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Optimal Pollution Control, Irreversibilities, and the Value of Future Information

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Abstract:

Optimal intertemporal pollution control with irreversibilities is investigated under conditions of certainty and uncertainty. When the assimilative capacity is positive, irreversible pollution levels are shown to be optimal only if the social time preference is sufficiently high. The irreversible destruction of environmental resource characteristics is assumed to occur when the environmental quality drops below some threshold value. If the impact of destroying a characteristic is uncertain and there is the prospect of better information, this future information carries a non–negative quasi–option value as in the Arrow–Fisher–Henry type of nature conservation model – adapted to pollution control.

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1. Introduction

Irreversible destruction of natural or cultural resources can take the form of species extinction by (over)harvesting – which may be the result of optimizing (Smith 1977, Sinn 1982) or not[1] – or of natural resource deletion by projects of industrial development with the Hells Canyon case as an early prototypical example in the literature (Krutilla and Cicchetti 1972). Most of the recent publications on irreversibilities (e.g. Clarke and Reed 1988; Smith 1986; Miller and Lad 1984) focused on the conflict between land development and wilderness conservation. This issue is clearly similar to the problem at hand, but its framework of investigation does not lend itself easily to the study of irreversibilities caused by **gradually increasing accumulation** of persistent pollutants in environmental resources.

To provide such a framework, the first step is to build a simple model capturing intertemporal environmental degradation by linking the emission of pollutants inversely to the quality of an environmental resource. The environment has a limited capacity of assimilating pollutants. As long as the flow of released pollutants exceeds that capacity, the environmental quality is reduced until eventually nature's assimilative services are completely exhausted.

It is very important to be specific about what exactly is irreversible or irreversibly destroyed. For one thing, there is **pollution irreversibility** in the sense that the pollution

[1]There is some literature on irreversibilities associated with (inefficient) overharvesting of (common) property resources, as e.g. the fishery. In the present paper we disregard these positive approaches. Here the focus is, instead, on problems of irreversibility in **optimal** resource control.

itself is irreversible. This is always the case, when the environmental resource does not provide assimilative services (any more). Moreover, even when the assimilative capacity is **not** (yet) exhausted, a characteristic of the environmental resource may be irreversibly lost (**characteristic irreversibility**) when environmental degradation exceeds a threshold level. For example, pollution causes irreversible modifications for the habitat of plants or animals or the destruction of "cultural characteristics" like the Akropolis in Athens.

The present paper aims at addressing both types of irreversibilities. With the objective of providing basic theoretical support for correct cost–benefit analyses various conditions for the (non)optimality of irreversible decisions are investigated under certainty and uncertainty. In the latter case special emphasis is placed on the likely situation that decisions about irreversibilities have to be made today under uncertainty about its costs when – at the same time (i.e. today) – the decision maker faces the prospect of better information at some future point in time.

In **section 2** the model is developed and used to investigate the policy implications of **pollution irreversibility under certainty**. It is shown, among other things, that ever increasing and hence finally irreversible pollution is never optimal in the case of positive assimilative capacity, unless the rate of social time preference is positive and sufficiently high. Whenever the assimilative capacity is positive though possibly small, multiple steady states may arise and some of these are unstable.

After having obtained a comprehensive understanding of the conditions under which irreversible pollution is optimal, **section 3** turns to the issue of **when it is optimal, under conditions of certainty, to irreversibly destroy a resource characteristic**. The loss of a resource characteristic is assumed to cause parametric changes and hence discontinuities in the production function, the welfare function and/or in nature's assimilative capacity. As a consequence, whether or not the preservation strategy is superior to the destructive strategy cannot be determined by conventional marginal analysis. It is an important result (to be used in the subsequent analysis of section 4) that if it is optimal at all to destroy the resource characteristic, then it is optimal to do so as soon as the optimal trajectory reaches the threshold value of environmental quality whose passing causes the destruction.

Section 4 focuses on **uncertainty about the (social) evaluation of the resource characteristic** (demand uncertainty). Taking expected welfare as the relevant objective, the introduction of uncertainty does not change the results from section 3 conceptually. A marked difference emerges, however, when we follow Weisbrod (1964) in making the additional assumption that there is the **prospect of better information on the social evaluation of the resource characteristics at some future point in time**. Now the question arises as to whether the resource characteristic should be preserved, temporarily at least, as long as the new information becomes available. Is sequential decision making of this kind a more appropriate planning procedure than a once–and–for–all decision? Is there a bias pro or contra preservation when these two decision making strategies are compared?

The literature on that issue deals with the concepts of "option value" and "quasi–option value" (Arrow and Fisher 1974; Henry 1974; see also footnote 4 below) in the context of (industrial) land development. The basic arguments of this literature are shown to be applicable for the problem under review in the present paper. The major result is that a **non–negative quasi–option value exists** implying that a decision maker is mistaken if he or she does not, in today's decision, account for the new information emerging in the future. The mistaken decision maker has an **anti–preservation bias**. The principal message is that "good" decisions about irreversibilities cannot be reached unless the best possible use of all available information is made which may imply sequential decision making when new information is in the offing. Technically speaking, the mistaken decision maker applies a certainty equivalence policy of the open–loop variety, whereas the optimal policy is of a closed loop or feedback nature (Clarke and Reed 1988).

In the final part of section 4 it is investigated how the quasi–option value depends on the length of the time period until the new information emerges. Confirming one's intuition, the value of new future information for the present decision making problem turns out to be the smaller the longer it takes to obtain the new information. Another extension of the model is to introduce uncertainty with respect to the future date of new information. It turns out that this type of uncertainty tends to favor the preservation strategy in the sense that an anti–preservation bias results from accounting for that uncertainty simply by taking the expected time of the new information as if it were known with certainty.

2. Pollution irreversibilities

Consider an economy in which two outputs are produced: a **consumption good** (good Y with quantity y) representing the national product and a **pollutant** which is generated as an undesired joint product along with good Y. Conventional productive factors are **labor** (ℓ) and **capital** (k), and the production of Y depends positively on the **environmental quality** which is measured by an index q. Labor can be used both for the production of good Y and for the reduction of pollutants through **intra–industrial waste abatement**. All these relationships are captured (Pethig 1979) by the production function Y: $D_y \to \mathbb{R}_+$,

$$y = Y(\underset{+}{\ell},\underset{+}{m},\underset{+}{k},\underset{+}{q}). \tag{1}$$

The convex set $D_y := \{(k,\ell,m,q) \in \mathbb{R}^3_+ \times \mathbb{R} \mid q \le q_m,\ m \le H(k,\ell,q)\}$ is the domain of function Y and $q_m \in \mathbb{R}_{++}$ denotes the quality of the unpolluted environment. As an externality, q creates a 'fundamental non–convexity' of the production set. But we proceed by assuming tacitly that on the relevant part of its domain, a subset of D_y, the function Y is concave and twice differentiable. In (1) the variable 'm' denotes the amount of pollutants which is generated as a by–product of good Y and then discharged into the environment. Observe that in function Y the emission is treated as if it were an input even though it is clearly an undesired output. But the input interpretation is both appropriate and convenient, because in addition to being an output the emission constitutes the industry's demand for a productive factor, namely for the **waste assimilation services** of the environment.

