VOLKSWIRTSCHAFTLICHE DISKUSSIONSBEITRÄGE

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Illegal Pollution and Monitoring of Unknown Quality
– A Signaling Game Approach –

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Discussion Paper No. 15–90

Revised version August 1991

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Abstract: In this paper a game model is considered whose strategically interacting agents are a polluting firm that can save abatement costs by illegal waste emissions and a monitoring agent (controller) whose job it is to prevent such pollution. When deciding on whether to dispose of its waste legally or illegally the firm does not know for sure whether the controller is sufficiently qualified and/or motivated to detect the firm's illegal releases of pollutants. The firm has the option of undertaking a small–scale (deliberate) "exploratory pollution accident" to get a hint about the controller's qualification before deciding on how to dispose of its waste. The controller may or may not respond to that "accident" by a thorough investigation thus perhaps revealing his or her type to the firm. It is this sequential decision process along with the asymmetric distribution of information that constitutes a signaling game whose equilibrium points may but need not signal the type of the controller to the firm.

In Part I of the paper the formal introduction of the game model is followed by an extensive discussion of four different equilibrium scenarios which are non–degenerate submodels whose (generic) equilibria are considered typical and especially interesting for the monitoring issue at hand. Having set up a rather complex game model the price to be paid is (as in many applications in other fields) the multiplicity of equilibria – even within one and the same equilibrium scenario. This multiplicity clearly weakens the predictive capacity of the model. To overcome it Part II addresses concepts of equilibrium refinement and selection on a fairly technical level. It is shown that the set of equilibria is reduced – not to a singleton, though – by applying the refinement concept of uniformly perfect pure strategy equilibria. Unique solutions are obtained by reference to the equilibrium selection theoretic concepts of cell and truncation consistency, of payoff dominance and of risk dominance.

1. Introduction

Pollution is typically a public bad since it results from the economic activities of some agents but bothers a large number of individuals in the society. Correspondingly, preventing pollution is a public good. When exploring pollution one can therefore rely on results in the literature on public goods (Blümel, Pethig, and von dem Hagen, 1986).

Most studies in the public goods literature assume that some public authority is in the position to impose the rules according to which self–interested individual agents interact (e.g. Hurwicz, 1973). Such set of rules is usually called a *mechanism*, i.e. a *strategic game*.

[1] Comments by participants at the conference and by an anonymous referee are gratefully acknowledged.

This approach could be used to design some reasonable legal rules to prevent river or air pollution, to limit the noise level of factories, trucks etc. Other studies (like Güth and Hellwig, 1986 and 1987, as well as Rob 1989) rely on private supply of public goods, i.e. the economic agents themselves decide whether one of the proposed mechanisms for providing the public good will be implemented or not.

Some of the studies mentioned above deal explicitly with the crucial fact that people have private information about how they are really affected by public goods or pollution, respectively. Privacy of information means that the public authority or the private agent does not know the utilities which their potential customers can obtain from the proposed mechanism. The usual approach to take care of this information deficit is the *revelation principle* (for instance Myerson 1979). According to the revelation principle one can find an allocation equivalent, incentive compatible direct mechanism for every possible public goods mechanism. A *direct mechanism* is a normal form game with payoffs determined by the true preferences and all possible preferences as strategies. A direct mechanism is *incentive compatible* if the vector of true preferences is an equilibrium point.

The revelation theorem states that for any equilibrium of any public goods mechanism there exists an incentive compatible direct mechanism for which general truth telling implies the same economic results. When looking for the possible results, e.g. some (welfare) optimal mechanism, one can therefore limit one's attention to incentive compatible direct mechanisms with general truth telling as the solution. One could say that the revelation theorem offsets the need to solve games with incomplete information.

The price one has to pay for using the revelation principle is that the result, e.g. the (welfare) optimal mechanism, is very sensitive to changes of the game structure. If, for instance, beliefs concerning private preferences change, one usually will have to rely on a different mechanism. This certainly contradicts the actual practice where one mechanism is used for many different situations which often enough were not envisaged when implementing the mechanism. A way to design mechanisms which are more robust to environmental changes is to look for mechanisms providing reasonable results for a large subset in the set of all possible preference profiles (Wilson, 1986).

Although incomplete information is also a crucial aspect of our model, we do not consider a problem of mechanism design by relying on the revelation principle. We rather introduce what we consider a natural *model of asymmetric information* and focus attention on how its results are influenced by the information deficits. Furthermore, we do not look at pollution in general but only at *illegal pollution*. More specifically, it is assumed that there exists already a set of rules, a legally codified mechanism, but polluters have an incentive to violate these rules.

Difficulties or even failures to enforce regulatory rules are not an exclusive problem in the area of environmental policy, of course. They rather seem to be widespread if not ubiquarian, since "violators" of rules can safely be expected whenever a piece of legislation passes the parliament. Not all enforcement deficits have the same empirical significance, however. In many areas it may be a good approximation to proceed on the assumption of perfect compliance – or, equivalently, on the assumption of complete and costless enforcement. Russell argues in the present volume that this used to be the mainstream proposition of environmental economists as well. But in recent years much evidence accumulated pointing to considerable discrepancy between the paper form of environmental law and the true state of the environment. In Germany as in other countries the lack of enforcing existing environmental regulations has become a decisive issue in recent time. Hansmeyer (1989, p.75) considers the notion of "enforcement deficit" (in German "Vollzugsdefizit") a much conjured catch–word. It also became evident, that incomplete enforcement does not seem to be a transitory but rather a persistent phenomenon the explanation of which must be based on analysing the strategic interactions of the parties involved.

In the model to be set up in the present paper the strategically interacting agents are (i) the polluters who can save considerable costs by disposing waste illegally and (ii) the controllers who are hired to prevent illegal waste disposal. The issue of who controls the controllers which is considered relevant in the public choice literature on bureaucracy would suggest to also consider the interactions between controllers and their supervisors (e.g. Pethig, 1991). But this is beyond the scope of the present paper. Given this limitation, our study can therefore be seen as a contribution to the game theoretic tradition of *strategic inspection analyses* (Maschler, 1966; Avenhaus, 1990; Avenhaus, Okada, and Zamir, 1991). Compared to such previous studies we explore a much more complex situation with multiple equilibria which is also the reason why we have to rely on a different game theoretic methodology.

For the sake of simplicity we assume that there is just one potential violator, a *firm* whose production process yields waste which can be legally disposed of only at high costs, and a simple *monitoring agency* called the *controller*. Detecting the source of illegal waste disposal is rather difficult and requires a lot of expertise which the agency may or may not have. When deciding on whether to dispose of its waste legally or illegally the firm does not know for sure whether the controller is qualified enough to detect illegal waste disposal, i.e. we assume asymmetric information concerning the qualification of the agency.

To be more certain about the qualification of the agency the firm has the option of undertaking a small–scale "exploratory" discharge of pollutants called *exploratory (pollution) accident* hereafter. Since a thorough investigation of this accident will be more costly for an unqualified agency than for a qualified one, this might yield a hint about the controller's qualification before deciding on whether to dispose of the waste legally or illegally. Because of the *sequential decision process* and the firm's information deficit concerning the controller's qualification our model is a *signaling game* with equilibria signaling the type, respectively the qualification of the monitoring agency.

In spite of its specific assumptions our game model defines a rather large class of sequential games with three subsequent decision stages, not including the fictitious initial chance move representing the firm's incomplete information concerning the qualification of the agency. In Section 2 the game model is formally introduced. Equilibrium scenarios, i.e. typical or especially interesting results of generic equilibria, are discussed in Section 3. To derive more specific results we apply the refinement concept of uniformly perfect pure strategy equilibria (Section 4) and equilibrium selection theory (Section 5). Finally the results are summarised and some potential lines of future research are indicated.

