastic and elastic demand functions of the consumers as well as to circular and linear one-dimensional market areas. Thus we have an approach of high generality. The authors alternatively assume as conjectural price variations either the Löschian-competition, HS-competition or GO-competition, and they explicitly point out that their model allows symmetrical as well as asymmetrical behavioral assumptions. This statement is formally correct, the questions however arise, whether different conjectural price reactions in a market can be justified for example by different production costs and locations, and more generally, whether certain conjectural reaction coefficients, for example $\phi = 1, 0, -1$, can be derived from the model at all. In other words, may the assumed conjectural behavior patterns be explained endogenously and thus be consistent with the rational profit-maximizing behavior of the firm? The intent of this short note is to investigate this important question. For this purpose, we adopt the designations of the variables as well as the assumptions from the model of Mulligan and Fik, with the exception of those assumptions relating to the conjectural behavior patterns.

II. Assumptions and Model

For our argumentation the limitation to perfectly inelastic demand functions of the consumers and circular one-dimensional market areas is sufficient. The results can easily be transferred to all other cases. The necessary assumptions of the model are as follows:

A 1: Within the market area there is demand for exactly one unit of the good per distance unit, independent of the price.

A 2: The consumers buy the good at the lowest delivered price p^d available at their place of consumption. The price p_i^d of firm i is computed by the mill price p_i and the transport cost t per distance unit: $p_i^d = p_i + tx$.

A 3: The firm i produces the good under variable and fixed costs: $C_i = k_iQ_i + F_i$, where C_i is the total cost, k_i represents the constant marginal cost, F_i is the fixed cost, and Q_i stands for the output quantity or sales quantity, respectively.

A 4: Viewed from its location, the market area a_i of the firm i is composed of a left-hand area $a_{i,L}$ and a right-hand area $a_{i,R}$: $a_i = a_{i,L} + a_{i,R}$.

At the borders of the non-overlapping market areas B, the delivered prices p^d of neighboring firms must be identical due to the assumption A 2. For the left-hand market area border of the firm i we have: $p_i^d(B_{i,L}) = p_{i-1}^d(B_{i-1,R})$ and for the right-hand market area border $p_i^d(B_{i,R}) = p_{i+1}^d(B_{i+1,L})$. Considering the locations X_{i-1} and X_{i+1} for the neighboring firms as well as X_i for the firm under consideration, the left-hand border may be expressed by:

$$B_{i,L} = (p_i - p_{i-1} + tX_i + tX_{i-1})/2t \tag{1}$$

and the right-hand border by

$$B_{i,R} = (p_{i+1} - p_i + tX_{i+1} + tX_i)/2t \tag{2}$$

so that the total market area of the firm i is

$$a_i = B_{i,R} - B_{i,L} = (p_{i-1} - 2p_i + p_{i+1} - tX_{i-1} + tX_{i+1})/2t \tag{3}$$

When the assumptions A 1 ($a_i = Q_i$) and A 3 are taken into account, the profit function of the firm i can be expressed by

$$\pi_i = Q_ip_i - Q_ik_i - F_i \tag{4}$$

$$= (p_i - k_i)[(p_{i-1} - 2p_i + p_{i+1} - t(X_{i-1} - X_{i+1})]/2t - F_i$$

This profit function has to be maximized with respect to p_i. If, to simplify the assumptions of the conjectural price reactions of the firm i, we write $\partial p_{i-1}/\partial p_i = \phi_{i,i-1}$ or $\partial p_{i+1}/\partial p_i = \phi_{i,i+1}$, the first-order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = (p_i - k_i)(\phi_{i,i-1} - 2 + \phi_{i,i+1})(1/2t) + Q_i = 0 \tag{5}$$

or

$$p_i(4 - \phi_{i,i-1} - \phi_{i,i+1}) = p_{i-1} + p_{i+1} - t(X_{i-1} - X_{i+1}) + (2 - \phi_{i,i-1} - \phi_{i,i+1})k_i$$

The second-order condition is:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = (\phi_{i,i-1} - 2 + \phi_{i,i+1})/t < 0 \tag{6}$$

or

$$\phi_{i,i-1} + \phi_{i,i+1} < 2$$

Up to this point of the investigation we follow the model of Mulligan and Fik (1989a). In the next section the conjectural reaction coefficients are endogenous variables of the model and therefore consistent conjectural reactions.

