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ECOLOGICAL DYNAMICS AND THE VALUATION OF ENVIRONMENTAL CHANGE

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1. Introduction

Environmental quality is reduced by the discharge of pollutants if the emission flow exceeds the resource's assimilative capacity. Excess demand or supply for assimilative services constitutes, therefore, a dynamic ecological disequilibrium with intertemporal changes of environmental quality. There are several contributions to the environmental economics literature that address environmental policy formation under explicit consideration of these ecological dynamics (e.g. Forrester (1971), d'Arge and Kogiku (1973), Mäler (1974), Siebert (1987), Pethig (1988), Barbier and Markandya (1990)). But the bulk of the literature seems to ignore the important intertemporal stock–flow aspect of pollution.[2] Even though environmental benefit–cost analysis has an established tradition to account for discounted flows of future benefits and cost, the analytical foundations of valuing disequilibrium time paths are not yet satisfactory, in our view. The present paper aims at shedding some light on the difficulties of valuation when ecological dynamics play an important role.

The first part of the paper focusses on the implications of optimal intertemporal pollution control under alternative hypotheses about the assimilative capacity of environmental resources. Then it is shown how the optimal intertemporal allocation could be achieved by an emission tax policy. As is well–known, the informational requirements for implementing such an optimal pollution control are unsurmountable. In particular, there is no hope to get all the information about marginal individual valuations necessary for the fine tuning of "optimal" tax rates along the time path to the optimal steady state. Therefore the paper proceeds with investigating a number of

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[1] I am grateful to the participants of the ESF task force on valuation, risk and uncertainty for their valuable comments and suggestions on an earlier version of this paper.

[2] Tietenberg (1992) distinguishes between stock pollution and fund pollution in an attempt to justify the use of static analysis at least for the subclass of fund pollution. In the context of this paper Tietenberg's 'fund pollution' should be seen as the special case where the stock of pollution is zero and assimilative capacity is in excess supply.

environmental policy strategies "for the better" which are less demanding with respect to information on individual preferences, but which nevertheless use some information on individual marginal valuations and at the same time account for the ecological dynamics – to some extend, at least. It is shown that ignoring the ecological dynamics may lead to ill–defined policy options and to significant biases in measuring willingness to pay either with indirect or direct valuation methods.

2. The model

Suppose some environmental resource is fixed in its quantitative dimension but its quality varies if pollutants accumulate in that resource. Consider an **environmental quality** indicator $q \in \mathbb{R}$ for this resource which attains higher values with decreasing pollution concentration. The indicator q is not bounded from below but it clearly has a "natural" upper bound $q_u > 0$ attainted in the absence of anthropogenous pollution. A **consumption good** (quantity: y) is produced along with a pollutant (quantity discharged: e) with the help of a productive factor, say **labor** (quantity: ℓ). The production process is impaired by decreasing environmental quality. Hence in formal terms the **production function** is $Y\colon D_y \to \mathbb{R}_+$ where D_y is the set of all triples $(e, \ell, q) \in \mathbb{R}^3$ satisfying $0 \leq e \leq \alpha \cdot \ell$, $\alpha > 0$, $\ell \geq 0$, and $q \in (-\infty, q_u]$. The function Y determines the output of good Y as

$$(1) \qquad y = Y(e, \ell, q).$$

For any given q function Y is assumed to be concave and increasing in e and ℓ. (For details see Siebert, Gronych, Eichberger, Pethig 1981). $\alpha \cdot \ell$ is the maximum amount of waste products that can be generated with labor input ℓ. Environmental quality is a positive production externality[3], satisfying $Y_q := (\partial Y/\partial q) > 0$ and $Y_{qq} := (\partial^2 Y/\partial q^2) \leq 0$ Even though in (1) the emission e is formally treated as an input it is clearly an output (by–product). But the input interpretation is also sensible, because the producer of good Y substitutes his own effort of disposing of the waste – which has the marginal opportunity cost Y_e – by using nature's waste disposal services: the environment is used as a waste receptacle.

For given q and ℓ the production function (1) represents a transformation function of the two outputs, the consumption good and the waste product. Suppose that (for given q

[3] It is well–known from the literature that the externality q makes the production function non–concave. But for the subsequent analysis it is convenient to proceed on the assumption that function Y is concave in q 'in its relevant domain'.

and ℓ) the emissions are successively reduced. Then the transformation function implies that labor is not forced into unemployment. Labor is rather withdrawn from production (in the narrow sense) and shifted towards abatement processes which are implicitly considered in (1).

Emissions are a flow variable measured in units of residual per period. Waste products released into the environment diffuse and accumulate. In addition they change their consistency by various chemical and biological processes, and/or are assimilated by nature's self–cleansing forces. That is, nature tends to reduce or absorb part of the waste which is discharged into the environment. We refer to nature's self–cleansing forces as to its **assimilative services.** The supply of these services, called **assimilative capacity** depends on the prevailing environmental quality. Denote the assimilative capacity by A(q). The demand for assimilative services is represented by the flow of emissions so that the positive or negative excess demand for assimilative services is given by

$$z := e - A(q).$$

Obviously, the quality of an environmental resource deteriorates (remains unchanged / improves) if the excess demand for assimilative services is positive (zero / negative). In formal terms one has[4]

$$(2) \qquad \frac{dq}{dt} := \dot{q} = \mathring{Q}(z) \quad \text{with } \mathring{Q}(0) = 0 \text{ and } \mathring{Q}_z < 0.$$

For the investigation of optimal environmental control in Section 3 it is convenient to assume $\mathring{Q}$ to be linear[5]. For analytical convenience we set $\mathring{Q}_z = -1$ turning (2) into

$$(2') \qquad \dot{q} = -z = A(q) - e.$$

There is considerable disagreement in the literature with respect to the concept of assimilative capacity. The following **hypotheses** have been put forward in the literature:

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[4] In this paper all variables refer to some point in time t. But the subscript t will be suppressed whenever this simplified notation does not cause ambiguities.

[5] Barbier and Markandya (1990) assume $\mathring{Q}$ to be strictly concave. But in their paper changes in environmental quality are considered essentially the same as changes in the quantity of environmental resources and the exess demand of assimilative services is only one of several determinants of these changes.

(H1) $A(q) = c \geq 0$ for all q (e.g. Siebert 1987)

(H2) Function A satisfies $A(q_u) = 0$, and it is concave and strictly decreasing on the entire domain of q (e.g. Forrester 1971, Bender 1976)

(H3) There is $q_\ell < q_u$ such that function A satisfies $A(q) = 0$ for all $q \leq q_\ell$; moreover, A is strictly concave on the interval (q_ℓ, q_u) and satisfies $\max A(q) > A(q_u) \geq 0$ (e.g. Pethig 1988)

(H4) As in (3c) there is $q_\ell < q_u$ such that function A satisfies $A(q) = 0$ for all $q \leq q_\ell$; moreover, A is concave and strictly increasing on the interval (q_ℓ, q_u) (e.g. Barbier and Markandya 1990).