PART 1: EQUILIBRIUM SCENARIOS WITH POOLING AND SIGNALING BEHAVIOR

2. The game model

For the sake of simplicity we consider a situation where one single *firm f*, e.g. a major factory, can save considerable costs by discharging waste illegally instead of abating it as required by law. As an example imagine a chemical factory which can simply release toxic waste into a water course. Other examples are illegal disposal of dangerous substances on declared or undeclared waste deposits, illegal release of waste in international waters for saving transportation costs, release of toxic smoke etc.

In order to prevent firm f from illegal pollution the government has hired a *controller c* whose task is to detect illegal pollution. In the case of river pollution controller c would be, for instance, a water protection agency. Usually, jobs in such agencies are poorly paid as compared to jobs in private industries, especially if sophisticated techniques and a lot of expertise are required to prove that firm f has caused the pollution. Therefore it is open to question, of course, whether controller c is really fit to detect a polluter. Another reason to cast doubt on c's qualities is shirking, i.e. controller c may not find out a polluter simply because he does not care about his duties but rather prefers to have some leisure time during his official working hours, instead. These arguments lead us to assume that *firm f is not sure about controller c's qualification* for detecting illegal pollution.

First of all, the controller is of course hired to detect regional (or local) increases in ambient pollution resulting from illegal releases of pollutants. But it is also his task to find out the (point) source of pollution which caused that increase, i.e. he has to trace back the pollution to the illegal polluter. As argued above, in both cases the controller's success depends on his expertise and/or determination and effort in doing his job. To simplify the subsequent analysis we assume, however, that (local) detrimental increases of ambient pollution are always detected by the controller. Hence potential polluters such as firm f are only uncertain about the controller's ability to identify them as the source of pollution.

In game theoretic terminology this means that our game model is one of asymmetric incomplete information since only controller c himself knows his own qualification to prove the discharge of pollutants by firm f whereas firm f has only probabilistic beliefs concerning c's prospects of finding out illegal behavior. In order to keep things simple we will distinguish only *two types* of controller c, as expected by f: *type* e(xpert) and *type* n(on–expert). Firm f expects type e with probability $w \in (0, 1)$ and type n with the complementary probability $1-w$. Firm f's beliefs are assumed to be common knowledge.

To capture firm f's information deficit about c's type we introduce a *fictitious initial chance move* whose result is type e with probability w and type n with probability $1-w$ respectively. While c learns about the result of the chance move, firm f only knows the probabilities for the two possible results. As a result of this fictitious initial chance move f's incomplete information concerning the type of its opponent is transformed into strategically equivalent imperfect information. With the fictitious initial chance move we obtain a game with complete but imperfect information.

In Figure 2.1 the fictitious initial chance move is the first move at the origin o (the top decision node) of the game tree. Player 0 is the chance player since at decision nodes of this player the choice behavior is determined not strategically but according to predetermined probabilities. That the result of this move is revealed to the existing type e or n of controller c but not to firm f can be seen from the information conditions of these players at later moves as graphically illustrated by their information sets. An *information set* of player i is a set of decision nodes where i has to decide. When deciding player i only knows that he is at one of the decision nodes in the information set but he doesn't know at which one exactly. Graphically the information sets in Figure 2.1 are illustrated by encircling all decision nodes belonging to the same information set.

After the fictitious initial chance move firm f – not knowing the chosen type of controller c – can initiate an exploratory small pollution "accident" with the intention to trigger an investigation of its production facilities by the controller in which the firm expects to learn whether controller c is of the e– or the n–type. This is *f's decision I* in Figure 2.1. Decision I implies that the firm deliberately and unlawfully discharges pollutants for the purpose of checking controller c's ability and/or reluctance to detect the source of that "accidental" spill over. The main idea of such a move of firm f is to take advantage of the fact that due to their different qualification it is more difficult for the non–expert n than for the expert e to find out whether harmful pollutants have been discharged by firm f or

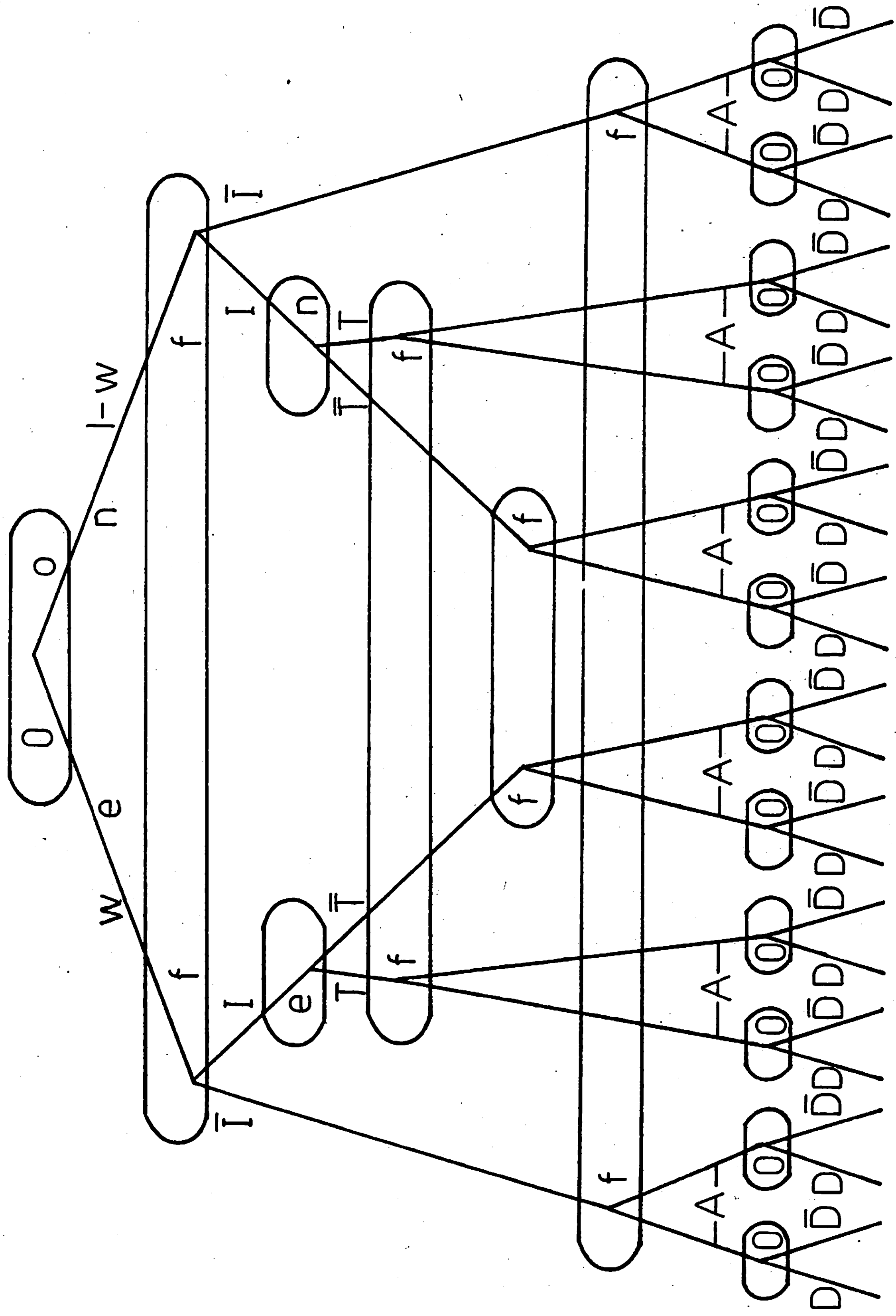

Figure 2.1: The game tree of the game model.

not. In the case of river pollution one can, for instance, imagine the release of non–toxic but easily observable waste water whose chemical substances are difficult to determine. The decision not to embark on such an exploratory discharge is denoted by $\bar{I}$.