The **supply of labor**, ℓ_o, is assumed to be time invariant. The **capital stock changes**

over time according to

$$\dot{k} = y - c - \beta k \qquad \text{with } k_t = k_o \geq 0 \text{ for } t = 0, \beta \geq 0, \tag{2}$$

where c is (aggregate) consumption of good Y and where β is the rate of capital depreciation. **Environmental quality** is an indicator of the ambient concentration of pollution, but inversely proportional to the stock of pollution. Its **changes over time** are

$$\dot{q} = Q(z) \qquad \text{with } q_t = q_o \leq q_m \text{ for } t = 0, \tag{3}$$

where z is the industry's excess demand for nature's assimilative services. In the sequel, the function Q is specified by $Q_z = -1$ and $Q(0) = 0$, but the more general notation in (3) is maintained when it is desirable to keep track of the proper dimensions of terms. The excess demand for assimilative services is

$$z = m - M(q), \tag{4}$$

where the assimilation function M: $(-\infty,q_m] \rightarrow \mathbb{R}_+$ is assumed to have the following properties:[2] There is $q_u \leq q_m$ such that $M(q_m) = 0$ and $M(q) = 0$ for all $q \leq q_u$. Moreover, M is continuously differentiable on $(-\infty,q_m]$, it is positively valued on (q_u,q_m), and there is an inflection point $q_f \in (q_u,q_m)$ such that $M_{qq}(q) > 0$ for $q \in (q_u,q_f)$ and $M_{qq}(q) < 0$ for $q \in (q_f,q_m)$. The **decision maker's objective** is to maximise

$$\int_0^\infty e^{-\delta t} W(c,q)dt, \tag{5}$$

[2]Conceptually, these assumptions are supported by evidence from the natural sciences (Fiedler 1989) even though they have been stylized here for convenience of exposition. Alternative plausible assumptions are, e.g. that $M(q_m)$ is positive or that M is strictly concave on $[q_u,q_m]$ and hence not differentiable in point q_u. While the consideration of these cases would certainly increase complexity, it would not change the qualitative results of the subsequent analysis.

where δ is a positive and constant rate of (social) time preference and where the strictly concave function W represents the decision maker's evaluation of the bundle (c,q) for each point in time.

An **optimal pollution control program** consists of time paths for the control variables c, m, and y and the state variables q and y maximising (5) subject to (1) – (4). Under the simplifying separability conditions

$$Y_{mk} = Y_{mq} = Y_{kq} = 0 \quad \text{and} \quad W_{cq} = 0 \tag{6}$$

it is easy to derive from the Hamiltonian $H = W(c,q) + \lambda Q[m - M(q)] + \pi[Y(k,\ell_0,m,q) - c - \beta k]$

the following necessary optimality conditions:

$$\pi = W_c > 0, \tag{7a}$$

$$\lambda = - W_c Y_m Q_m^{-1} = W_c Y_m > 0, \tag{7b}$$

$$\hat{\pi} = \delta - (Y_k - \beta) = - \delta \left[\frac{MR_k}{\delta} - MC_k\right], \tag{7c}$$

$$\hat{\lambda} = \delta + M_q Q_z + \frac{W_c Y_q Q_z}{W_c Y_m} + \frac{W_q Q_z}{W_c Y_m} = \frac{\delta}{MB_m}\left[MB_m - \frac{MC_m}{\delta}\right]. \tag{7d}$$

In (7d) $MB_m := W_c Y_m$ is the marginal benefit from releasing pollutants (at time t) and $MC_m := - Q_z(W_c Y_q + W_q + W_c Y_m M_q) > 0$ is the **instantaneous** cost of emitting that same marginal unit of pollutants. More specifically, $|W_c Y_q Q_z|$ is the indirect marginal damage of pollution that emerges because the emission of pollutants diminishes the environmental quality. This quality reduction decreases the output of the consumption good whose marginal evaluation is positive. $|W_q Q_z|$ is the direct damage of increased pollution which arises because consumers suffer under the reduction in environmental quality, and $|W_c Y_m M_q Q_z|$ represents the cost of the change in nature's assimilative capacity induced by emitting the last unit of waste. This cost factor is positive [negative], if $M_q > 0$ [if $M_q < 0$].

Observe that in (7d) the sign of $\hat{\lambda}$ is equal to the sign of the difference between marginal benefit and cost of emission. If we consider emission at point in time t, the associated instantaneous cost is MC_{mt}. But since emission in t causes a **permanent** reduction in q, we have $MC_{mt'} = MC_{mt}$ for all t' > t. Hence the **overall marginal cost of emission in t is the present value of all future marginal costs:** MC_m/δ.

The second equation in (7c) has a similar interpretation. The marginal cost (in terms of the consumption good) of capital investment is $MC_k = 1$. Its instantaneous marginal revenue is $MR_k = Y_k - \beta$. Therefore, the overall marginal revenue from investment at point in time t is MR_k/δ, because investment in t generates the extra output $Y_{kt} - \beta = Y_{kt} - \beta$ for all t' > t.

Suppose, the optimal program converges to a steady state characterised by $k = q = \lambda = \pi = 0$. Owing to (7c) and (7d), such a state satisfies

$$\frac{MR_k}{\delta} = MC_k \quad \text{and} \quad MB_m = \frac{MC_m}{\delta}. \tag{7e}$$

As is well–known, the first condition requires to expand capital formation to the point, where (overall) marginal revenue of investment equals its marginal cost. According to the second condition the environmental quality must be reduced until the (overall) marginal cost of pollution balances its marginal benefit. The optimal capital stock, k_s, is only affected by the marginal productivity of capital which depends on k only in view of (6). Therefore, one has $c = y - \beta k_s$ for $\dot{k} = 0$ and it is possible to use the function

$$V(m,q) := W[Y(m,k_s,q)-\beta k_s,\ q] \tag{8}$$

for the evaluation of steady states. This function clearly satisfies $V_m = W_c Y_m > 0$, $V_q = W_c Y_q + W_q > 0$, $V_{mm} = Y_m^2 W_{cc} + W_c Y_{mm} < 0$, $V_{qq} = Y_q^2 W_{cc} + W_{qq} < 0$, and V_{mq}

$= W_{cc}Y_mY_q < 0$. For $\hat{\lambda} = 0$ equation (7d) yields $(\delta - M_q)W_cY_m + W_cY_q$ or

$$[\delta - M_q(q)] \cdot V_m(m,q) = V_q(m,q). \tag{9}$$

Define, in addition,

$$\varphi(q) := [\delta - M_q(q)] \cdot V_m[M(q),q] \quad \text{and} \tag{10}$$

$$\psi(q) := V_q[M(q),q], \tag{11}$$

whose derivatives are given by

$$\varphi_q = (\delta - M_q)(V_{mm}M_q + V_{mq}) - M_{qq}V_m \quad \text{and} \tag{10'}$$

$$\psi_q = V_{qq} + V_{qm}M_q. \tag{11'}$$

The graph of function φ can be determined with the help of (10), (10') and the properties of function M. If there is $\underline{q}_\delta$ and $\overline{q}_\delta$, $\underline{q}_\delta < \overline{q}_\delta$, such that

$$M_q(q) > \delta \text{ if and only if } q \in (\underline{q}_\delta, \overline{q}_\delta), \tag{12}$$

then $\varphi(\underline{q}_\delta) = \varphi(\overline{q}_\delta) = 0$ and $\varphi(q) < 0$ if $q \in (\underline{q}_\delta, \overline{q}_\delta)$. Moreover, as drawn in figure 1, $\varphi_q < 0$ for all $q \leq \underline{q}_\delta$. The curvature of φ is indeterminate on the intervals $[\underline{q}_\delta, q_f]$ and $[\overline{q}_\delta, q_m]$. But suppose that the production externality is not too strong in the sense that there is $q_g > q_h :=$ arg max M(q) satisfying sgn $(V_{mm}M_q + V_{mq}) =$ sgn $V_{mm}M_q$ for all $q \in [q_g, q_m]$. Then φ_q is positive on $[q_g, q_m]$. If M does not fulfill condition (12) then the graph of function φ is as indicated by the line $ABLP_4G$ in figure 1.