In (H1) the assimilative capacity is assumed to be constant; in particular, it is independent of the prevailing environmental quality. In contrast, under hypothesis (H2) the assimilative capacity increases indefinitely with decreasing environmental quality. Finally, if (H3) applies, the regeneration rate attains a maximum at a moderate level of environmental quality; for very low quality levels ($q < q_\ell$) the environmental resource does not recover anymore: pollution has become irreversible. Basically, (H3) represents the standard natural growth function of animal or fish populations with thhe major difference that the concept of distinction is replaced by that of irreersibility. Observe that assumption (H4) is qualitatively the same as (H3) for low values of environmental quality and (H2) resembles (H3) for low qualities.

Figure 1 illustrates these four concepts of ecological change and shows that they differ markedly from each other with respect to their implications about ecological steady states ($\dot{q} = 0$).

– **Figure 1**: Assimilative capacity of environmental resources –

On the basis of ecological information available from natural sciences, especially from ecology, biology, and chemistry, all of these hypotheses seem to approximately describe nature's assimilative services for some class of empirically relevant pollutants (Fiedler 1992)[6]. Therefore we will refer to all of them in our subsequent analysis with the main emphasis being placed on (H3), partly because this hypothesis seems to be relevant for major pollutants and partly because this functional form contains (H2) and (H4) in some subset of its domain. Hence it is easy to extend the analysis of (H3) to these cases.

[6] Fiedler (1992) did not find natural science support for (H4). but it is possible, of course, to interprete (H4) as a limiting case of (H3).

To complete our basic model we need to specify the consumers' evaluation of environmental quality. Individual preferences are defined over the consumption good Y and the environmental quality Q. They are represented by a quasi–concave and strictly increasing utility function $U^i : \mathbb{R}^2 \to \mathbb{R}$. The utility is intertemporally invariant so that $e^{-\delta t}U^i(y_{it}, q_{it})$ denotes consumer i's 'present value' utility derived from the bundle (y_{it}, q_{it}) at any point in time t. $\delta \geq 0$ is the individual time preference rate assumed to be the same across consumers. During the time interval [0, T] consumer i's present value of utility is

$$(3) \qquad u_i = \int_0^T e^{-\delta t} \cdot U^i(y_{it}, q_{it})\, dt.$$

Including environmental quality into the individual utility function does not imply that the respective individual is in control of that variable. In fact, very often consumers have to "passively" accommodate to the prevailing level of environmental quality. Consumers jointly consume q, they cannot be excluded from consumption of q, and they cannot exclude themselves from its consumption which they might wish to do if q is very low. In other words, $q_{it} = q_t$ for all i and all t. The same arguments hold for environmental quality as a production externality introduced in equation (1) above.

3. Intertemporal optimisation

Since the environment–economy relationship modelled in (2) is dynamic, an appropriate analysis of the benefits and costs of pollution control must take the time dimension explicitly into account. For expository purposes suppose the number of consumers, s, and the aggregate labor supply, ℓ_o, are invariant in time[7]. An optimal intertemporal allocation of resources consists of time paths for the control variables y_t and e_t and the state variable q_t which solve the problem

$$(4) \qquad \text{Maximise} \int_0^T \Sigma_i\, e^{-\delta t} \cdot U^i(y_{it}, q_{it})\, dt \quad \text{subject to (1) – (3)}$$

[7] Ideally, the long–term analysis should also include growth of productive factors, in particular capital accumulation and technical change. But manageability of the analysis suggests to follow this more modest approach.

when the initial value of environmental quality is given by $q_{t=0} = q_a \in (-\infty, q_u]$. For $T = \infty$ the Lagrangean

$$L(\cdot) \;= \Sigma_j U^j(q_j, y_j) + \mu\cdot \hat{Q}[e\text{-}A(q)] + \lambda\cdot[Y(e, \ell_o, q_y) - y] + \lambda_y\cdot(y - \Sigma_j y_j) + \\ + \Sigma_j \lambda_{qj}\cdot(q - q_j) + \lambda_q\cdot(q - q_y)$$

yields the following necessary optimality conditions:

$$(5) \qquad U^i_y = \lambda_y; \;\; U^i_q = \lambda_{qi}; \;\; -\mu\hat{Q}_z = \lambda Y_e; \;\; \lambda_q = \lambda Y_q; \;\; \lambda = \lambda_y;$$

$$(6) \qquad \hat{\mu} := \frac{\dot{\mu}}{\mu} = \delta + A_q\hat{Q}_z + \frac{\Sigma_j U^j_q \hat{Q}_z}{U^i_y Y_e} + \frac{U^i_y Y_q \hat{Q}_z}{U^i_y Y_e} =$$

$$= \frac{\delta}{MB(e)}\cdot\left[MB(e) - \frac{MC(e)}{\delta}\right] = \delta - \frac{MC(e)}{MB(e)},$$

where $MB(e) := \mu = -U^i_y\, Y_e/\hat{Q}_z = U^i_y Y_e > 0$ represents the **marginal benefits of emissions** and where $MC(e) := -\hat{Q}_z(\Sigma_j U^j_q + U^i_y Y_q + U^i_y Y_e A_q) > 0$ is the **instantaneous marginal cost of emissions**. This marginal cost consists of the following three components: The term $|\Sigma_j U^j_q \hat{Q}_z|$ is the marginal damage of emissions which arises because environmental quality is a consumption externality. $|U^i_y Y_q \hat{Q}_z|$ is the marginal damage of pollution that emerges because environmental quality is a production externality. As a consequence of declining quality the output of the consumption good decreases, and this causes a loss in utility, measured by U^i_y. Finally, $|U^i_y Y_e A_q \hat{Q}_z|$ represents the marginal cost (or benefit) of emissions attributable to the change in nature's assimilative capacity induced by the emissions. The sign of this cost factor is the same as the sign of the derivative A_q.

Note that $MC(e_t)$ is the marginal cost of e_t at point in time t. Since e_t causes a permanent reduction of environmental quality, the release of e_t causes the marginal damage $MC(e_{t'}) = MC(e_t)$ for all $t' > t$. Hence for $T = \infty$ the overall marginal cost of emissions in t is the present value of all future marginal costs: $MC(e)/\delta$.

The next step is to investigate the properties of an optimal steady state ($\dot{q} = 0$ and $\dot{\mu} = 0$). If such a state exists as part of the optimal time path, it satisfies

(7) (a) $MB(e) = \dfrac{MC(e)}{\delta}$ or, equivalently, (b) $\sum_j \left[\dfrac{U_q^j}{U_y^j} \right] - \delta Y_e = -(Y_e A_q + Y_q)$,

i.e. in the long run it is optimal to adopt that particular environmental quality for which the (overall) marginal cost of emissions balances its marginal benefit (7a). The RHS of equation (7b) represents the long–term ecologically stable marginal rate of transformation as implied by the transformation function $y = Y[A(q), \ell_o, q]$ while the LHS is the sum of all individuals' marginal willingness to pay for environmental quality reduced by the fraction δ of the marginal abatement cost Y_e. Clearly, if the term δY_e was absent, equation (7b) would be exactly Samuelson's condition for the optimal allocation of a public good in a static framework of analysis.