Suppose decision $\bar{I}$ has been taken. Then firm f has to *choose the level* $A \in [0, \bar{A}]$, $\bar{A} > 0$, *of illegal waste release* without any further hint about c's qualification. In other words, when deciding on A, firm f can only rely on its a priori beliefs as expressed by the probability parameter w. In Figure 2.1 we have graphically illustrated by the line '–A–' that the level A of illegal waste disposal is a continuous variable.

In case of I the actually chosen type of controller c can decide whether to investigate the "accident" thoroughly (the *decision* T) or not (the *decision* $\bar{T}$). It is assumed that both types have the ability to thoroughly investigate the spill over although such an investigation is more costly for n than for e. After the investigation firm f is informed about the result, i.e. firm f learns about the decision T or $\bar{T}$ before the game continues.

When deciding about A, firm f therefore knows in case of I whether controller c has chosen T or $\bar{T}$. But it still does not know the result of the initial chance move. It is because of this information condition that our model is a *signaling game*. Suppose the two types react differently to the exploratory accident I, for example, e chooses T and n chooses $\bar{T}$ (signaling behavior). Then firm f can infer from its observation T or $\bar{T}$ whether c's true type is e or n. This situation exhibits the typical structure of signaling games which require asymmetric information and a sequential decision process with earlier decisions of the more informed players allowing the less informed players to make inferences. Observe, however, that if both types of controller c do not react differently to the exploratory accident (pooling behavior), then firm f does not know c's qualification when choosing A in the interval $[0, \bar{A}]$.

After firm f's choice of A it is stochastically decided whether in case of $A > 0$ firm f is detected as the illegal polluter (the *chance move* D) or not (the *chance move* $\bar{D}$). The probability for the result D is denoted by $W(A, t)$ with $A \in [0, \bar{A}]$ and $t \in \{e, n\}$ i.e., the detection probability of illegal pollution depends on the amount A of illegal pollution and on controller c's qualification. A simple probability function $W(A, t)$ satisfying the two obvious requirements,

$$W(\bar{A}, t) > W(A, t) \text{ for } \bar{A} > A \text{ and } t = e, n \quad \text{and} \tag{1}$$

$$W(A, e) > W(A, n) \text{ for all } A \in (0, \bar{A}), \tag{2}$$

is the linear probability function

$$W(A, t) = \begin{cases} MA/\bar{A} & \text{for } t = e, \\ NA/\bar{A} & \text{for } t = n, \end{cases} \quad \text{with} \tag{3}$$

$$0 < N < M < 1. \tag{4}$$

This completes the interpretation of the game tree as graphically visualised in Figure 2.1. We refrained from including the probabilities $W(A,t)$ and $1-W(A,t)$ for the final chance moves D and $\bar{D}$ in Figure 2.1 in order not to overburden the graphical presentation of the game decision process and the information conditions.

As for the description of the game model, it only remains to specify how firm f and the two types e and n of controller c *evaluate* the different plays. We follow the usual convention of assigning to a non–existing type of a player the payoff of its existing type. In case of no exploratory investigation, $\bar{I}$, and in case of "I *and* $\bar{T}$" the controller has not invested in special work effort so that his cost of effort can be neglected.

Illegal pollution is often detected because of its disastrous environmental consequences, e.g. dead fish in case of illegal river pollution. But it is quite a different problem to find out the polluter – which is assumed to be the controller's job. How good this job is done depends on the controller's qualification and incentives which in turn are largely determined by his or her payoff. In what follows the elements constituting this payoff are successively described.

First we assume that tracing illegal waste disposals back to the polluter will promote the controller's career. This has an impact on the payoff u_t of both types $t = e$ and $t = n$ of controller c. In case of $A > 0$ the payoff level u_t of type t is simply the probability $W(A,t)$ of being able to prove that firm f has caused the pollution. The payoff u_t for $A = 0$ should not be smaller than the payoff in case of a detection. Therefore we assume $u_t = 1$ for $A = 0$. To formalise this hypothesis, define

$$\delta_A = \begin{cases} 1, & \text{if } A > 0, \\ 0 & \text{otherwise.} \end{cases}$$

With the help of the variable δ_A type t's payoff for $\bar{I}$ and $(I, \bar{T})$ can be expressed as

$$u_t = 1-\delta_A + W(A,t) \qquad \text{for } t = e,\, n. \tag{5}$$

After I and T occurred type t's payoff is the difference of the payoff in (5) and the cost of a thorough investigation of the exploratory accident:

$$u_t = \begin{cases} 1-\delta_A + W(A,t) - H & \text{for } t = n, \\ 1-\delta_A + W(A,t) - L & \text{for } t = e, \end{cases} \tag{6}$$

where the assumption

$$H > L > 0 \tag{7}$$

reflects the higher qualification of the expert e as compared to the non–expert n. The inequalities (7) may be alternatively or complementary interpreted to reflect n's greater disutility from working.

Define in addition

$$\delta_I = \begin{cases} 1, \text{ if } f \text{ chooses } I, \\ 0, \text{ if } f \text{ chooses } \bar{I}, \end{cases} \quad \text{and} \quad \delta_T^t = \begin{cases} 1, \text{ if } t \text{ chooses } T, \\ 0, \text{ if } t \text{ chooses } \bar{T}. \end{cases}$$

With the help of this notation the payoff functions (5) and (6) can be comprehensively written as

$$u_t = \begin{cases} 1-\delta_A + W(A,t) - \delta_I\delta_T^t H & \text{for } t = n, \\ 1-\delta_A + W(A,t) - \delta_I\delta_T^t L & \text{for } t = e. \end{cases} \tag{8}$$

For the firm,

$$K(A) = p + FA \quad \text{with } F \geq 0 \text{ and } p > 0 \tag{9}$$

is the *cost per unit of waste* by which legal disposal exceeds the cost level of illegal disposal. The assumption $p > 0$ means that illegal disposal is always cheaper. If F is positive, the discrepancy between legal and illegal disposal increases with the amount of waste which is illegally disposed. The cost advantage $A \cdot K(A)$ of illegal waste disposal equals the firm's abatement cost which has to be juxtaposed to the *fine* $B + PA$ with $P > p$ to be paid in case of detection. We suppose that the fine P per unit of illegal disposal is independent of the amount A since most such per unit fines do not depend on amounts. The fine $B \geq 0$

does not depend on A and hence expresses that part of the penalty which is caused by any detected illegal waste disposal. In case of $\bar{I}$ firm f's payoff u_f is therefore

$$u_f = (p + FA)A - W(A,t)(B + PA)(10) \tag{10}$$

with $P > p > 0$ and $F \geq 0$. Let $C \geq 0$ denote the costs of firm f to induce and perform the exploratory accident I and $E \geq 0$ firm f's costs implied by the thorough investigation T of such an accident. In case of I and $\bar{T}$ the cost C, and in case of I and T the cost $C+E$, have to be subtracted from the payoff level in (10). With the help of the variables δ_I and δ_T^t firm f's payoff function can therefore be summarised by

$$u_f = (p + FA)A - W(A,t)(B + PA) - \delta_I(C + \delta_T^t E) \text{ with} \tag{11}$$

$$P > p > 0,\ F \geq 0,\ C \geq 0,\ E \geq 0,\ B \geq 0. \tag{12}$$

The game tree in Figure 2.1 supplemented by the probability assignment (3) and by the payoff functions (8) and (11) defines our *game model.* Of course, all these structural relationships do not define just a single game but rather a multi–dimensional class of games with parameters $\bar{A}$, B, C, E, F, H, L, M, N, P, p, and w which can vary in certain (half–)intervals. Denote by $\Omega \subset \mathbb{R}^{12}$ the set of all feasible parameter values, the *parameter space* of our game model.

Some subregions in this parameter space Ω will have obvious results. Nevertheless, to solve all games in this large parameter space is certainly beyond the scope of this paper. Therefore we will not explore all subregions in the same systematic way but restrict the most advanced game theoretic exercise to some subregions of this large parameter space that we consider to be especially interesting.