– **Figure** 1: Steady states in optimal pollution control –

The main results are summarised in

Proposition 1: *(i) If there is a unique optimal program implying ever increasing pollution (no steady state), then M does not satisfy (12). In other words, ever increasing pollution is never optimal in the case of positive assimilative capacity unless the rate of social time preference is positive and sufficiently high. If (12) holds, then there is always an optimal steady state in which pollution is reversible.*

(ii) Consider a separable welfare function $\bar{W}$ *satisfying* $\bar{\varphi}(q_w) = \bar{\psi}(q_w)$ *for some* $q_w < q_u$. *Then there is* $\bar{\sigma} > 1$ *such that there is no optimal steady state with irreversible pollution, if* $\psi = \sigma\bar{\psi}$ *with* $\sigma \geq \bar{\sigma}$. *But in this case there exists an optimal steady state* q_s *such that* $M_q(q_s) < \delta$.

(iii) There are at least two steady states, if $\psi = \sigma\bar{\psi}$ *with* $\sigma < \bar{\sigma}$ *and if (12) holds. One of these states is characterised by* $q_s \in (q_u, \underline{q}_\delta)$ *and the other one by* $q_s \in (\bar{q}_\delta, q_m]$. *A third steady state with irreversible pollution exists, unless* $\varphi(q) < \psi(q)$ *for all* $q \leq q_u$.

(iv) Suppose that $\psi = \sigma\bar{\psi}$ *with* $\sigma < \bar{\sigma}$ *and there is* $q_s < q_u$ *such that* $\varphi(q_s) = \psi(q_s)$. *Then* q_s *is the more likely to be the unique steady state, the flatter is the assimilation function. A necessary condition for uniqueness is that (12) doesn't hold.*

Proposition 1i makes it clear that pollution irreversibility crucially depends on "high" social time preference as long as the objective function is of the utilitarian variety with aggregating discounted flow benefits. In view of eq. (7e) the marginal cost of waste emission becomes unbounded when the rate of time preference tends to zero implying that the levels of ambient pollution tolerated in the optimal steady state are the lower the smaller is the social rate of time preference. Conversely, capital formation is encouraged by diminishing social time preference because that increases the overall revenue to investment (MR_k/δ).

For supplementary information about the optimal program it is convenient to develop a phase diagram. The locus of all tuples (m,q) satisfying $\hat{\lambda} = 0$ is implicitly given by (9). Its curvature depends heavily on the properties of M and W. Giving priority to concreteness over completeness and generality, suppose that (12) doesn't hold, that the production externality is weak (hence $|Y_{mq}|$ is small) and that M_{qq} is small on $[q_u,q_f)$[3]. Then condition (9) can be represented by a function F such that $m = F(q) \geq 0$ if and only if (m,q) satisfies (9), and it is true that $F_q(q) > 0$ whenever $F(q) > 0$. Moreover, the condition $V_{qm} > (\delta - M_q)V_{mm}$ also guarantees that $\hat{\lambda} \gtreqless 0$ if and only if $m \lesseqgtr F(q)$.

Differentiating (7a) and (7b) with respect to time yields

$$\hat{\pi} = \omega_c \hat{c} \quad \text{or} \quad \hat{c} = \frac{\delta - (Y_k - \beta)}{\omega_c}, \tag{13}$$

$$\hat{\lambda} = \hat{\pi} + \eta_m \hat{m}, \tag{14}$$

where $\omega_c := cW_{cc}/W_c < 0$ and $\eta_m := mY_{mm}/Y_m < 0$. Suppose the initial capital stock k_o is so small that $Y_k(k_o) < \delta + \beta$ holds which appears to be more plausible than the opposite inequality. Then $\hat{\pi} < 0$ or $MR_k/\delta > MC_k$ at $t = 0$, and it is optimal to continue capital investment until $\hat{\pi} = 0$ is reached in the steady state. The next step is to show that if it is optimal to approach a steady state monotonely with $\hat{\pi} < 0$ and $\hat{q} < 0$, then it must be also true that $\hat{\lambda} > 0$ until the steady state is reached (with a minor restriction on the upper bound of M_{qq}). To see this, differentiate (7d) with respect to time. This yields

$$\frac{d\hat{\lambda}}{dt} = -\frac{d}{dt}\left[M_q + \frac{W_q}{W_c Y_m} + \frac{Y_q}{Y_m}\right] = \left[\frac{W_q}{W_c Y_m} + \frac{Y_q}{Y_m}\right]\eta_m \hat{m} - b\hat{q} + \frac{W_q}{W_c Y_m}\omega_c \hat{c}, \tag{15}$$

[3]More precisely, the conditions to be required are $M_{qq} < -V_{qq}/V_q$ and $V_{qm} > \max[(M_{qq}V_q + V_{qq})/(\delta - M_q), (\delta - M_q)V_{mm}]$ with $\delta > M_q$.

with $b := \frac{q\, W_{qq}}{W_c Y_m} + \frac{qY_{qq}}{Y_m} + qM_{qq}$. From the equations (7c), (7d), and (13) one obtains $\eta_m \hat{m} = Y_k - \beta - \left[M_q + \frac{W_q}{W_c Y_m} + \frac{Y_q}{Y_m}\right]$ and $\omega \hat{c} = \delta + \beta - Y_k$. Substitute these terms in (15) and write after some rearrangement of terms

$$\frac{d\hat{\lambda}}{dt} = -\left[\frac{W_q}{W_c Y_m} + \frac{Y_q}{Y_m}\right]\hat{\lambda} - b\hat{q} - \frac{Y_q}{Y_m \omega_c}\hat{\pi}. \tag{15a}$$

Suppose now, $\hat{\pi} < 0$, $\hat{q} < 0$, and there is $\tau \geq 0$ such that $\hat{\lambda}_\tau < 0$. Then $\hat{\lambda}_t < 0$ for all $t > \tau$ (for $b < 0$) contradicting the presupposition that a steady state is reached. Hence $\hat{\lambda} > 0$ which determines, in turn, $\hat{m} > 0$ via (14).