Consider the following three conditions:

(a) There is an upper bound $\gamma > 0$ for the marginal opportunity cost of abatement Y_e in all feasible allocations;

(b) There is a lower bound $\mu > 0$ for the aggregate willingness to pay $\Sigma_j(U_q^j/U_y^j)$ in all feasible allocations;

(c) $\mu > \delta\gamma$.

If these conditions hold and an optimal steady state exists, the following observations are straightforward:

(i) Under hypothesis (H1) or (H4) there is a unique optimal steady state at the maximum environmental quality q_u. This observation shows that under the hypothesis (H1) and (H4) the long–term optimum is ery likely a completely unpolluted enironment (in particular since $\mu > \delta\gamma$ is not necessary but only sufficient for this to happen). It is therefore ery likely that with (H1) or (H3) there is no long–run goal conflict between material well–being and envvironmenatal quality. Pollution control does not require hhard social choices between competing ends. Conflicts only arise in case of policy mistakes, in particular, if myopic polititians cede to short–run special interests.

(ii) Under (H3) the optimal steady state environmental quality satisfies $A_q(q) < 0$. In this case – as in the case of (H2), of course – a long–run goal conflict between material well–being and environmental quality is ineitable, and the optimal compromise is a serious issue of public choice.

(iii) The inequality $\mu > \delta\gamma$ (condition c) trivially holds for $\delta = 0$. But in that case the objective function in (4) is unbounded so that the optimisation problem (4) is ill defined. It is shown in the literature (e.g. Feichtinger (...., p....)), however, that this complication can be handled by slightly modifying the optimality criterion. In spite of this cange the marginal conditions (5) and (6) turn out to be also necessary for satisfying the modified optimality criterion. Hence (7b) still characterises the optimal steady state if it exists.

According to (7b) the steady–state marginal willingness to pay must be larger than the marginal rate of transformation if $\delta > 0$. For hypothesis (H3) this is illustrated in Figure 2 where condition (7b) with $\delta = 0$ is satisfied at point Q ($\tan \alpha = \Sigma_j(U_q^j/U_y^j) = -(Y_eA_q + Y_q)$. Hence the optimal steady state must be a point on the curve $Y[A(q), \ell_o, q]$ to the left of point Q, such as point P, because in P we have

$$\tan \gamma = \sum_j \left[\frac{U_q^j}{U_y^j} \right] > -Y_eA_q - Y_q = \tan \beta.$$

Observe that with increasing discount rates the steady state quality becomes lower until the optimal steady state eventually moves to the upward sloping part of the transformation function.

– **Figure 2**: Impact of time preference on the optimal steady state –

If $Y_q = 0$ holds it can be considered realistic that $e_m := \max A(q) < e_o := \alpha \cdot \ell_o$. In this case $y_o = Y(e_o, \ell_o)$ is the output in the absence of any waste abatement. But by assumption, the maximum sustainable output is $Y(\ell_o, e_m) < y_o$. Since in all steady states (e, q) the condition $e = A(q) < e_m = A(q_o)$ holds, the pertinent allocations clearly imply waste abatement.

To characterise the optimal steady state in more detail we simplify the model as follows;

- utility functions are separable: $U_{qy}^i = 0$ all i;
- consumers are identical and treated equally, i.e.

(8) $\quad \Sigma_j U^j(y_j, q) = s \cdot U(\frac{y}{s}, q) = s \cdot U[Y(e, \ell_o, q)/s, q] =: V(e, q).$

With $U_{qy} = 0$ function V from (8) satisfies $V_e = U_yY_e > 0$; $V_q = sU_q + U_yY_q > 0$; $V_{ee} = U_{yy}Y_e^2/s + U_yY_{ee} < 0$, $V_{qq} = sU_{qq} + U_{yy}Y_q + U_yY_{qq} < 0$, $V_{eq} = 0$. With the help of (8) equation (7) can be rewritten as

(7c) $\quad [\delta - A_q(q)] \cdot V_e(e, q) = V_q(e, q).$

Define, in addition, the functions φ and ψ by

(9) $\varphi(q) := [\delta - A_q(q)] \cdot V_e[A(q), q]$,

(10) $\psi(q) := V_q[A(q), q]$,

and consider their derivatives

(9a) $\varphi_q = (\delta - A_q)V_{ee}A_q - A_{qq}V_e$,

(10a) $\psi_q = V_{qq} < 0$.

In what follows we wish to characterise the graph of function φ with the help of (9), (9a) and the properties of function A. Suppose first function A satisfies condition (11):

(11) There is $q_\delta > q_\ell$ satisfying $A_q(q_\delta) = \delta$.

Clearly, this condition cannot be satisfied unless A is given by (H3) or (H4). If it holds along with hypothesis (H3), $\varphi(q_\delta) = 0$ and $\varphi(q) < 0$, $\varphi_q(q) < 0$, for $q \in (q_\ell, q_\delta)$. φ is discontinuous at q_ℓ with $\varphi(q) > 0$ and constant for $q \in (-\infty, q_\ell)$. Moreover, we also find $\varphi(q) > 0$ on the interval $(q_\delta, q_u]$ with $\varphi_q(q) > 0$ on $[q_m, q_u]$, $q_m :=$ arg max $A(q)$. The sign of φ_q is indeterminate on (q_δ, q_m).

– **Figure 3**: Optimal steady states –

The line ABCD in Figure 3 is the graph of function φ from (9), if condition (11) is satisfied. Suppose, function ψ is given by GH. Then there are two steady states: P_1 and P_2.[8] Moreover, there are conditions under which the graph of φ is below that of ψ for all $q \geq q_\ell$. In that case no steady state with irreversible pollution exists. As an illustration, suppose that the marginal valuation of environmental quality, V_q, increases successively to the effect that the line GH shifts upward. Then the number of steady states eventually reduces to one, e.g. P_3, the only intersection of ABCD and KL. If V_q is sufficiently high, the optimum long–run environmental quality may be even equal to the maximum one.

[8] It cannot be excluded that under hypothesis (H3) an even number of additional steady states exists on (q_δ, q_m) since φ need not be monotone increasing on that interval. In case of hypothesis (H4) the number of additional steady states on (q_δ, q_u) may be even or uneven. See also Barbier and Markandya (1990).

Suppose now that (11) is not satisfied. Then φ does not attain negative values; in this case, its graph is represented by a line such as A'B'C'D'. Again, depending on the position of the V_q–curve, there may exist a steady state with either reversible or irreversible pollution or both. It is also interesting to observe that the shift from ABCD to A'B'C'D' could have been induced, ceteris paribus, by a parametric increase in δ.

If the graphs of φ and ψ are given by A'B'C'D' and GH , respectively, then P_4 will be the only optimal steady state. In other words, there is a rate of time preference, δ, sufficiently high and a marginal evaluation of environmental quality, V_q, sufficiently low so that no optimal steady state exists in which the assimilative capacity of the environmental resource is positive. In this case high accumulation of pollution and the depletion of the resource's assimilative capacity is optimal.