3. A gallery of equilibrium scenarios

Game theoretic studies of real–life decision problems are often attempts to provide consistent explanations of observed behavior, especially if the observed behavior appears to be paradoxical at first sight. A naive observer might be surprised, for instance, that there is a lot of littering on highways in spite of the fact that it would be much cheaper to dispose garbage properly and that one finally has to pay for the more expensive clean up, either via higher road tolls or tax rates. But a game theoretic analysis may show that the situation is actually a *prisoner's dilemma game* with littering (resembling confessing of the prisoners) as a unique dominant strategy in spite of the inefficiency which it implies. This illustrates the purpose of game theoretic studies as attempts to resolve cognitive dissonance about seemingly paradoxical observed behavior which becomes more understandable if one can find a game model, reflecting the main strategic aspects, whose equilibrium result is consistent with the observed behavior.

This way of using game theoretic models, namely to provide consistent explanations for observed behavior, has been called the *method of equilibrium scenarios* (Güth, 1984; Avenhaus, Güth, and Huber, 1991; as well as Gardner and Güth, 1991). According to the method of equilibrium scenarios one does not determine all equilibria of a game model but only those whose implied play corresponds to some really observed behavior. An *equilibrium (point)* is a strategy vector $s^* := (s_1^*, \dots, s_n^*)$ specifying a strategy s_i^* for every player i, such that no player i can profitably deviate unilaterally. To put it differently, for all players i and all alternative strategies s_i of i one must have[1]

$$u_i(s^*) \geq u_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*), \tag{1}$$

where u_i is player i's payoff function which is transformed from the set of end points/plays to the set of strategy vectors in the usual way. The strategy vector s^* is called an *equilibrium scenario* if there exists a *full–dimensional subspace* Ω' of the parameter space Ω such that s^* is an equilibrium point for all games in Ω'. In other words, for s^* to be an equilibrium scenario there has to exist a generic game with s^* as an equilibrium.

The method of equilibrium scenario is most useful for very complex game models like the game model described in Section 2, for which a more thorough game theoretic analysis is often practically impossible. This is especially true if one is more interested in exploring certain kinds of equilibria than in determining all possible equilibrium results.

We will not check the equilibrium condition for the normal form with players *f*, *e*, and *n*. We rather check it for the so–called *agent normal form* (Selten, 1975, as well as Güth and Kalkofen, 1989), where there are as many players as information sets of personal players. For the case at hand this means that we have four *F*–players rather than a single firm *f*, namely the *F*–player who decides between I and $\bar{I}$ as well as the three *F*–players

[1] For the sake of notational simplicity, the numbering of equations starts with (1) in each of the subsequent sections. If in some section of the present paper we make a reference in the text to an equation by writing, say, 'Equation (14)', then it is understood that we mean Equation (14) of the very same section where this reference appears. On the other hand, if we wish to refer to Equation (14) from Section X in the text of Section Y, X ≠ Y, then we write 'Equation (X.14)'.

choosing A after $\bar{I}$, T, and $\bar{T}$. These four F–players are called the *agents of firm f*. Formally the agent normal form is a normal form game with all agents of all (normal form) players as players whose strategies are the moves in their information set and whose payoffs are those of their (normal form) player. In our game model the agent normal form has 6 players, namely the 4 F–players in addition to players e and n.

The basic idea of the agent normal form is that a given player's decision in one of his information sets should only be governed by the future consequences of his move, i.e. what one will do in such a situation is determined by what one can still achieve and not by what one initially tended or promised to do. What we exclude is therefore that a player can threaten to behave non–optimally later. Thus the agent normal form relies on decentralised decision making of local players. See Güth (1990) for a discussion of the various notions of a player.

In order to describe equilibrium scenarios based on the agent normal form it is convenient to introduce the following pieces of notation. We denote a strategy vector by

$$s = (s_f, s_e, s_n) \tag{2}$$

with component s_f, s_e, and s_n being the strategy of firm f, type e, and type n of controller c, respectively. Firm f's strategy has to specify the choice δ_I (with $\delta_I = 1$ meaning the move I and $\delta_I = 0$ meaning move $\bar{I}$) as well as the choice of A for its three later information sets. Both types of controller c have only to decide between $\delta_T^t = 1$ (the move T) and $\delta_T^t = 0$ (the move $\bar{T}$). Therefore the strategies s_e and s_n are completely described by the move δ_T^e and δ_T^n of the two types of c. With this notation firm f's strategy s_f can be described as

$$s_f = (\delta_I, A|_{\delta_I\delta_T^t=1}, A|_{\delta_I(1-\delta_T^t)=1}, A|_{\delta_I=0}) \tag{3}$$

where '$A|_*$' stands for the level of A chosen after the previous moves described by '*'. For the sake of notational simplicity we will often write

$$s_f = (\delta_I, A_1, A_2, A_3) \tag{3'}$$

instead of (3) and correspondingly

$$s = ((\delta_I, A_1, A_2, A_3), \delta_T^e, \delta_T^n) \tag{2'}$$

for the strategy vector s.

3.1. Pooled shirking and illegal waste disposal: 'polluter's paradise' scenario

It certainly is not an unrealistic situation that both types of controller c are shirking and that firm f will react to this by maximal illegal pollution $A = \bar{A}$. This situation is described by the strategy vector

$$s^a := ((0, \bar{A}, \bar{A}, \bar{A}), 0, 0). \tag{4}$$

This strategy vector (4) describes a situation in which firm f does not invest into an exploratory accident $(\delta_I = 0)$. Therefore the response of controller c to a (hypothetical) accident is irrelevant but the reactions $\delta_T^e = \delta_T^n = 0$ as reported in s^a to that hypothetical situation indicate, of course, the controller's reluctance to investigate: If an exploratory accident would have had occurred, both types of c would not have reacted by a thorough investigation. In fact, according to s^a the enforcement of the zero–emission rule is so ineffective that the firm ignores the legal constraint completely: Not only doesn't it consider it worthwhile to explore the controller's type, but regardless of previous moves firm f finds it even advantageous to choose the maximum amount of emissions $\bar{A}$ – based on its a priori beliefs about the controller. If s^a can be shown to be an equilibrium for a full–dimensional parameter region, it characterises doubtlessly a *"polluter's paradise scenario"*.

In order to demonstrate that s^a is, indeed, a scenario we first show that it is an equilibrium. For s^a to be in equilibrium the following condition must be satisfied for all levels $A \in [0, \bar{A}]$:

$$(p + F\bar{A})\bar{A} - [wM + (1-w)N](B + P\bar{A}) \geq (p + FA)A - [wM + (1-w)N]\frac{A}{\bar{A}}(B + PA)$$

or $$p(\bar{A}-A) + F(\bar{A}^2-A^2) \geq [wM + (1-w)N][B(1-\frac{A}{\bar{A}}) + P(\bar{A} - \frac{A^2}{\bar{A}})]. \tag{5}$$

Inequality (5) requires that the lower risk of detection by choosing smaller levels $A < \bar{A}$ of illegal disposal must be (over)compensated by the increase of disposal costs. No other conditions have to be satisfied for two reasons: firstly, an exploratory accident does not pay if both types of controller c react in the same way and therefore do not signal their qualification and secondly, the decisions δ_T^e and δ_T^n do not matter in case of $\delta_I = 0$.

For $\bar{A} > A$ inequality (5) can be simplified to

$$\frac{\bar{A}(p + F(\bar{A} + A))}{B + P(\bar{A} + A)} \geq wM + (1-w)N. \tag{5'}$$

The term on the right hand side of (5') is the a priori detection probability for $A = \bar{A}$ as expected by firm f. The left hand side of (5') is a relation of the cost advantage and the penalty for illegal waste disposal. Due to (2.12) the left hand side of (5') is strictly positive. Thus there exists a full–dimensional non–empty sub space Ω^a of the 12–dimensional parameter space Ω of games, described in Section 2, such that s^a is an equilibrium in all games with a parameter vector $\omega = (\bar{A},B,C,E,F,H,L,M,N,P,p,w)$ in $\Omega^a \subset \Omega$.