– **Figure 2**: Steady states and optimal trajectories –

Figure 2 provides the phase diagram corresponding to figure 1, when the graphs of φ and ψ are given by the lines ABMG and HH', respectively. The preceding investigation clarified that the optimal trajectory is to approach P_3 [P_1] from a point like R [Q], if $q_o \in [q_3, q_m)$ [if $q_o \in (q_1, q_2)$]. The third steady state P_2 is unstable. For $q_o \in (q_2, q_3)$ it is optimal to approach P_3.

3. Irreversible loss of resource characteristics under certainty

Suppose now there is a characteristic of the environmental resource and a **threshold value of environmental quality,** say $q = q_r$, such that this characteristic disappears for good as soon as the environmental quality drops below q_r for the very first time. The loss of the resource characteristic is assumed to **cause parametric changes** (hence discontinuities) in at least one of the three building blocks of the model: in the production function, in the welfare function and/or in the assimilation function. To formalise these changes let $x \in X \subseteq \mathbb{R}^n$ and rewrite the functions Y, M, and W to include x as an argument. Then the analysis of the preceding section can be reinterpreted to hold for some specific vector x, say $x_t = x_o$ for all t. This modified setup allows to describe the impact of the loss of a resource characteristic by

$$x_t \begin{cases} = x_o & \text{if } q_\tau \geq q_r \text{ for all } \tau \leq t, \\ = x_1 & \text{otherwise.} \end{cases} \tag{16}$$

The decision maker faces now the **control problem A**: Maximise (5) subject to (1) – (4) and (16) with $Y(\cdot)$, $M(\cdot)$, and $W(\cdot)$ from (1), (4), and (5) replaced by $Y(k,\ell_o,q,x)$, $M(q,x)$, and $W(c,q,x)$, respectively. In what follows assume without loss of generality, that $q_o > q_r$ and that the solution to control problem A yields an optimal program with q strictly monotone decreasing from q_o towards a steady state $q_s < q_r$, **if it were true** that $x_1 = x_o$. In the general and more relevant case $x_1 \neq x_o$ it is then no longer a priori clear whether the solution to problem A is the preservation strategy or the strategy of destroying the resource characteristic (destructive strategy). The preservation strategy is defined as the solution of **control problem P**: Solve control problem A under the additional constraint

$$q_t \geq q_r \text{ for all t.} \tag{17}$$

The solution to this problem exhibits the following property: *There is $\tau > 0$ in the solution of control problem P such that $q_t > q_r$ for $t \in [0,\tau)$ and $q_t = q_r$ for all $t \geq \tau$. On $[0,\tau]$ the optimal program is independent of the value of x_1.* To proof this claim observe that according to Jacobson, Lele, and Speyer (1971) the constraint (17) can be taken care of by considering the Lagrangean

$$L = W(c,q,x) + \lambda[m - M(q,x)] + \pi[Y(\ell_0,m,k,q,x) - c - \beta k] + \rho(q - q_r).$$

The marginal condition (7a) remains unchanged but (7d) is substituted by

$$\hat{\lambda} = \delta - M_q - \frac{W_q}{W_c Y_m} - \frac{Y_q}{Y_m} - \frac{\rho}{\lambda}. \tag{7d'}$$

It is known, in addition, that $\rho \geq 0$ and $\rho(q - q_r) = 0$ and that the co–state variable λ may jump in that particular point in time, say τ, when the constraint (17) becomes binding. In fact, for $q_\tau = q_r$ we have $\dot{\lambda}(\tau-) > 0$. Therefore ρ jumps from zero to $\rho(\tau+) = \lambda(\tau+)[\delta - M_q - (W_q/W_c Y_m) - (Y_q/Y_m)]$, and $\lambda(t) = \lambda(\tau+)$ for all $t > \tau$ to keep $\dot{\lambda}(t) = 0$ for all $t > \tau$. It follows that (7d') is equal to (7d) for $\rho = 0$ so that the optimal program is independent of the value of x_1 during the time interval $[0,\tau]$.

The next step is to define the **destructive strategy** as the solution of **control problem D**: Solve control problem A under the additional constraint $q_t < q_r$ for some t. Clearly, during the time interval $[0,\tau]$ the solution to this problem must be the same as that of problem P. For convenience, the comparison between both strategies is therefore restricted to the period $[\tau,\infty)$ in what follows.

– **Figure 3**: Optimal pollution control under certainty when a destructible resource characteristic is present –

So far, it is an open question whether the destructive strategy implies, as suggested by figure 3, that $q_t < q_r = 0H$ for all $t > \tau$ (path BCD), or that the destruction of the characteristic is postponed to some point in time $\theta > \tau$, i.e. to the end of the time interval $\alpha := (\tau, \theta] \neq \emptyset$ (path BEF). To investigate this issue denote by B_0 the value of the objective function (5) attached to the solution of control problem D. Call d = 1 the decision not to destroy the characteristic at all, call d = 0 the decision to destroy the characteristic at point in time τ, and denote by B(d) the associated value of the objective function. B(1) is clearly the value of the preservation strategy whereas B(0) is the value attached to the solution of control problem D when the additional constraint [$q_t < q_r$ for all $t > \tau$] is imposed.

Proposition 2: *If it is optimal to destroy the resource characteristic, then it is optimal to destroy it at time $t = \tau$, i.e.*

$$max\,[B(1),B_0] = max\,[B(1),B(0)]. \tag{18}$$

To prove that proposition observe first that $B(0) \leq B_0$ by definition of these terms. Therefore (18) always holds, if max $[B(1),B_0] = B(1)$. We have to show, therefore, that $B(0) = B_0$ if max $[B(1),B_0] = B_0$. Suppose not. Then there is $\theta > \tau$ such that

$$B_0 = B^{\alpha}(1) + B^{\beta}(1,0) > B^{\alpha}(0) + B^{\beta}(0,0) =: B(0), \tag{19}$$

where the superscripts α and β refer to the time intervals α and β as indicated in figure 3. $B^{\alpha}(d_{\alpha})$ and $B^{\beta}(d_{\alpha},d_{\beta})$ in (19) denote values of the objective function during the time intervals α and β depending on the decisions d_{α} and d_{β} about the preservation of the resource characteristic during those time intervals. $d_{\gamma} = 0$ [$d_{\gamma} = 1$] indicates that $q_t < q_r$ [that $q_t \geq q_r$] for all $t \in \gamma$ and for $\gamma = \alpha, \beta$. Observe that $B^{\alpha}(1) = (1 - e^{-\delta\theta})B(1)$ and $B^{\beta}(1,0) = e^{-\delta\theta}B(0)$. Hence (19) can be rewritten as

$$(1 - e^{-\delta\theta})B_0 + e^{-\delta\theta}[B_0 - B(0)] = (1 - e^{-\delta\theta})B(1) > (1 - e^{-\delta\theta})\, B(0). \tag{19'}$$

Since $B_0 > B(0)$ by presupposition, (19') implies $B_0 < B(1)$. This contradiction of the premise max $[B(1),B_0] = B_0$ proves the proposition, so that the optimal decision regarding the preservation or the destruction of the resource characteristic is

$$d^* = \arg\max_{d} B(d). \tag{20}$$

4. Characteristics irreversibility, uncertainty, and its future disappearance

Uncertainty about the future benefits of preserving or losing an environmental characteristic can be easily incorporated into our formal model by interpreting the term x in $Y(\cdot,x)$, $M(\cdot,x)$, and $W(\cdot,x)$ as a **random variable with known distribution**. Consequently, the value of the optimal program **under uncertainty** with its implied decision $d = 0$ or $d = 1$ is now considered to be its **expected value** $E_x[B(d,x)]$. With these qualifications the analysis of section 3 carries over. In particular, the equivalent of equation (20) reads

$$d^* = \arg\max_{d} E_x[B(d,x)]. \tag{20'}$$

As the comparison of the equations (20) and (20') shows, the effect of uncertainty simply consists of replacing discounted welfare streams by their expected discounted values.