The following observations summarise our preceding discussion (and are easily verified in Figure 3):

(a) There is a positive threshold value $\bar{\delta}$ such that, ceteris paribus, at least one optimal steady state with reversible pollution exists whenever $\delta < \bar{\delta}$. (e.g. P_2 for ABCD and GH).

(b) There is a marginal valuation of environmental quality sufficiently high such that there is a **unique** optimal steady state for which the assimilation function E is downward sloping (e.g. P_3 for ABCD and KL).

(c) If δ tends to zero, then the optimal control tends to imply a unique steady state in which the assimilation function E is downward sloping, because in this case $\varphi(q) \leq 0$ for all $q \in (-\infty, q_m)$.

(d) If (e, q) is the unique optimal steady state with $q < q_u$ and $e > 0$, then an increase in the rate of social time preference δ reduces the optimal steady–state environmental quality.

To obtain supplementary information about the optimal time path it is convenient to develop a phase diagram. The locus of all tuples (e, q) satisfying $\hat{\lambda} = 0$ is implicitly given by (7c). Suppose, for convenience, that conditions (8) and (11) hold. Then equation (7) can be represented by a function $F : (-\infty, q_u] \to \mathbb{R}_+$ where, by definition, $e = F(q)$, if and only if (e, q) satisfies (7c). Total differentiation of (7c) yields

(12) $$F_q = \frac{V_{qq} + A_{qq}V_e}{(\delta - A_q)V_{ee}}.$$

– **Figure 4**: Steady states and optimal trajectories –

In Figure 4 the graph of function F is ABCDGH if condition (11) is satisfied. Optimal steady states are given by the points A and K.

Recall from (5) and (6) that $\hat{\mu} = \delta - A_q(q) - \frac{s \cdot U_q[Y(\ell_o, e)/s\,,\ q]}{U_y[Y(e\,,\ \ell_o)/s,\ q] \cdot Y_e(e,\ \ell_o)}$. Hence $\frac{\partial\hat{\mu}}{\partial e} = -s \cdot U_y(U_{yy}Y_e/s + U_yY_{ee})/\mu^2 > 0$. In other words, any move upward [downward] from the $\dot{\mu} = 0$ locus makes $\hat{\mu}$ positive [netgative]. Moreover, in view of (5) one calculates $\dot{\mu} = (U_{yy}Y_e^2 + U_yY_{ee}) \cdot \dot{e}$ implying $\dot{\mu} \gtreqless 0 \Leftrightarrow \dot{e} \lesseqgtr 0$.

Suppose we choose a point to the right of the graph of F. Then $\dot{\mu} > 0$ and therefore $\dot{e} < 0$. Conversely, to the left of the graph of F we have $\dot{\mu} < 0$, and therefore $\dot{e} > 0$. When combined with (2) this explains the arrows in Figure 4. For the situation depicted in Figure 4 we now summarise the following qualitive information about the optimal time path:

(a) If the initial environmental quality, q_a, is in the interval $(q_K,\ q_u]$ then the steady state K is approached with emission levels greater than q_KK. Along the path towards K emissions are strictly decreasing while the shadow price of environmental quality change (μ) rises.

(b) If $q_a \in (q_\ell\,,\ q_K)$, then the steady state K is also approached with emission levels $e \in (F(q),\ A(q))$ for all $q \in [q_a,\ q_K)$. In this case the path towards K is characterised by strictly decreasing shadow price μ and (hence) strictly increasing emissions.

(c) If $q_a \in (q_2,\ q_\ell)$, then the steady state A is approached with decreasing emission levels $e \in (F(q),\ 0)$ for all $q \in [q_a,\ q_2)$.

(d) For any initial environmental quality $q_a < q_2$ it is optimal to choose a path of ever increasing pollution.

It is worthwhile emphasising that irreversible accumulation of pollution – and hence the exhaustion of the resource's assimilative capacity – may be an optimal solution to problem (4). Such an outcome is the more likely the higher the (social) rate of discount and the smaller the marginal valuation of environmental quality. Depletion of assimilative capacity appears to contradict the notion of sustainable development which requires according to Pearce and Turner (1990, p 44) to always keep waste flows to the environment at or below the positive assimilative capacity. Pearce and Turner (1990, p. 225) suggest to supplement the normal cost–benefit approach by the constraint that the assimilative capacity should be kept constant. This proposal introduces a new source of valuation which is completely ad hoc and unspecified, unfortunately. The idea of introducing an additional constraint of 'ecological stock maintanance' seems to be rooted in a deep mistrust of the neoclassical optimality concept. Pearce and Turner probably fail to see that if an environmental resource along with its assimilative capacity **is** considered valuable the solution of (4) **will not** imply its depletion.

4. Implementation and evaluation of optimal pollution control

Suppose the optimal time path $\{(e_t^o, q_t^o, y_t^o)\}$ is to be implemented with the help of emission taxes in an economy with perfectly competitive markets for labor and the consumption good. To show how this is done conceptually, we denote by p_{et}, $p_{\ell t}$, and p_{yt} the prices for emissions, for labor and for good Y, respectively, and write $p_t := (p_{et}, p_{\ell t}, p_{yt})$. An **intertemporal competitive equilibrium with emission taxes** is constituted by a price path $\{p_t\}$ and an allocation path $\{e_t^c, \ell_t^c, q_t^c, y_t^c, (y_{it}^c)\}$, such that for all t:

(a) $\{(e_t^c, \ell_t^c, y_t^c)\}$ maximises $\int_o^T e^{-\delta t} \cdot (p_{yt} y_t - p_{et} e_t - p_{\ell t} \ell_t)\, dt$

subject to $y_t = Y(e_t, \ell_t, q_t)$;

(b) for all i: y_{it}^c maximises $U^i(q_t, y_{it})$ subject to $\ell_{oi} + \theta_i[G(p_t) + p_{et} e_t^c] \leq p_{yt} y_{it}$,

where $\theta_i \in [0, 1]$, $\Sigma_i \theta_i = 1$ and where $G(p_t)$ is the maximum of $p_{yt} Y(e_t, \ell_t, q_t) - p_{et} e_t - p_{\ell t} \ell_t$ with respect to e_t and ℓ_t;

(c1) $y_t^c = \Sigma_i y_{it}^c$, (c2) $\ell_o = \Sigma_i \ell_{oi} = \ell_t^c$, (c3) $\dot{q}_t^c = A(q_t^c) - e_t^c$.

Suppose for convenience of exposition condition (c2) holds. Then condition (a) implies for all t :

(13) $p_{et} = p_{yt} Y_e(e_t^c, \ell_o, q_t^c)$.

Define the **real emission tax** in terms of the consumption good Y by $\pi_t := p_{et}/p_{yt}$ and restate equation (13) as

(14) $e_t^c = E(\pi_t, q_t^c; \ell_o)$.