The most dramatic examples of the polluter's paradise equilibrium scenario s^a seem to be the nuclear power plants in their early development. Before the uprise of the anti–nuclear or green movement such plants were neither regularly turned off in case of small accidents (although this caused environmental problems as, for instance, radioactive cooling water and radioactive steam and increased the risk of major accidents), nor did the control agencies always pay attention to such accidents. By now nuclear power plants are monitored much more intensively so that even minor accidents induce an immediate stop of the process followed by a thorough investigation of the event.

An empirical situation where s^a still seems to be an equilibrium scenario is the pollution of national and international waters. Here the detection probability of illegal polluters, even of major emissions such as the release of oil into a river, is still too small to prevent illegal disposal. Partly, this might be due to the fact that detection techniques are not well developed as is the case, for instance, when ships release oil into rivers. Moreover, fines for detected polluters are often too low to be deterring. But certainly to some extent it may also be the shirking of water authorities and water police departments that must be held responsible for very low detection rates.

Observe, however, that in the polluter's paradise scenario the controller's qualification cannot be blamed for the complete breakdown of enforcement (provided that society doesn't have at its disposal controllers who are better qualified than type e of controller c), since even type e of controller c cannot cope with his or her task. Suppose, the probability of detection cannot be improved (i.e. for fixed w, M, and N) and the abatement costs of firm f (as characterised by the parameters p and F) are exogenous. Then the right hand side of (5') and the numerator of the term on the left hand side of (5') cannot be manipulated. Consequently any escape from "polluter's paradise" must rely on raising the fine for unlawful emissions thus increasing the denominator of the term on the left hand side of (5') until the inequality sign is reversed.

3.2. Exploratory accidents and illegal waste disposal due to unqualified control: 'signaling scenarios'

Signaling takes place when $\delta_I = 1$ is followed by $\delta_T^e = 1$ and $\delta_T^n = 0$, since in that case after the move T, firm f would conclude that controller c is of the e–type whereas the move $\bar{T}$ signals the n–type. We consider two strategy vectors implying such a behavior:

$$s^b := ((1,\ 0,\ \bar{A},\ 0),\ 1,\ 0) \quad \text{and} \tag{6}$$

$$s^\beta := ((1,\ 0,\ \bar{A},\ \bar{A}),\ 1,\ 0). \tag{7}$$

Observe first that s^b and s^β differ only in that $A_3 = 0$ in s^b, but $A_3 = 1$ in s^β. The interpretation of this difference is that if the firm is asked to choose its emissions based on its a priori beliefs about controller c, then it would not emit at all in situation s^b while it would choose the maximum emission level $\bar{A}$ in situation s^β. But in both situations firm f prefers to launch an exploratory accident rendering the value of A_3 irrelevant.

As a reaction to $\delta_I = 1$ the two types of controller c exhibit signaling behavior in that the expert launches an investigation $(\delta_T^e = 1)$ but not the non–expert $(\delta_T^n = 0)$. After this signal of the controller's type, firm f responds in both situations s^b and s^β as follows:
If the controller turned out to be the expert, firm f abides completely to the law: $A_1 = 0$; but if the controller turned out to be the non–expert, firm f chooses to completely ignore the law: $A_2 = \bar{A}$.

Hence the polluter is extremely sensitive in his or her reaction to the controller's qualification. We now want to prove that s^b and s^β are equilibrium scenarios of our game model. For s^b to be in equilibrium for all $A \in [0,\ \bar{A}]$ one must have

$$(p + F\bar{A})\bar{A} - N(B + P\bar{A}) \geq (p + FA)A - N\frac{A}{\bar{A}}(B + PA), \tag{8}$$

$$(p + FA)A \leq M\frac{A}{\bar{A}}(B + PA), \tag{9}$$

$$(p + FA)A \leq [wM + (1-w)N]\frac{A}{\bar{A}}(B + PA), \tag{10}$$

$$(1-w)[(p + F\bar{A})\bar{A} - N(B + P\bar{A})] \geq C + wE, \tag{11}$$

$$H + N \geq 1 \geq L + M. \tag{12}$$

The first three inequalities ensure that firm f cannot gain by choosing a different disposal level $A \in [0, \bar{A}]$ in the three information conditions $(I,\bar{T})$, (I,T), and $(\bar{I})$, which can equivalently be expressed by $\delta_I(1-\delta_T^t) = 1$, $\delta_I\delta_T^t = 1$, and $\delta_I = 0$. Condition (11) implies the choice of I to be better than that of $\bar{I}$ for strategy vector s^b. The left hand side [right hand side] inequality in (12) is the best reply condition for $t = n$ $[t = e]$. For $\bar{A} > A$, condition (8) can be simplified to

$$\frac{\bar{A}[p + F(\bar{A} + A)]}{B + P(\bar{A} + A)} \geq N. \tag{8'}$$

Obviously, this condition is similar to (5'). Due to $M > N \geq 0$ and $w \in (0, 1)$, inequality (9) holds for a certain level $A > 0$, whenever this is true for condition (10). Thus to prove that s^b is an equilibrium scenario we only have to demonstrate that conditions (8'), (10), (11), and (12) can be simultaneously satisfied in a non–degenerate parameter region.

Owing to $\bar{A} > 0$ and $P > p > 0$ the left–hand side of (8') is well–defined and positive. Since the condition $H > 1 > L + M$ is consistent with the parameter restrictions (2.7), conditions (8') and (12) can be simultaneously satisfied by choosing N (≥ 0) sufficiently small. For rather low values of $N \geq 0$ inequality (11) is obviously true for low parameter values $C \geq 0$ and $E \geq 0$. For $A > 0$ condition (10) can be rewritten as

$$wM + (1-w)N \geq \frac{\bar{A}(p + FA)}{B + PA}. \tag{10'}$$

Since the left hand side of (10') can be chosen near to 1 by choosing w and M near to 1, it suffices to show that the right hand side of (10') can be generically smaller than 1. This is obviously true if B, the part of the fine not depending on A, is sufficiently large. Since all conditions stated so far are mutually independent, *s^b is demonstrated to be an equilibrium scenario.*

The equilibrium conditions for s^β are (5) [replacing (10)], (8), (9) , (12), and

$$w[M(B + P\bar{A}) - E - (p + F\bar{A})\bar{A}] \geq C \tag{13}$$

[replacing (11)]. To satisfy (13) assume the parameters $E, \bar{A}$, and C to be so small that the coefficient $[\cdot]$ of w is positive and that the left hand side of (13) is larger than C. Similarly, sufficiently small values of w and $N \in (0, M)$ ensure that (5) holds. As before it is possible to choose $\bar{A}p < B$ and $F\bar{A} < P$ such that the condition

$$\frac{\bar{A}(p' + FA)}{B + PA} < M \tag{9'}$$

can be satisfied for $M \in (0, 1)$. Except for (9') and the requirement $H > 1 \geq L + M$ all our conditions are mutually independent. $1 \geq L + M$ is clearly compatible with (2.4) and (2.7) because for every $M \in (0, 1)$ there is $L \in (0, 1)$ to satisfy this inequality. Thus all our conditions can be simultaneously satisfied for a non–degenerate parameter region which proves that s^{β} *is also an equilibrium scenario* of our game model.

The illegal release of waste water, or chemical substances, or oil into water resources are important examples of detrimental waste disposal in case of 'unqualified control'. Unlawful polluters can take advantage of the darkness and/or of poorly controlled parts of the river system, and one can well imagine polluters starting with the emission of minor amounts, as modeled by our 'exploratory accident I'. They may then be encouraged to choose larger amounts and in some cases even more dangerous substances if they experienced that their first unlawful action did not cause a thorough investigation. It is this reinforcement of illegal behavior which causes so dramatic environmental damage if unqualified control cannot be excluded.