III. Consistent Conjectural Price Reactions

The assumptions of the firm i on the price reactions of the firms $i+1$ and $i-1$ are consistent in those cases where they coincide with the actual price reactions of their neighboring firms ($\hat{\phi}_{i-1} = \phi_{i,i-1}$, $\hat{\phi}_{i+1} = \phi_{i,i+1}$) [s. Bresnahan (1981), Boyer/Moreaux (1984), Schöler (1989)]. The actual reactions of rationally acting firms take place on the basis of their particular structural conditions (demand and cost functions) and their objectives (profit maximization). The actual reactions can therefore be derived from the reaction function of the firm $i+1$

$$\begin{aligned} R_{i+1} = p_{i+1}(4 - \phi_{i+1,i} - \phi_{i+1,i+2}) - p_i - p_{i+2} + t(X_i - X_{i+2}) \\ -(2 - \phi_{i+1,i} - \phi_{i+1,i+2})k_{i+1} \end{aligned} \tag{7}$$

and the firm $i-1$

$$\begin{aligned} R_{i-1} = p_{i-1}(4 - \phi_{i-1,i-2} - \phi_{i-1,i}) - p_{i-2} - p_i + t(X_{i-2} - X_i) \\ -(2 - \phi_{i-1,i-2} - \phi_{i-1,i})k_{i-1} \end{aligned} \tag{8}$$

If we differentiate the implicit function $R_{i+1}(p_{i+1}, p_i)$ to p_{i+1} and p_i, or the implicit function $R_{i-1}(p_{i-1}, p_i)$ to p_{i-1} and p_i, the conjectural reaction coefficients contained in the derivation of firm i must be equal to the actual reaction coefficients. This enables us to proceed from consistent, i.e. endogenous reactions:

$$\hat{\phi}_{i-1} = -\frac{\partial R_{i-1}/\partial p_i}{\partial R_{i-1}/\partial p_{i-1}} \tag{9}$$

$$= -\frac{\phi_{i,i-1}(4 - \phi_{i-1,i-2} - \phi_{i-1,i}) - \phi_{i-1,i-2}\phi_{i,i-1} - 1}{4 - 2\phi_{i-1,i-2} - 2\phi_{i-1,i}}$$

$$\hat{\phi}_{i+1} = -\frac{\partial R_{i+1}/\partial p_i}{\partial R_{i+1}/\partial p_{i+1}} \tag{10}$$

$$= -\frac{\phi_{i,i+1}(4 - \phi_{i+1,i+2} - \phi_{i+1,i}) - \phi_{i+1,i+2}\phi_{i,i+1} - 1}{4 - 2\phi_{i+1,i+2} - 2\phi_{i+1,i}}$$

As neither the different locations ($X_{i+j}, \quad j = -2, -1, 0, 1, 2$) nor the different marginal production costs ($k_{i+j}, \quad j = -1, 0, 1$) have an impact on the determination of the actual price reactions $\hat{\phi}_{i-1}$ and $\hat{\phi}_{i+1}$, there is no reason to assume that price reactions vary from firm to firm. Consequently, we may proceed on the assumption that

$$\hat{\phi}_{i-1} = \hat{\phi}_i = \hat{\phi}_{i+1} = \hat{\phi} \tag{11}$$

and therefore also

$$\phi_{i,i-1} = \phi_{i-1,i-2} = \phi_{i-1,i} = \phi_{i,i+1} = \phi_{i+1,i+2} = \phi_{i+1,i} = \phi \tag{12}$$

are valid. The equations (9) and (10) can therefore be reduced to

$$\hat{\phi} = -\frac{-3\phi^2 + 4\phi - 1}{4 - 4\phi} \tag{13}$$

or, by applying the demand for consistency $\hat{\phi} = \phi$, to

$$-7\phi^2 + 8\phi - 1 = 0. \tag{14}$$

The solution of this equation contains $\phi_1^* = 1$ and $\phi_2^* = 1/7$. As the second-order condition excludes ϕ_1^*, the consistent conjectural reaction coefficient is $\phi^* = 1/7$.