Observe that (14) combined with the equilibrium condition (c3) completely determines the ecological dynamics for every path of real emission taxes $\{\pi_t\}$. It is easy to show that an intertemporal equilibrium can be associated with **any** path $\{\pi_t\}$. To see this, observe that equilibrium condition (b) trivially yields $y_{it} = [\ell_{oi} + \theta_i \cdot (G(p_t) + p_{et}e_t^c)]/p_{yt}$ so that (c1) can be turned into

$$\sum_i \frac{\ell_{oi} + \theta_i \cdot [G(p_t) + p_{et}e_t^c]}{p_{yt}} = \frac{\ell_o + G(p_t) + p_{et}e_t^c}{p_{yt}} = Y(e_t, \ell_o, q_t).$$

The last equation holds by definition of $G(p_t)$. In other words, associated with any time path $\{\pi_t\}$ is a time path of instantanous equilibrium allocations

$$\{e_t(\pi_t), q_t(\pi_t), y_t(\pi_t)=Y[e_t(\pi_t), \ell_o, q_t(\pi_t)]\}.$$

It is therefore feasible to construct a sequence $\{\pi_t^o\}$ such that π_t^o satisfies the equaltion $e_t^o = E(\pi_t^o, q_t^o; \ell_o)$ for all t with e_t^o and q_t^o taken from the optimal path $\{(e_t^o, q_t^o, y_t^o)\}$ as determined on Section 3. Hence we showed that the optimal intertemporal allocation can be implemented by a tax policy $\{\pi_t^o\}$ with continuous tax rate adjustments as specified in our discussion of the phase diagram (Figure 4).

To assess the **value** of optimal dynamic pollution control one needs to specify the value measure. If the criterion for value is **social welfare** as defined by the integral in (4) the value of an intertemporal competitive equilibrium with emission taxes $\{\pi_t\}$ is clearly

(15) $V(\{\pi_t\}) := \int_o^T \Sigma_i e^{-\delta t} \cdot U^i[q_t(\pi_t), y_{it}(\pi_t)]\, dt.$

In particular, the value of optimal pollution control is

$$V(\{\pi_t^o\}) := \int_o^T \Sigma_i e^{-\delta t} \cdot U^i[q_t(\pi_t^o), y_{it}(\pi_t^o)]\, dt = \int_o^T \Sigma_i e^{-\delta t} \cdot U^i(q_t^o, y_{it}^o)\, dt.$$

where $\{e^o_t, q^o_t, y^o_t, (y^o_{it})\}$ is the optimal intertemporal allocation as determined above. Generally, one is less interested in $V(\{\pi^o_t\})$ than in the value of a **policy change** from some given suboptimal policy to optimal pollution control. But for the discussion of intertemporal optimisation in Section 3 we only determined the initial value of environmental quality $q_a \in (-\infty, q_u)$, but we did not – and did not need to – specify any environmental control (or its absence) in the initial state. For determining the increment in welfare attained by switching to optimal control, it is necessary to specify the (suboptimal) **status–quo policy**.

To keep the argument simple, suppose environmental quality is no production externality ($Y_q = 0$) and the status–quo policy consists of an emission tax policy $\{\pi^a_t\}$ such that $\pi^a_t = \pi_a$ for all t (where $\pi_a = 0$ is included as a limiting case). In view of (14) such a policy yields a constant flow of emissions $e_a = E(\pi_a)$ over time (for $Y_q = 0$). The initial situation (at $t = 0$) may happen to be an ecological steady state. By definition of status–quo policy the economy will then remain in that state if no change in policy occurs. But suppose that at $t = 0$ the excess demand $z_a = e_a - A(q_a)$ of assimilative services is non–zero. Depending on the sign of z_a **and** the assimilation hypothesis applied, the intertemporal allocation $\{e_a, q^a_t, y^a_t, (y^a_{it})\}$ associated to the status–quo policy is either represented by a time path towards a steady state (at $q = q_u$ or $q < q_u$) or by a path with indefinitely accumulating (and possibly irreversible) pollution. The social value of the status–quo policy is uniquely given by

$$(16) \quad V(\{\pi^a_t\}) = V(\pi_a) = \int_0^T \Sigma_i e^{-\delta t} U^i(y^a_{it}, q^a_t)\, dt.$$

Hence the welfare increment of adopting the optimal pollution control policy is $V(\{\pi^o_t\}) - V(\pi_a)$. Clearly, this difference is non–negative for all status–quo policies (and zero only in exceptional cases).

Since the measurement of welfare is not operational one would like to substitute welfare by a money value measure of the policy switch. To do this, consider the compensating variation $CV^{it}(\{\pi^o_t\}, \pi_a)$ defined as the amount of money, consumer i is willing to pay at time t for substituting the status–quo policy π_a by the optimal pollution control $\{\pi^o_t\}$. Using the private consumption good as the numeraire, CV^{it} is readily determined by

$$U^i[y^o_{it}-CV^{it}(\{\pi^o_t\}, \pi_a), q^o_t] = U^i(y^a_{it}, q^a_t).$$

The pertaining total present money value of that policy switch is

$$(17) \quad CV(\{\pi^o_t\}, \pi_a) := \Sigma_i \int_o^T e^{-\delta t} \cdot CV^{it}(\{\pi^o_t\}, \pi_a)\, dt.$$

Obviously, any attempt to measure the value $CV(\{\pi^o_t\}, \pi_a)$ empirically is extremely demanding. There is simply no hope of ever getting the information necessary to determine the optimal tax policy $\{\pi^o_t\}$. Indeed, one would have to calculate the values $CV(\{\pi_t\}, \pi_a)$ for all feasible tax strategies $\{\pi_t\}$ in order to find out for which one the function $CV(\cdot)$ attains its maximum. It is begging for too much imagination to recommend suc an approach to practitioners using empirical valuation methods, e.g. the contingent valuation procedure. This is still true if continuous time would be replaced by discrete time intervals. As a concession to operationality one might think of choosing these discrete time intervals fairly large. But even with this approximation respondents would have to place a money value on changes between many pairs of hypothetical situations witout having adequate information on ecological dynamics and economic repercussions.

5. Second–best approaches for policy and valuation

Since the determination and implementation of "fine tuned" optimal intertemporal pollution control is informationally infeasible one might want to search for less demanding second–best approaches. Since simplicity is always an important precondition for political feasibility, it is quite appealing to restrict attention to those pollution controls in which real emission taxes are kept intertemporally constant. Since we know that along the optimal path emissions are not constant, in general, confining the focus on constant real tax rates introduces clearly allocative inefficiency. The constant–tax strategy has additional drawbacks depending on which hypothesis about assimilative capacity applies:

(a) Suppose, (H1) or (H4) holds and $q_a < q_u$. Then constant–tax policies have one of te following implications: With a high tax rate, an inefficient steady state (e, q_u) with $e < A(q_u)$ is reached; with a low tax rate, the environmental resource is indefinitely degraded and no steady state is reached; with a particular intermediate tax rate the steady state $(A(q_a), q_a)$ is supported as a 'knife–edge solution'.

(b) Suppose (H3) applies and at q_a the assimilation function is positively sloped. Then wit a high ta rate a steady state $(A(q), q)$ with $q > q_a$ and $A(q) < A(q_a)$ is reached; otherwise the implications are as described in point (a) above. If q_a is in the interval where the assimilation function is negatively sloped, very low tax rates lead to indefinite environmental degradation. Otherwise, any steady state $(A(q), q)$ with $A_q(q) > 0$ can be attained but no steady state $(A(q), q)$ with $A_q(q) > 0$.