Since in both "signaling equilibrium scenarios" s^b and s^{β} firm f reacts sharply on which type of controller c is revealed, it is exclusively the controller's insufficient qualification which has to be blamed for any enforcement deficit. In fact, as far as these scenarios provide an adequate characterisation of the empirical situation, the enforcement problem could be easily solved by firing all non–expert controllers. Observe, however, that this recommendation leads us to the issue of controlling the controllers which is beyond the scope of the present paper.

3.3 Absence of illegal pollution due to efficient control: "controller's paradise scenario"

All strategy vectors with the consistent choice of $A = 0$ by firm f can easily be shown to be equilibrium scenarios since one can always prevent illegal pollution in our model by assuming sufficiently large fines and/or high detection probabilities. To design a more interesting scenario in which no illegal pollution takes place consider

$$s^c := ((0, 0, \bar{A}, 0), 1, 1). \tag{14}$$

According to the strategy vector (14) both types of controller c would launch an investigation if an (exploratory) accident should occur, in which case it would be optimal for the firm not to pollute at all $(A_1 = A_3 = 0)$. Owing to the controller's types' pooling behavior the firm is not interested to choose I in the first place. But even then, monitoring and fines are so effective that zero pollution $(A_3 = 0)$ is the firm's best strategy. Such a situation is, in fact, the "controller's paradise".

For s^c to be in equilibrium the following condition must hold for all $A \in (0, \bar{A}]$:

$$wM + (1-w)N \geq \frac{\bar{A}(p + FA)}{B + PA}. \tag{15}$$

For the two types e and n of controller c no equilibrium condition has to be imposed since due to the move $\bar{I}$ by firm f their decision does not matter. But for s^c to be a perfect equilibrium (Selten, 1975, and Section 4.2 below) one must also require in addition to (15)

$$1 \geq H + N, \tag{16}$$

$$1 \geq L + M, \tag{17}$$

and (8) hold for all $A \in [0, \bar{A})$. To satisfy (16) and (17) we assume

$$1 > max\{H + N, L + M\}. \tag{18}$$

Furthermore, by choosing N small enough one can guarantee (8'). To satisfy (15) we assume values of w and M near to 1 and values of $\bar{A}$ and B such that

$$1 > \frac{\bar{A}(p + FA)}{B + PA} \tag{19}$$

for all $A \in (0, \bar{A}]$. Consequently s^c *is an equilibrium scenario* of our game model.

Although the equilibrium scenario s^c is not often mentioned in the political debate about environmental effects, it may be an empirically important one. It covers all situations where we do not experience illegal pollution since this is no profitable activity due to the efficient control system. Notwithstanding some exceptions like the case of the radioactive material from a run–down hospital in Brazil that had not properly been disposed of, radioactive material is nearly always legally abated where, of course, legal disposal may not always be environmentally adequate. Similarly, most extremely hazardous chemical waste and military weapons are properly disposed of. Most accidents with such substances were not caused by deliberate unlawful action.

One might argue that it is the aim of environmental policy to design a legal system for waste disposal such that the strategy vector s^c is an equilibrium and hopefully the only one. It is obvious from (15) that it is always possible to charge such a high fine B – or to increase P – as to satisfy (15) for any feasible set of values A, $\bar{A}$, F, M, N, and w. Hence in the tradition of "crime and punishment" (Becker, 1968) the obvious policy advice seems to be draconic monetary disincentive (punishment) to deter agents from polluting the environment. However, there are political, legal, and social reasons why societies might want to place upper bounds on the parameters B and P to the effect that the inequality (15) cannot be achieved. The political risk associated with high penalty rates as well as horizontal equity considerations prevent the government from increasing the seemingly costless penalty (Kolm, 1973). Lawyers point to the requirement of keeping means (here: fines) in reasonable proportion to the ends. In particular, with high values of B or P the punishment of low–wealth violators would be unacceptably high. It is for these reasons that incomplete enforcement of environmental standards still is an important feature in many empirical scenarios and cannot be easily overcome by the principle of "crime and punishment".

Even though the "controller's paradise scenario" is characterised by effective enforcement it is not true that the absence of law violations can be attributed to the controller's qualification. Recall that the two types of controller c show pooling behavior. The low expertise of non–experts is sufficient to deter illegal pollution. Therefore, the monitoring agency should pursue a policy of substituting expert controllers by non–experts.

3.4. Intermediate illegal pollution: "constrained polluter's paradise scenario"

Till now only scenarios with either $A = \bar{A}$ or $A = 0$ have been considered. In the following we want to show that intermediate levels $A \in (0, \bar{A})$ of illegal waste disposal can also constitute equilibrium scenarios. To see that, consider strategy vectors of the form

$$s^d := ((0, A^*, A^*, A^*), 0, 0) \quad \text{with } A^* \in (0, \bar{A}). \tag{20}$$

Observe that s^d is exactly like s^a from (4) except that $\bar{A}$ is everywhere replaced by $A^* < \bar{A}$.It implies that both types of controller c would not react to I by a thorough investigation T so that an exploratory accident I cannot provide any information concerning the actually existing type of c.

For s^d to be an equilibrium one must have

$$A^* := \left[\arg \max_{A \in [0,\bar{A}]} g(A)\right] \in (0, \bar{A}), \quad \text{where} \tag{21}$$

$$g(A) := (p + FA)A - [wM + (1-w)N](B + PA)\frac{A}{\bar{A}}. \tag{22}$$

It remains to be demonstrated that the function $g(A)$ assumes its maximal value A^* in the interior of the interval $[0, \bar{A}]$. The first and second order conditions for an interior maximum of function g are

$$g'(A) = p + 2FA^* - \frac{wM + (1-w)N}{\bar{A}} \cdot (B + 2PA^*) = 0, \tag{23}$$

$$\text{and} \quad g''(A) = 2(F - \frac{wM + (1-w)N}{\bar{A}} \cdot P) < 0. \tag{24}$$

These two conditions easily translate into

$$[wM + (1-w)N]P > F\bar{A}, \quad \text{and} \tag{25}$$

$$A^* = \frac{\bar{A}p - [wM + (1-w)N]B}{2\{[wM + (1-w)N]P - F\bar{A}\}}. \tag{26}$$

Obviously, (25) can be satisfied by choosing P large enough. Due to (25) the denominator of the right hand side of (26) is positive. A sufficiently small value of $B \geq 0$ generates therefore $A^* > 0$. The condition $A^* < \bar{A}$ is equivalent to

$$wM + (1-w)N > \frac{\bar{A}(p + 2F\bar{A})}{B + 2P\bar{A}} \tag{27}$$

and can be satisfied by choosing $p > 0$ and $F \geq 0$ small enough. Since P has to be large, the denominator on the right hand side of (27) is not very small. Thus we have shown that intermediate levels of illegal pollution *also constitute an equilibrium scenario.* Equation (26) lends itself to an easy exercise in comparative statics analysis: The firm will raise its emissions level A^* if, ceteris paribus,

i) abatement costs increase: $\frac{\partial A^*}{\partial p} > 0$, $\frac{\partial A^*}{\partial F} > 0$;

ii) the fine for illegal emissions decrease: $\frac{\partial A^*}{\partial p} < 0$, $\frac{\partial A^*}{\partial B} < 0$;

iii) the probability w with which firm f expects to meet an expert controller decreases: $\frac{\partial A^*}{\partial w} < 0$

iv) the detection probability $W(A, t)$ of illegal pollution decreases: $\frac{\partial A^*}{\partial M} < 0$, $\frac{\partial A*}{\partial N} < 0$.

All these changes in unlawful pollution conform to one's intuition.

3.5. Equilibrium scenarios and the multiplicity of equilibria

According to the method of equilibrium scenarios one tries to demonstrate that a certain type of behavior is consistent with what we consider the most basic requirement of individually rational decision behavior, namely the equilibrium property, by showing that such a behavior is implied by a generic equilibrium point. What is totally disregarded is whether this equilibrium point is the only one.