The preceding arguments implicitly presupposed that with the passage of time nothing further is learned about the value of x. An alternative, more plausible assumption appears to be that **the value of x becomes known at some point in time $\theta > \tau$**. For that case the structure of the problem is easily illustrated with the help of Figure 3. The path BCD represents the optimal program conditional on deletion of the characteristic at time τ. When point C is reached on that path at time θ, the new information is useless. The alternative option to preserve the characteristic at least until time θ means to choose the path BE of environmental quality during the time interval $[\tau,\theta]$. This strategy leaves the agency with the option of following either EG or EF, once the new information will have emerged. These considerations suggest, therefore, to partition the entire planning horizon $[\tau,\infty)$ into the time intervals $\alpha := [\tau,\theta]$ and $\beta := [\theta,\infty)$ and then to raise the **decisive question whether or not the characteristic should be preserved during the time interval α.** More formally, denote by

$$E_x[B(d_\alpha,d_\beta,x)] = B^\alpha(d_\alpha,x_0) + E_x[B^\beta(d_\alpha,d_\beta,x)]$$

the expected value of a pollution control program that involves the decisions d_α and d_β, where (as before) $d_\gamma = 0$ [$d_\gamma = 1$] indicates that $q_t < q_r$ [that $q_t \geq q_r$] for all $t \in \gamma$ and for $\gamma = \alpha, \beta$. Note that the decision sequence $d_\alpha = 0$ and $d_\beta = 1$ is infeasible, since irreversibility implies $d_\beta \leq d_\alpha$. Therefore, the decision d_β is constrained to the set $D(d_\alpha) := \{0,d_\alpha\}$ with $D(1) = \{0,1\}$ and $D(0) = \{0\}$. Using this notation we now compare the decisions about deleting the resource characteristic when the decision maker has different attitudes towards the future information (Fisher and Hanemann 1986).

Suppose first, the decision maker **disregards** (today) the new information emerging in θ. Then the optimal decision about d_α is the maximizer of

$$V^*(d_\alpha) := B^\alpha(d_\alpha,x_0) + \max_{d_\beta \in D(d_\alpha)} E_x[B^\beta(d_\alpha,d_\beta,x)]. \tag{21}$$

That is, the environmental characteristic is preserved during the time interval α, if $V^*(1) \geq V^*(0)$. On the other hand, if the decision maker **does not ignore** the prospect of new information, then d_α has to be chosen as to maximize

$$\hat{V}(d_\alpha) := B^\alpha(d_\alpha,x_0) + E_x\left[\max_{d_\beta \in D(d_\alpha)} B^\beta(d_\alpha,d_\beta,x)\right]. \tag{22}$$

Hence the preservation strategy is optimal, if $\hat{V}(1) \geq \hat{V}(0)$. Comparing (21) and (22) for d_α

$= 0$ reveals that $V^*(0) = \hat{V}(0)$. Therefore the information–neglecting decision procedure may result in a program different from that selected by the information–regarding procedure if and only if the **quasi–option value**, defined[4] as

$$QOV := \hat{V}(1) - V^*(1), \tag{23}$$

is equal to zero. In fact, the literature (Arrow and Fisher, 1974; Henry 1974; see also Freixas and Laffont 1984; Fisher and Hanemann 1986) established

Proposition 3: *The quasi–option value is non–negative ($QOV \geq 0$).*

A few comments on the relevance of Proposition 3 are in order. First, it should be emphasized that (22) is the **correct objective function** for decision making on d_α. Decision makers who ignore the prospect of new information are mistaken, and since $QOV \geq 0$, they tend to underestimate the value of preserving the characteristic during the time interval α. If their **wrong decision making procedure** leads to an incorrect decision – which need not inevitably be the case – it is always the decision to destroy the characteristic when it should have been preserved during the time interval α.

Note, however, that the resource characteristic will not necessarily be preserved for ever, if $d_\alpha = 1$ turns out to be the optimal decision. In the ex ante perspective, i.e. at the time of decision τ, the optimal decision $d_\alpha = 1$ may well imply the intention to destroy the characteristic at point in time $\theta > \tau$. This point is clarified by a simple numerical example

[4]Some authors like Henry (1974) and Fisher and Hanemann (1986) refer to QOV as the "option value", while others use the term "option value" for a phenomenon distinctly different from QOV. This semantic confusion has its origin in the historical development of these concepts. Fortunately, there is no more confusion in substantive issues, as is demonstrated by Bishop (1986) and Freeman (1986). The option value as defined in these two articles will not be discussed in the present paper, because that option value "focuses attention on the individual economic agent as s/he evaluates alternatives under uncertainty" (Bishop 1986, p. 147) while for quasi–option value (QOV), the focus ought to be, as it is in our paper, "on the public decision maker who is evaluating public policies or projects under uncertainty" (ibidem). This point is also forcefully made by Freeman (1986).

in Pethig (1989). Of course, when the new information has become known at time θ, the ex ante decision to destroy the characteristic in θ may again be changed in the light of that new information.

– **Figure 4**: A scenario of decision making –

A simple but illuminating scenario of decision making is depicted in figure 4 for the case that the random variable has two realisations. The assimilation curve is given by A_0NPQA_4, the initial environmental quality is $q_m = 0A_0$, and $q_r = 0A_2$ is the threshold value beyond which the resource characteristic is destroyed. Suppose for a moment that the variable x remained unchanged after destruction of the characteristic and that the associated locus for $\lambda = 0$ is YA_7. Then the optimal time path for q would be A_0DBC approaching the steady state value $q = 0A_7$, and m would decline from a positive value less than A_0Y to zero taking values in the area bounded by YA_7 and the assimilation curve. But since x becomes uncertain for $q < q_r$ rather than remains constant, at point in time τ a decision must be made with regard to preservation.

If the optimal decision is $d_\alpha = 1$, we have $q_t = q_r$ and $m_t = PA_2$ for all $t \in [\tau,\theta]$. Reaching the point in time θ (along the straight line DE in figure 4) there are two options: either to keep $q_t = q_r$ and $m_t = PA_2$ for all t (line EF) or to dip below q_r and then follow the path being optimal for the realisation of x. Let us assume that $x = x_1$ prevails shifting the $\lambda = 0$ curve to XA_8. Then m must be given a value such as A_2P'', and m would have to decline from that value to zero while q decreases to $0A_8$ along the line EL. On the other hand, if $x = x_2$ is the realisation of x and ZA_3 is the associated $\lambda = 0$ line, then m would have to be assigned a value such as A_2P' (after being slightly raised above A_2P to push q marginally below q_r) in order to approach the point N on the assimilation curve. Simultaneously, the environmental quality rises from $0A_2$ to $0A_1$ along the curve EG.