(c) In case of hypothesis (H2) any steady state $(A(q), q)$ can be reached by a policy of constant real tax rates.

These observations suggest that – unless the relevant hypothesis is (H2) – the constant–tax approach constrains the search for efficient pollution control so severely that it cannot be recommended as an "operational approximation" to dynamic optimal control.

An alternative second–best strategy is to ignore the adjustment path altogether and to restrict valuations to ecological steady states. This amounts to determining an ecological steady state $(A(q), q)$ as a political environmental quality target, and an environmental authority must be assigned the power and discretion to achieve this goal by trial and error. Observe that this proposal is strongly reminiscent of Baumol's and Oates' (1971) price–and–standard approach. But our explicit consideration of the ecological dynamics introduces a number of important qualifications and difficulties for that pollution control policy:

(a) The environmental quality standard to be achieved and then maintained has got to be an ecological steady state, i.e. the standards for emissions and environmental quality must be simultanously determined via the assimilation function A.

(b) If the currently prevailing environmental quality happens to become the policy standard, its implementation requires a change of status–quo policy whenever the prevailing situation is characterised by an ecological disequilibrium.

(c) If the quality standard differs from the initial environmental quality, it depends on the initial excess demand for assimilation services $z_a = e_a - A(q_a)$ and the functional form of A (hypotheses (H1) – (H4)) whether the real emission tax rate must be raised or reduced. Suppose for example the quality standard is $q_s \in (q_m, q_u)$ in Figure 4 and the initial situation is characterised by $q_a \in (q_\ell, q_\delta)$ and $e_a > A(q_a)$. Then the tax rate must first be increased but then decreased again, and it may even be smaller than π_a when the environmental quality target is eventually reached.

(d) If the adjustment path to the targeted ecological steady state takes much time, ignoring the welfare along that path is an unwarranted coarse approximation, in particular, since the possibility cannot be excluded that the environmental authority deviates significantly from the optimal path (which exists for any predetermined quality standard but which the authority does not know).

All these argumenta suggest that the trial and error dynamics of a tax–and–standard policy à la Baumol and Oates are much more complex than suggested by static and comparative static analysis.[9] The approximation bias introduced by this approach depends on the adjustment path chosen and, given this path, it is the smaller, the smaller is the social rate of discount and the sooner the optimal steady state is reached.

When environmental policy switches are valued by their implied ecological steady states only, the status–quo policy must be also reduced to its long–term steady–state which obviously excludes those status–quo policies which do not approach any steady state[10]. With this proviso the valuation problem is essentially reduced to an exercise in static analysis: There is an initial competitive equilibrium with prices $(p_{ea}, p_{\ell a}, p_{ya})$ and quantities $(e_a{=}A(q_a), q_a, y_a{=}\ Y(e_a, \ell_a, q_a), (y_{ia}))$ as well as a target equilibrium with prices (p_e, p_ℓ, p_y) and quantities $(e{=}A(q), q, y{=}Y\ (e, \ell_o, q), (y_i))$. Since environmental policies are now completely characterised by their steady state quality standards, we refer to $CV_i(q, q_a)$ as consumer i's compensating variation for a substitution of policy q_a by policy q. This money measure is defined by

$$U^i[y_i - CV_i(q, q_a), q] = U^i(y_{ia}, q_a).$$

Hence the aggregate value of switching from policy q_a to policy q is

$$CV(q, q_a) := \Sigma_i\ CV_i(q, q_a),$$

and the environmental authority would choose q^* such that $CV(q^*, q_a) \geq CV(q, q_a)$ for all $q \in (-\infty, q_u)$. If the marginal willingness to pay is to be elicited by direct valuation methods, one encounters considerable difficulties, however:

(a) In case that the present situation does not constitute a steady state the respondents are asked to compare two hypothetical scenarios. Usually the basis of assessing the value of canges is the present situation.

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[9] If marketable emission permits are used as a policy instrument rather than emission taxes, it is also non–trivial to take the ecological dynamics into account.

[10] This is the case if the status–quo policy satisfies $e_{t=0} > A(q)$ for all $q \leq q_a$. One may then take the steady state $(A(q_n), q_n)$ as a basis of comparison.

(b) The respondents must be informed that their target steady state will not be attained immediately so that they should indicate their willingness to pay according to their 'long term preferences'. In other words, their responses should reflect their own rates of time preference.

6. Concluding remarks

This paper focussed on the impact of dynamic ecological processes on both (optimal) environmental policy formation and valuation of environmental change. Having clarified the conceptual aspects of this issue, it must be acknowledged that the informational requirements for 'fine–tuned' intertemporal optimal pollution control are prohibitive. But unfortunately, less ambitious approaches like the policy of a constant emissions tax rate or the tax–and–standard concept introduced by Baumol and Oates cannot be recommended either witout reservations. Constant tax rates are certainly appealing for their simplicity, but depending on the shape of the assimilation function thhey can be severely inefficient. As for the tax–and–standard approach, achieving the target steady state (standard) by emission tax adjustments in a trial and error procedure is not as straight–forward as suggested by Baumol and Oates when the assimilative capacity and the associated dynamics are explicitly taken into account. Moreover, preference revelation for long term environmental changes poses difficult problems for both indirect and direct valuation methods, in particular, if the initial situation does not correspond to a stationary ecological state.

Another message of this investigation is tat the valuation of largr–scale environmental changes in which ecological disequilibrium processes matter are difficult if not impossible to assess with the valuation methods available. Environmental valuation studies in the contet of small and well–defined projects provide important information for public decision makers. But large projects in which intertemporal changes of environmental quality as well as changes in prices and quantities of goods and services are significant cannot be tackled with any known valuation technique. The temptation of applying review techniques to such complex situations should be resisted.