In what follows we will first demonstrate the generic multiplicity of equilibria focusing particular attention on so–called *pooling equilibria* and *signaling equilibria*. An equilibrium is said to be a pooling equilibrium, if both types of controller c react in the same way to the exploratory accident I so that the choice of I gives the firm no clue and will therefore be avoided. In contrast, a signaling equilibrium implies, by definition, that the equilibrium solution signals controller c's true type because his two types e and n react differently to the exploratory accident I and that firm f will choose I.

Due to our basic interest in signaling behavior we want to illustrate the generic multiplicity of equilibria by showing that in a generic class of games both types of equilibria, *signaling* ones and *pooling* ones, coexist. More specifically we want to prove that there exists a full–dimensional subset Ω' of Ω such that the two strategy vectors

$$s^s := ((1, 0, \bar{A}, \bar{A}), 1, 0) \quad [= s^\beta \text{ from } (3.7)] \tag{28}$$

and $$s^p := ((0, \bar{A}, \bar{A}, \bar{A}), 0, 0) \quad [= s^\alpha \text{ from } (3.4)] \tag{29}$$

are equilibria of all games in Ω'. If the signaling equilibrium s^s turns out to be the solution, firm f chooses I. Having observed T, f then knows that it is facing type e whereas it infers that c is of the n–type after having observed $\bar{T}$. As can be readily seen from (29), in case of the pooling equilibrium s^p both types of c react to I by $\bar{T}$ so that firm f does not launch I.

Since $s^s = s^\beta$ and $s^p = s^a$, the equilibrium conditions for s^s are (5), (8), (9), (12), and (13), and for s^p the equilibrium condition is (5).

As mentioned above we now simplify the model by considering only values $A \in \{0, \bar{A}\}$ instead of $A \in [0, 1]$. Note that an alternative route to proceed would have been to state conditions for the function $g_k(A) := (p+FA)A - k(A/\bar{A})(B+PA)$ such that the maximum of $g_k(A)$ on $[0, \bar{A}]$ is either $A = 0$ or $A = \bar{A}$ for all 'relevant' coefficients k to be considered. Here we do not enter into the discussion of such conditions since to apply some of the more advanced game theoretic concepts, e.g. perfectness (Selten, 1975), we have to restrict players to finitely many strategies anyhow. Thus we study a finite extensive game to which some of the more advanced concepts can be applied (Selten 1975, Harsanyi and Selten 1988, and Güth and Kalkofen 1989). If only values $A \in \{0,\bar{A}\}$ are allowed, our equilibrium conditions for s^s and s^p are specified by

$$p\bar{A} + F\bar{A}^2 \geq [wM + (1-w)N](B + P\bar{A}), \tag{5"}$$

$$p\bar{A} + F\bar{A}^2 \geq N(B + P\bar{A}), \tag{8"}$$

$$p\bar{A} + F\bar{A}^2 \leq M(B + P\bar{A}), \tag{9"}$$

in addition to (12) and (13). We want to show that all these conditions can be simultaneously satisfied on a generic subset of the parameter space Ω defined by our game model in Section 2. Since (5") implies (8") conditions (5"), (8") and (9') together can be expressed as

$$M \geq \frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} \geq wM + (1-w)N. \tag{30}$$

Obviously one can simultaneously satisfy (12) and (30) by choosing $\bar{A}$ such that

$$1 - L > M \geq \frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} \geq wM + (1-w)N. \tag{31}$$

Moreover, due to $M > N$ and $0 < w < 1$ there exists a non–degenerate interval for the expression $\bar{A}(p + F\bar{A})/(B + P\bar{A})$.

Observe that inequality (13) is equivalent to (9') for $C = 0 = E$. Thus by choosing $C \geq 0$ and $E \geq 0$ sufficiently small we can satisfy (30) and (13) simultaneously if we rely on the more refined condition

$$1 - L > M > \frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} \geq wM + (1-w)N \tag{31'}$$

rather than on (31). This shows that all requirements for s^s and s^p to be equilibria can be satisfied for a full–dimensional subset Ω' of the parameter space Ω and that therefore *the coexistence of signaling and pooling equilibria is a generic phenomenon.*

This observation leads us to the question which meaning and relevance can be attached to equilibrium points, when in all (or many) games with the equilibrium point s^s the strategy vector s^p is also in equilibrium. From the earlier discussion of these equilibria we know that they characterise markedly different behavior. Obviously, the fact that a certain behavior is implied by one of the equilibria, say s^s; does not mean that this behavior is also the solution behavior. We simply do not know whether a certain model, defined by a non–degenerate sub space of parameters should be characterised as "polluter's paradise" or as scenario of signaling the controller's type.

It is possible to respond to this dilemma in different ways: Either one does not want to distinguish among equilibria. Then all, what one can claim, is that s^s might be the solution. Or one is willing to discriminate between equilibria and to apply more refined notions of individual rationality, e.g. refinement concepts as reviewed by van Damme (1987) or equilibrium selection theories as suggested by Harsanyi and Selten (1988) and by Güth and Kalkofen (1989).

In the remainder of the paper we choose the route to discriminate between equilibria by applying more refined game theoretic solution concepts. For the sake of simplicity and since some of the more refined solution concepts are not yet defined for continuous games, we will restrict the choice of A to $\{0, \bar{A}\}$ instead to $[0, \bar{A}]$ and totally neglect mixed strategy equilibria.

PART II: PERFECT EQUILIBRIA AND (UNIQUE) SOLUTIONS VIA EQUILIBRIUM SELECTION

4. Uniformly perfect pure strategy equilibria

The weakness of the equilibrium concept, defined by the mutual best reply property (3.1), is that it does not guarantee optimal decisions in information sets off the equilibrium play. For the case at hand the strategy vector s^c as specified in (3.14) can be, for instance, an equilibrium even if conditions (3.16) and (3.17) are not satisfied. The reason is that the relatively high costs of a thorough investigation T do not matter since, due to s^c, firm f chooses $\bar{I}$ so that no explorative accident occurs. But if (3.16) and (3.17) are not satisfied, the intention to choose T is a non–credible threat since, given the situation I, it is better to use $\bar{T}$ for both types of controller c.

In order to exclude non–optimal choices in unreached information sets, Selten (1975) has introduced the *concept of perfect equilibria* which often is colloquially described as trembling hand perfectness. The basic idea is to derive from the game at hand a so–called *perturbed game* which differs from the former in that each move has to be chosen with a small positive minimum probability (due to a trembling hand). The original game is then viewed as the limit of its perturbed games when all these (artificial) small positive minimum probabilities converge to zero. In a perturbed game all information sets are reached with positive probability so that the choices in all information sets have to be optimal. An equilibrium point of the original game is said to be perfect if it is an equilibrium point in all games of a sequence of perturbed games approaching the original game. A perfect equilibrium point is therefore immune against small perturbations in the sense of small positive minimum probabilities for all moves.

Selten's perfectness idea is a rather weak concept by requiring all minimum choice probabilities for a perturbed game to be positive. Selten wanted to define only a necessary condition for individual rationality, namely an equilibrium concept excluding non–optimal choices in unreached information sets. Some recent *refinement concepts* as those discussed by van Damme (1987) developed more selective equilibrium concepts by imposing more demanding requirements of how immune against perturbations a strategy vector has to be. An extreme requirement of this form is, for instance, to ask that a strategy vector should be immune against all small perturbations. But such extremely stable equilibria do not always exist.

Here we do not want to enter into a discussion of whether one should try to develop more selective equilibrium concepts, which still do not solve many games uniquely, or whether one should design equilibrium selection theories which yield unique solutions but are at least partly based on preliminary ideas. Both approaches are discussed in Harsanyi and Selten (1988) as well as in Güth and Kalkofen (1989). The refinement concept, which will be used in the following, has been suggested by equilibrium selection theory (Harsanyi and Selten, 1988) and seems to be the most attractive refinement since it is not an ad hoc–concept for a special class of games. It rather relies on what we consider a sound philosophical basis of defining individual rationality.