IV. Determination of Equilibrium Prices

If we do not, as Mulligan and Fik did in their papers, use different exogenous conjectural reaction coefficients in equation (5) of the model, but instead use the price reactions ϕ^*, which we calculated in the last section, we obtain the following price equation for firm i

$$\alpha p_i = p_{i-1} + p_{i+1} - t(X_{i-1} - X_{i+1}) + (\alpha - 2)k_i \qquad (15)$$

with

$$\alpha = 3.71429$$

For n firms in a one-dimensional circular market area we therefore get

$$CP = X \qquad (16)$$

with

$$C = \begin{pmatrix} \alpha & -1 & 0 & 0 & \dots & 0 & -1 \\ -1 & \alpha & -1 & 0 & \dots & 0 & 0 \\ 0 & -1 & \alpha & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \dots & -1 & \alpha \end{pmatrix}$$

$$P = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{pmatrix}$$

$$X = \begin{pmatrix} tX_2 - tX_n + (\alpha - 2)k_1 \\ tX_3 - tX_1 + (\alpha - 2)k_2 \\ tX_4 - tX_2 + (\alpha - 2)k_3 \\ \vdots \\ tX_1 - tX_{n-1} + (\alpha - 2)k_n \end{pmatrix}$$

Therefore the vector of the equilibrium prices is

$$P^* = C^{-1}X \qquad (18)$$

For example, in a market with 5 firms, the equilibrium price of firm 1 is:

$$p_1^* = 0.28554t(X_2 - X_3) + 0.06057t(X_3 - X_4) + 0.55006k_1 + 0.16440(k_2 + k_5)$$
$$+0.06057(k_3 + k_4) \quad (19)$$

The results for $n = 6$ and $n = 7$ are compiled in Table 1.

V. Summary

These considerations are based on the need for consistent models. If we look at the spatial oligopoly approach of Mulligan and Fik from this point of view, we draw two conclusions: First, their model does not indicate any reasons for asymmetrical conjectural or actual price reactions in the market, as they are not influenced by any different marginal production costs and different locational distributions. Second, exactly one conjectural reaction coefficient is consistent with the profit maximizing behavior of the firms, and this coefficient is identical for all firms in the market. Consequently, the alternative market results for different reaction coefficients in the paper of Mulligan and Fik are reduced to *one* result at *consistent* conjectural reaction coefficients.

References:

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Number of firms	5 firms	6 firms	7 firms
Location effects on price			
1st nearest	$0.26316t(X_2 - X_5)$	$0.26667t(X_2 - X_6)$	$0.26761t(X_2 - X_7)$
2nd nearest	$0.05263t(X_3 - X_4)$	$0.06667t(X_3 - X_5)$	$0.07042t(X_3 - X_6)$
3rd nearest	-	-	$0.01408t(X_4 - X_5)$
Cost effects on price			
Own	$0.57895k_1$	$0.57778k_1$	$0.57746k_1$
1st nearest	$0.15789(k_2 + k_5)$	$0.15556(k_2 + k_6)$	$0.15493(k_2 + k_7)$
2nd nearest	$0.05263(k_3 + k_4)$	$0.04444(k_3 + k_5)$	$0.04225(k_3 + k_6)$
3rd nearest	-	$0.02222k_4$	$0.01408(k_4 + k_5)$

Table 1: Components of firms 1's price in a circular market

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