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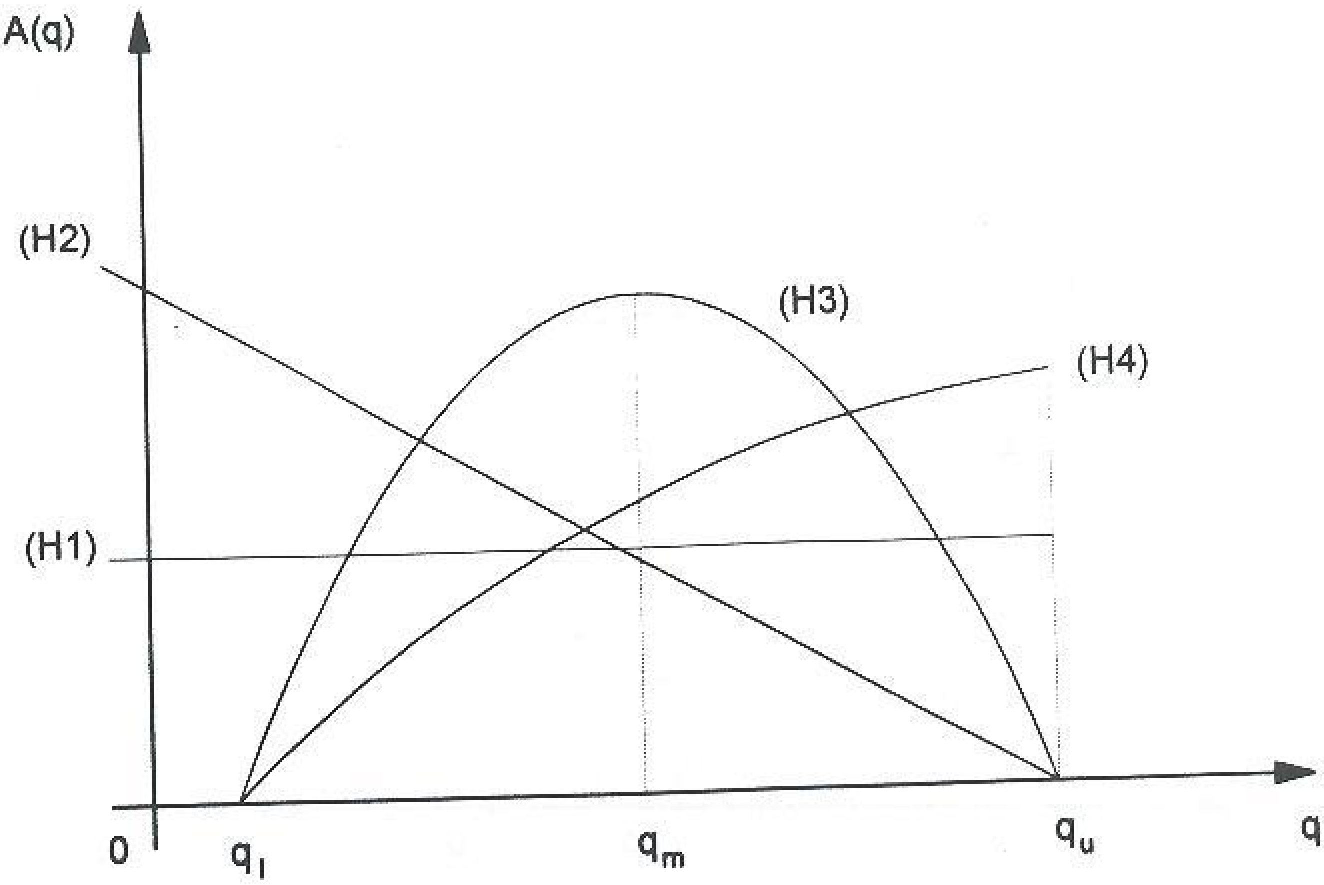