Suppose now, the optimal decision is $d_\alpha = 0$. Then there exists an optimal path for the time interval α such as DB in figure 4. At point in time θ (at point B) there is no option left. The decision maker has to calculate that program which is optimal for the realisation of x. It is either characterised by $q_t = 0A_5$ (and $m_t = 0$) for all $t \geq \theta$, if $x = x_1$, or by BM in case of $x = x_2$. If the assimilation curve is represented by A_0NPQA_6, it would be optimal to raise q from $0A_5$ to the steady state $0A_1$ following the line BH. This option became possible only because, by assumption, nature's assimilative capacity was not yet exhausted at $q = OA_5$, thus clarifying the close relationship between the issues of pollution irreversibility and the optimal decision about the preservation of a resource characteristic. kObserve, however, that owing to (16) the increase of environmental quality from OA_7 over $q_r = OA_2$ to $q = OA_1$ does not restore the resource characteristic.

The (positive) quasi–option value has been identified as a conditional **value of information**, conditional on retaining the decision leading to an irreversible state (Conrad 1980, Fisher and Hanemann 1986). It does not follow, however, that the quasi–option value should be considered a separate or additional component of benefit in applied benefit–cost analyses, as the earlier literature seemed to imply. The message of Proposition 3 is, instead, to avoid mistaken decision making. To the extent that conventional benefit–cost analysis neglected the (present) value of future information (Bishop 1986, p. 150) careful consideration of the information issue will have significant implications for applied research.

A straightforward extension of the previous analysis is to investigate the impact on the quasi–option value of **parametric changes of** θ. For that purpose recall the definition of QOV from (23), introduce the parameter θ in all pertaining functions, and define $ME(\theta) := \max\{E_x[B(1,1,x,\theta)], E_x[B(1,0,x,\theta)]\}$ and $EM(\theta) := E_x[\max\{B(1,1,x,\theta), B(1,0,x,\theta)\}]$ to obtain, after some rearrangement of terms,

$$QOV(\theta) = \hat{V}(1,\theta) - V^*(1,\theta) = e^{-\delta\theta}[EM(0) - ME(0)]. \tag{24}$$

Since EM(0) and ME(0) do not depend on θ, (24) yields immediately

Proposition 4: *Suppose, uncertainty about the social evaluation vanishes at the point of time θ with certainty and $QOV(\theta) > 0$. Then the quasi-option value shrinks monotonely to zero, ceteris paribus, when θ tends to infinity.*

This result conforms to the intuitive idea that the value of new information depreciates with the social time preference rate. With increasing values of θ the probability of mistaken decisions decreases so that the error from ignoring new information is the smaller the longer one has to wait for that new information.

It has been assumed, so far, that time θ is known with certainty. While it appears to be very realistic that the future will, in fact, reveal knowledge about resource characteristics and their proper evaluation, it is quite uncertain, in general, precisely when that new information will be available. Suppose, therefore, **θ itself is a random variable** with a known distribution. In that scenario a decision maker might wish to avoid complexity in the decision making process by simply substituting the previously certain time θ by its expected value $E(\theta)$. But then the following question arises: Does this procedure lead to erroneous decisions about preservation in the (now uncertain) time interval $\alpha(\theta)$, when a decision maker is considered who is willing to take the prospect of better information into account, when evaluating the preservation strategy. The answer is given by

Proposition 5: *Suppose, uncertainty about the social evaluation disappears at the point of time θ, but θ itself is a random variable with a known distribution. Then there is $\bar{\theta} \leq E_\theta(\theta)$ such that $E_\theta[\hat{V}(1,\theta)] = \hat{V}(1,\bar{\theta}) \geq \hat{V}[1,E_\theta(\theta)]$ and $E_\theta[QOV(\theta)] = QOV(\bar{\theta}) \geq QOV[E_\theta(\theta)]$.*

To substantiate this observation recall that the decision maker considers $\hat{V}(d_\alpha,\theta)$ from (22) for $d_\alpha = 1$ which can be transformed to read

$$\hat{V}(1,\theta) = P(x_0) + e^{-\delta\theta}[EM(0) - P(x_0)], \tag{25}$$

where $P(x_0)$ is the value of the preservation strategy over the period $[\tau,\infty)$. According to (25) either $EM(0) = P(x_0)$ or $EM(0) > P(x_0)$. Obviously, $EM(0) = P(x_0)$ yields $E_\theta[\hat{V}(1,\theta)] = \hat{V}[1,E_\theta(\theta)]$. Hence in this case the correct result is obtained by taking care of the uncertainty about θ simply through considering $E_\theta(\theta)$. For $EM(0) > P(x_0)$, however, $\hat{V}$ is strictly convex in θ so that $E_\theta[\hat{V}(1,\theta)] > \hat{V}[1,E_\theta(\theta)]$. The strategy of capturing uncertainty in $E(\theta)$ would now underestimate the net benefit of the preservation program. This underestimation effect caused by mistaken handling of uncertainty with respect to θ carries over to the quasi–option value, since (24) implies $E_\theta[QOV(\theta)] :=$
$=[EM(0) - ME(0)]\cdot E_\theta(e^{-\delta\theta}) \geq QOV[E_\theta(\theta)]$. Obviously, the strict inequality sign holds if and only if $EM(0) > ME(0)$ or $QOV(\theta) > 0$ which proves proposition 5.

This result suggests to interpret the effect of uncertainty about θ by reference to a "certainty equivalent": The certain time $\bar{\theta}$ that would lead to the same evaluation of the preservation strategy as the expected time $E_\theta(\theta)$ satisfies $\bar{\theta} \leq E_\theta(\theta)$. In that sense, uncertainty about θ tends to favor the preservation strategy.

5. Concluding remarks

The present paper aimed at clarifying the principles of optimal intertemporal pollution control involving decisions about irreversibilities of two types: irreversible pollution and the irreversible destruction of environmental resource characteristics. The latter problem turned out to cause the technical difficulty of discontinuities in the model's functional relations so that the conventional marginal analysis had to be supplemented by comparing total conditions. If the impact of irreversibly destroying a characteristic is uncertain, and there is, at the same time, the prospect of better information of that impact in the future, then one deals with the Arrow–Fisher–Henry type of nature conservation model – adapted to the issue of pollution control. Even though we still treated environmental management as an optimal control problem with infinite horizon, the decision about the preservation of resource characteristics boiled essentially down to a two–period matter.

Several extensions of the analysis are conceivable and desirable. For example, the threshold value of environmental quality beyond which the resource characteristic is deleted may be a random variable itself and new information about its true value may be available at some future point in time. A more fundamental suggestion for the future research agenda is an attempt to overcome both the all–or nothing nature of future information and its exogeneity. Since uncertainty is largely about the future benefits of preservation, the most appropriate procedure would seem to be the inclusion into the formal model of research activities that eventually result in removing uncertainty. Similar approaches have been discussed in the literature on backstop technologies e.g. by Dasgupta, Heal and Majumdar (1976) and by Kamien and Schwartz (1977). In the context of land development an interesting step in this direction has been taken by Clarke and Reed (1988).

It is certainly somewhat unsatisfactory to assume that the uncertainty about the future benefits of preservation vanishes altogether at some specific future point in time. One would like to see a model in which the variance of the probability distribution over the set of "possible valuations" decreases over time (and as a response to research efforts). In such a framework the optimal pollution control policy would have to be derived by methods of stochastic dynamic programming. Consequently, The length of the time period (α) for which the preservation decision has to be made would be an endogenous decision variable in its own right. The study by Clarke and Reed (1988) on land development gives some indications how the problem of pollution control could be extended in that direction.

Some people may have an ethical mental reservation against the proposition that creating irreversible environmental damage might be – or even might be called – an optimal decision. In the present paper the term optimal was used purely technically refering to the maximisation under constraints of a utilitarian objective function. The dispute over what the proper welfare function "should" be is old and controversial in environmental and resource economics. In my view this normative issue certainly applies also to the issue of irreversibities but not in a way differing from decision making that does not involve irreversible environmental damage.

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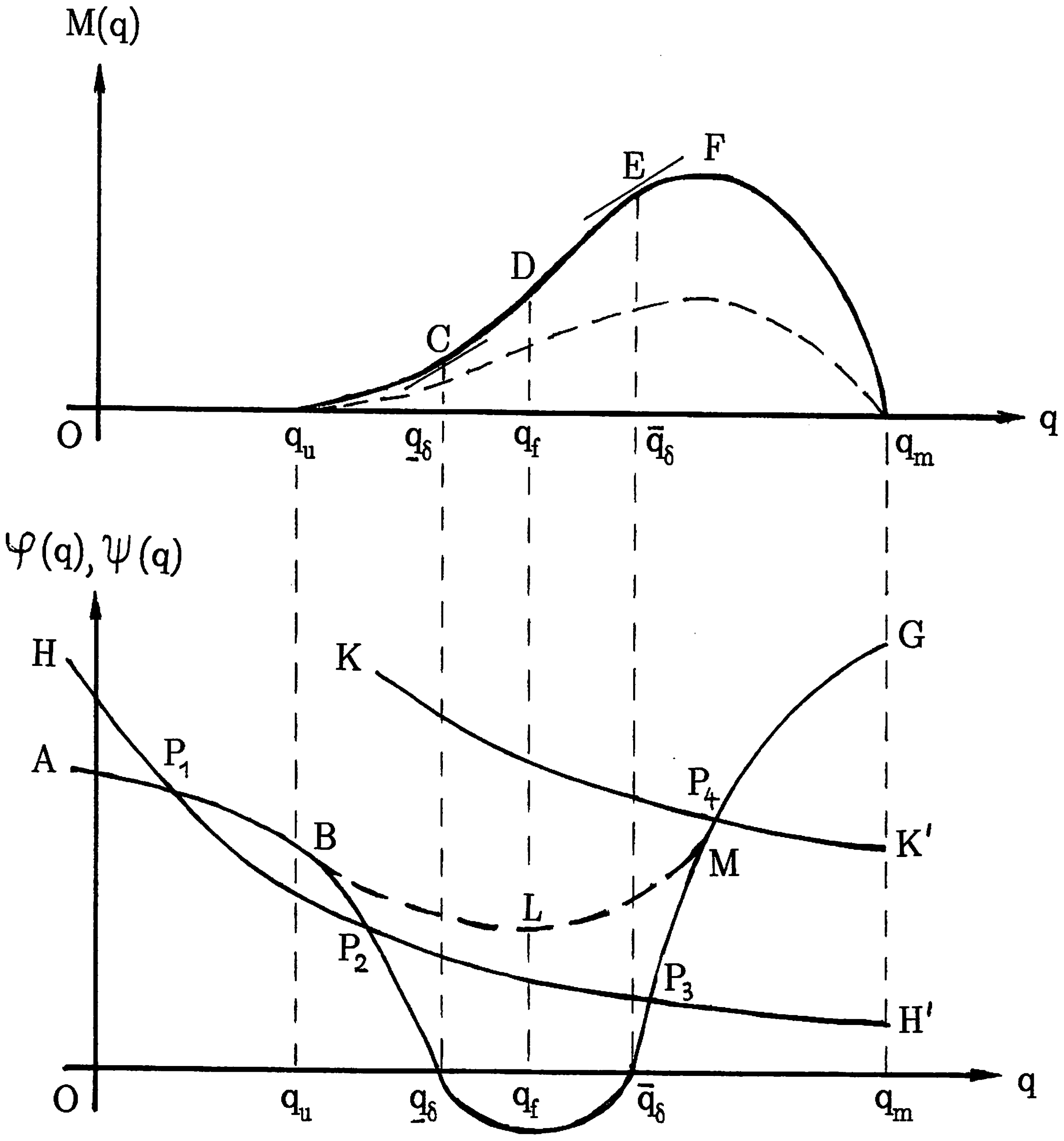