Figure 1: Assimilation capacity of environmental resources

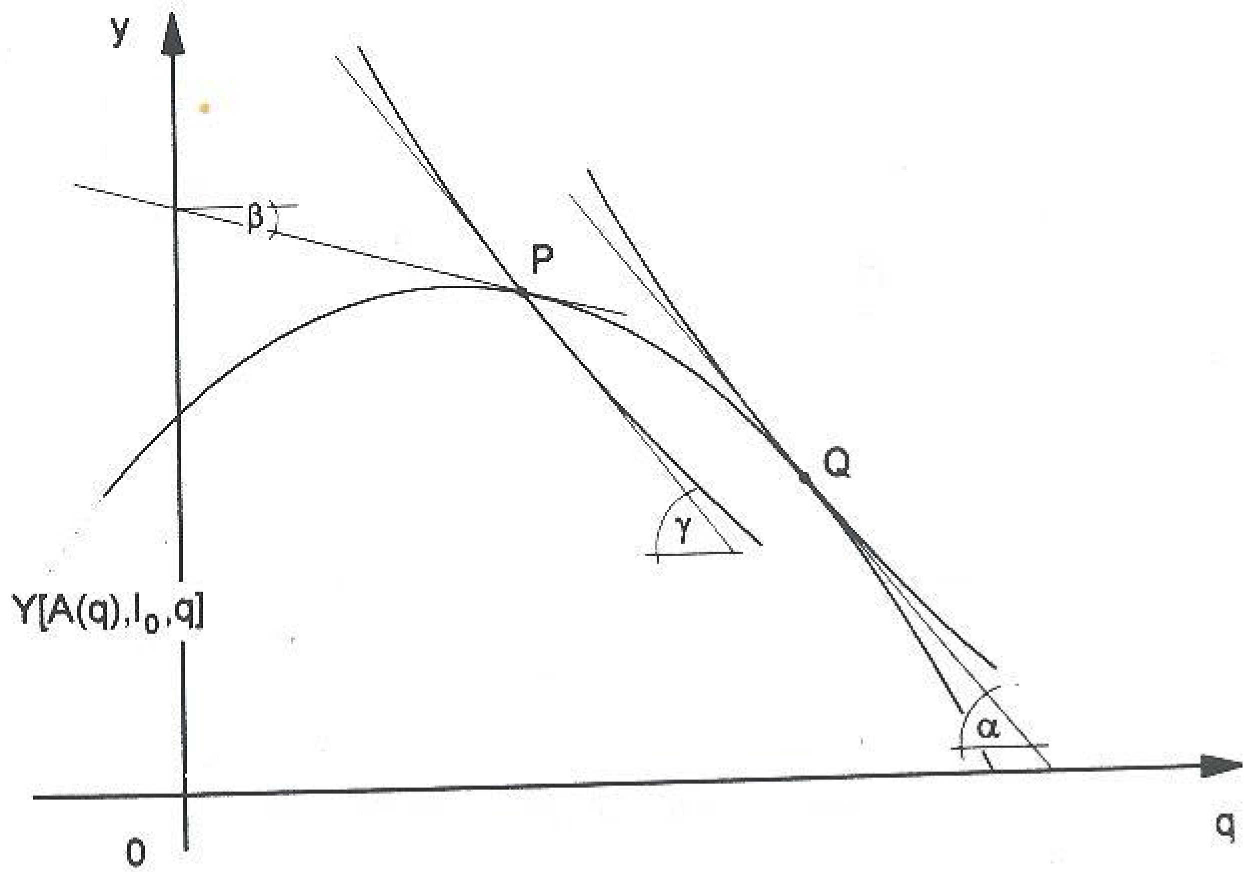

Figure 2: Impact of time preference on the optimal steady state

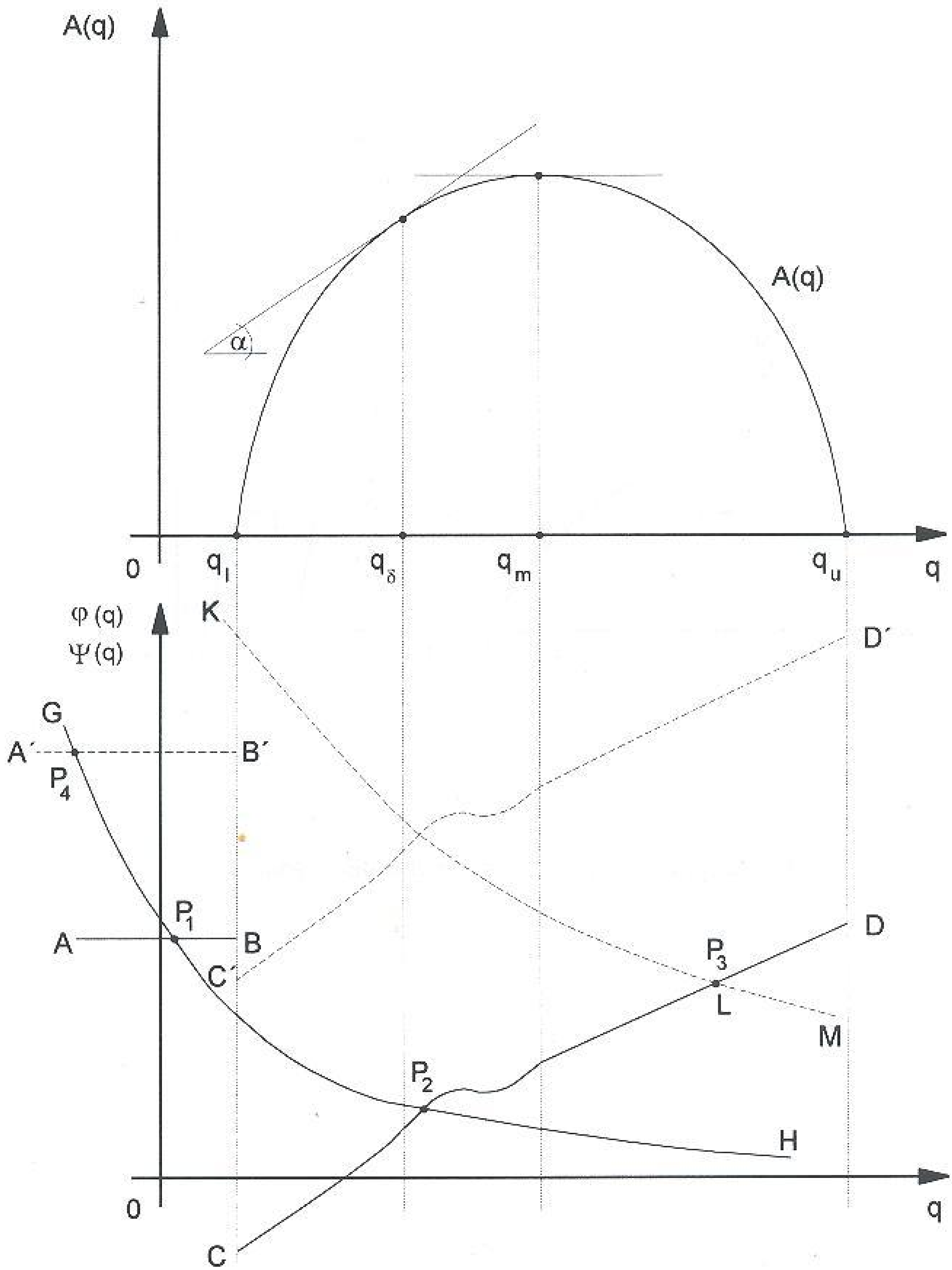

Figure 3: Optimal steady states

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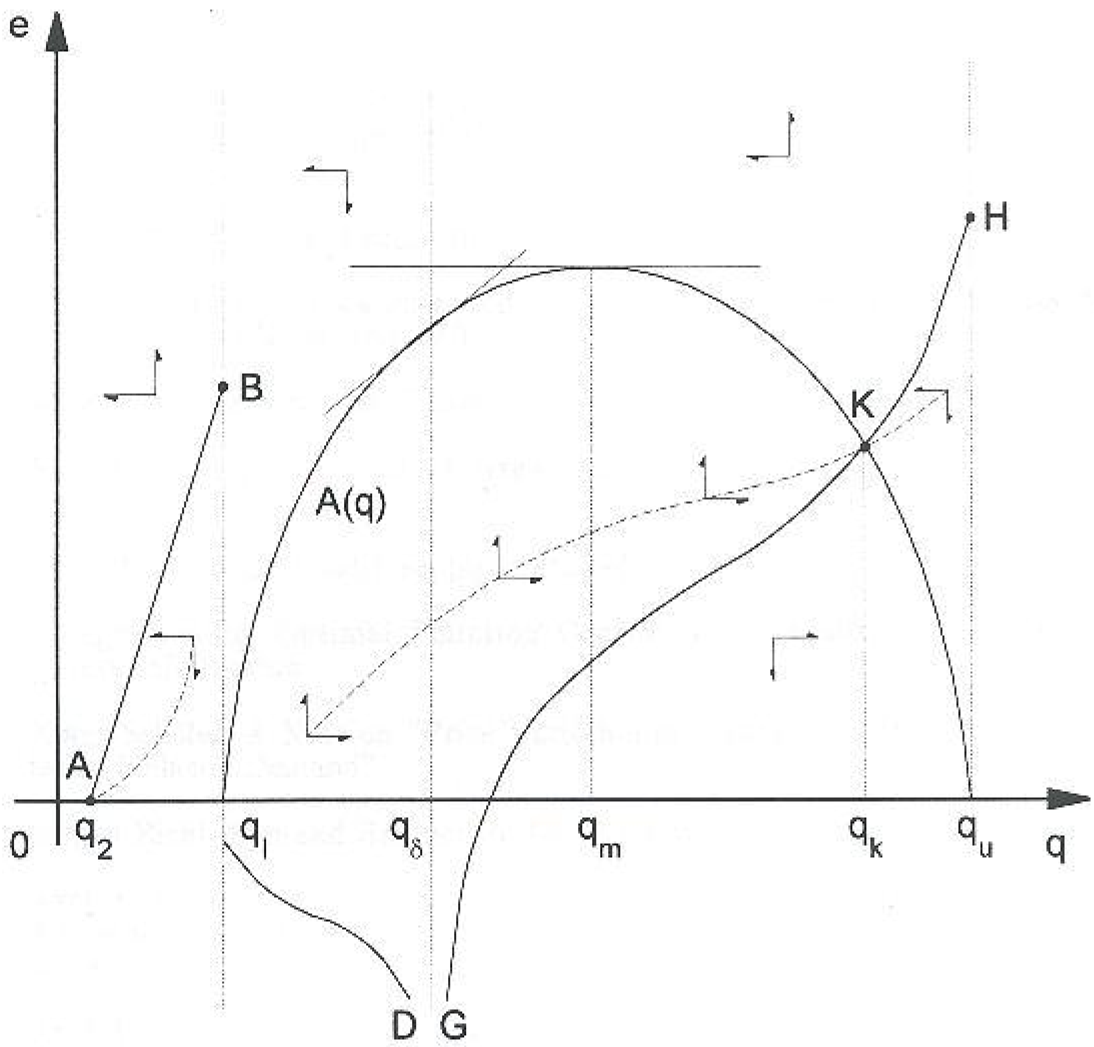

Figure 4: Steady states and optimal trajectories

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