Figure 1: Steady states in optimal pollution control

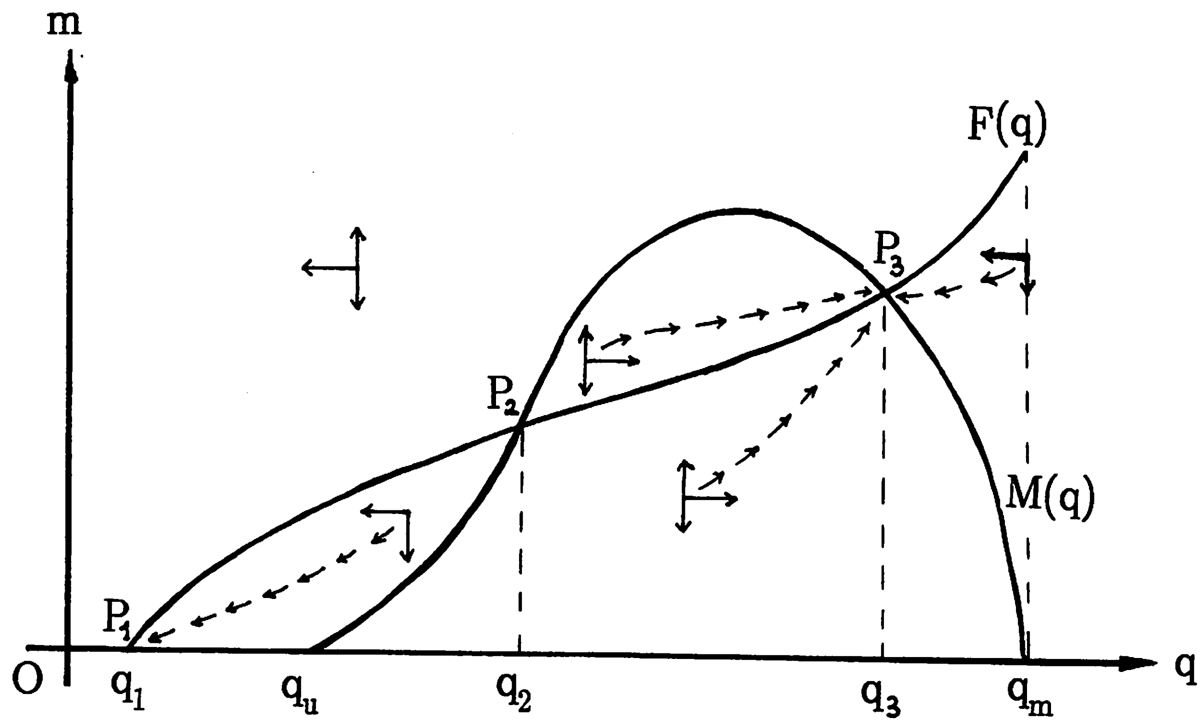

Figure 2: Steady states and optimal trajectories

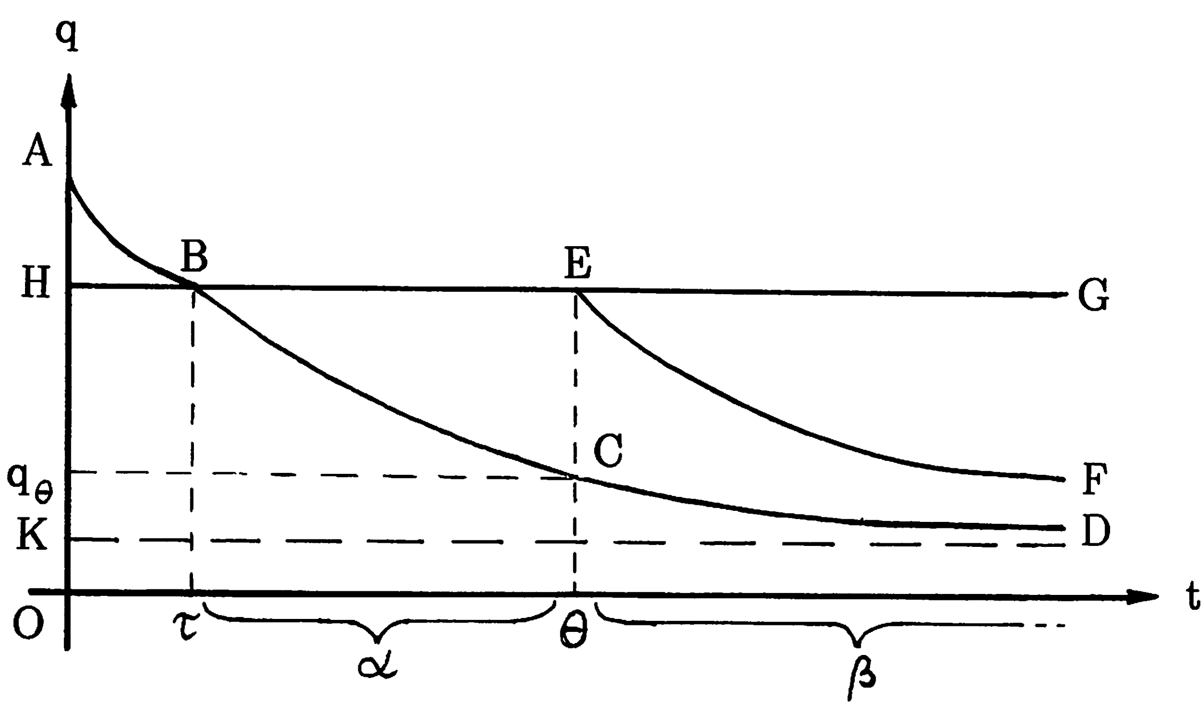

Figure 3: Optimal pollution control under certainty when a destructible resource characteristic is present

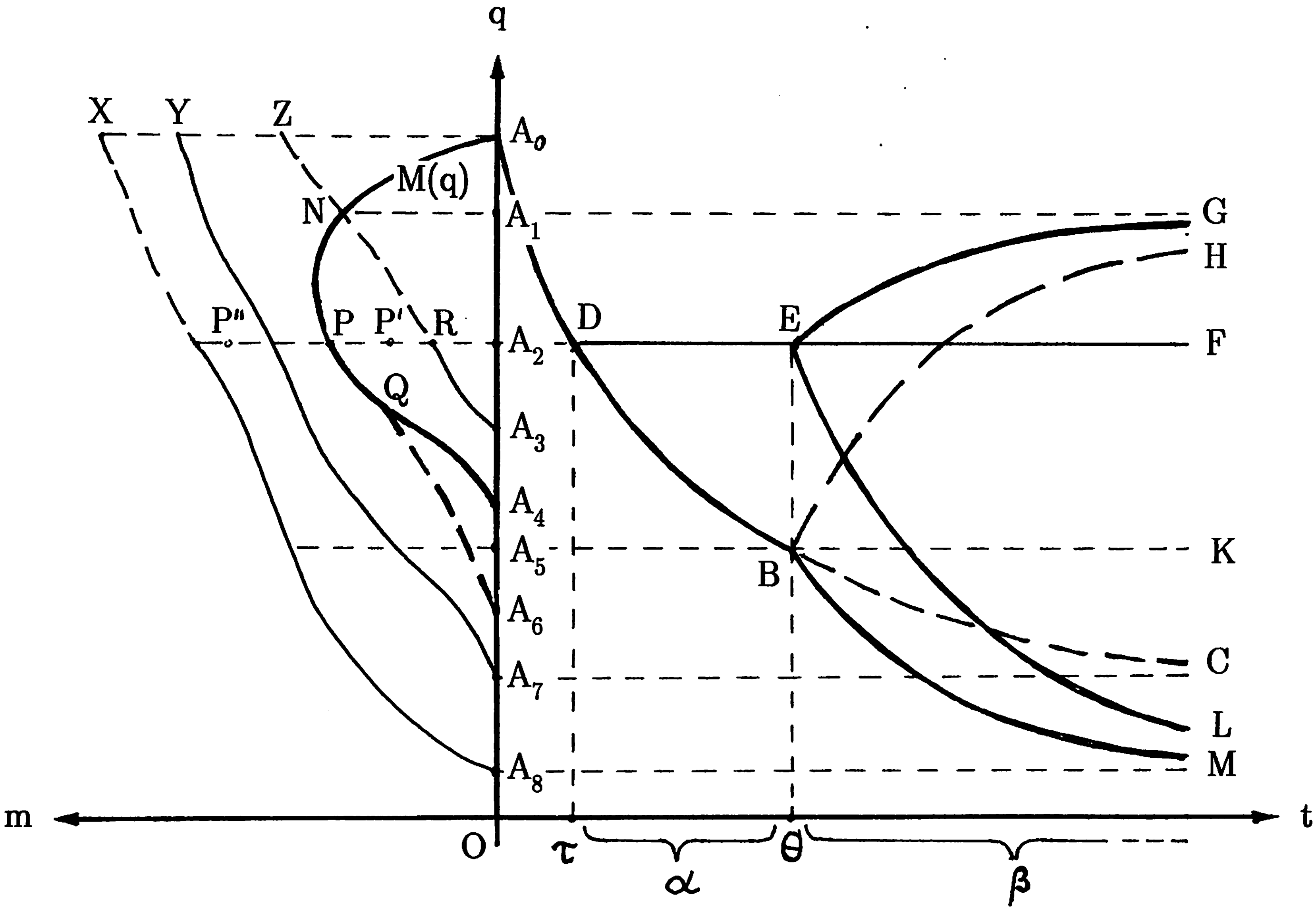

Figure 4: A scenario of decision making

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