Assume that there are no dominant strategies and that no player has superfluous moves in the sense that there are no two moves which always yield the same result. The idea of the trembling hand is that moves can be chosen by mistake i.e. involuntarily. Making mistakes is not an intentional act so that the probability of making a mistake should be the same for all moves. In an *ε–uniformly perturbed game* of the original game we require therefore the same small positive minimum choice probability ε for all moves in all information sets of personal players excluding chance moves. An *equilibrium* of the original game is called *uniformly perfect* if it is an equilibrium in all games of a sequence of ε–uniformly perturbed games converging to the original game in the sense of $\varepsilon \to 0$. Here it should be clear, of course, that to use a strategy or a strategy vector of the unperturbed game in a perturbed game means to choose it with maximal probability.

The main aim of this section is to determine the set of uniformly perfect pure strategy equilibria if only the two extreme levels $A = 0$ and $A = \bar{A} > 0$ of illegal waste disposal are feasible. Let G denote the game model described in Section 2 with $A \in \{0, \bar{A}\}$ and G^{ε} its ε–uniformly perturbed games with $\varepsilon > 0$. Clearly, after $\bar{I}$ firm f's beliefs concerning whether the controller is of type e or n are given by its a priori probabilities w for e and $1-w$ for n. The same is also true for the other agents of firm f, if the two types of controller c use identical strategies in G^{ε}. Let κ_t with $\kappa_t \in (0, 1)$ denote the probability that the necessary moves of firm f and type t of controller c lead to firm f's information after I and T or $\bar{I}$ and $\bar{T}$, respectively. The assumption that both types of controller c use the same strategy in G^{ε} implies $\kappa_e = \kappa_n = \kappa$. The probability for reaching firm f's information set is then given by $w\kappa + (1-w)\kappa$. Due to $\kappa \in (0, 1)$ this probability is positive. Thus firm f's conditional probability for encountering type e of controller c is given by

$$\frac{w\kappa}{w\kappa + (1-w)\kappa} = w,$$

i.e. the a priori probability for meeting type e of controller c. If *the two types of controller c use the same strategy* in G^{ε}, i.e. in case of pooling behavior, then firm f must choose $\bar{I}$ with maximal probability $1-\varepsilon$, and the optimal level A^* of illegal waste disposal is given by

$$A* = \begin{cases} 0 \;\; for \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} \leq wM + (1-w)N \\ \bar{A} \text{ for the reversed inequality} \end{cases} \tag{1}$$

If $A^* = 0$ is chosen with maximal probability, the decision T cannot be optimal if ε is positive but very small. If $A^* = 0$ is the firm's optimal strategy, controller c will therefore use $\bar{T}$ with maximal probability. This proves

Lemma 4.1: *For*

$$\frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} < wM + (1-w)N \tag{2}$$

the only uniformly perfect pooling equilibrium is

$$s_0^p = ((0,\ 0,\ 0,\ 0),\ 0,\ 0).\square \tag{3}$$

If $A^* = \bar{A}$ is firm f's optimal choice, the decision for T is never optimal since it does not pay to invest into a costly signal if firm f does not react to it. From this follows

Lemma 4.2: *For*

$$\frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} > wM + (1-w)N \tag{4}$$

the only uniformly perfect pooling equilibrium is

$$s_{\bar{A}}^p = ((0,\ \bar{A},\ \bar{A},\ \bar{A}),\ 0,\ 0).\square \tag{5}$$

Pooling equilibria of the type s^c with the choice of T by both types of controller c are not uniformly perfect since the different decisions for A after T and $\bar{T}$ cannot be justified by different beliefs after T and $\bar{T}$, respectively.

The case where (2) and (4), respectively, hold as an equality is neglected since we do not want to focus attention on degenerate games without special political interest.

If *the two types of controller c use different strategies* in G^{ε}, this always means that type $t = e$ uses T with higher probability than $t = n$. Let π_t denote the probability with which the move T is chosen by type t. Then for given π_e and π_n firm f's probability for observing move T, given its decision for I, is

$$\mu(T) = w\pi_e + (1-w)\pi_n. \tag{6}$$

In an equilibrium of an ε–uniformly perturbed game one has $\pi_e \geq \pi_n \geq \varepsilon > 0$ so that $\mu(T)$ is always positive. We can therefore define firm f's conditional or *posterior probability* for facing type $t = e$ after observing move T by using Bayes–rule as

$$\mu(e|\,T) = \frac{w\pi_e}{w\pi_e + (1-w)\pi_n}. \tag{7}$$

Analogously, the posteriori probability $\mu(e|\,\bar{T})$ is given by

$$\mu(e|\,\bar{T}) = \frac{w(1-\pi_e)}{w(1-\pi_e) + (1-w)(1-\pi_n)}. \tag{8}$$

With the help of this notation the optimal decision A^* of firm f can be derived as

$$A^*|_{\delta_T^t \delta_I = 1} = \begin{cases} 0 & \text{for } \mu(e|\,T)M + [1-\mu(e|\,T)]N \geq \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}, \\ \bar{A} & \text{for } \mu(e|\,T)M + [1-\mu(e|\,T)]N \leq \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} \end{cases} \tag{9}$$

$$A^*|_{\delta_I(1-\delta_T^t) = 1} = \begin{cases} 0 & \text{for } \mu(e|\,\bar{T})M + [1-\mu(e|\,\bar{T})]N \geq \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}, \\ \bar{A} & \text{for } \mu(e|\,\bar{T})M + [1-\mu(e|\,\bar{T})]N \leq \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}, \end{cases} \tag{10}$$

$$A^*|_{\delta_I = 0} = \begin{cases} 0 & \text{in case of (2)}, \\ \bar{A} & \text{in case of (4)}, \end{cases} \tag{11}$$

Let us now explore the possibility of signaling behavior in the sense that type e chooses T and type n chooses his move $\bar{T}$ with maximal probability, i.e. $\pi_e = 1-\varepsilon$ and $\pi_n = \varepsilon$. In an ε–uniformly perturbed game with $\varepsilon > 0$ this behavior is optimal if $1-L > M$ and $N > 1-H$. Moreover, one obtains $\mu(e|T) = w(1-\varepsilon)/[w(1-\varepsilon) + (1-w)\varepsilon]$ and $\mu(e|\bar{T}) = w\varepsilon/[w\varepsilon + (1-w)(1-\varepsilon)]$ so that the posteriori probability $\mu(e|T)$ increases when ε decreases and converges to 1 for $\varepsilon \to 0$ whereas $\mu(e|\bar{T})$ decreases with ε and converges to 0 for $\varepsilon \to 0$. For ε positive and sufficiently small the following moves are therefore chosen with maximal probability:

$$A^*|_{\delta_I \delta_T^t = 1} = \begin{cases} 0 & \text{for } M > \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}, \\ \bar{A} & \text{for } M \leq \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}, \end{cases} \tag{9'}$$

$$A^*|_{\delta_I (1-\delta_T^t) = 1} = \begin{cases} 0 & \text{for } N \geq \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}, \\ \bar{A} & \text{for } N < \dfrac{\bar{A}(p + F\bar{A})}{B + P\bar{A}}. \end{cases} \tag{10'}$$

Denote by

$$A^* = (A^*|_{\delta_I \delta_T^t = 1}, A^*|_{\delta_I (1-\delta_T) = 1}, A^*|_{\delta_I = 0}) \tag{12}$$

the vector of firm f's illegal waste disposal levels for its three information sets when only levels $A \in \{0, \bar{A}\}$ are possible. The implications of conditions (9'), (10'), and (11) are graphically illustrated in Figure 4.1 from which it follows that uniformly perfect pure strategy *signaling* equilibria can only exist in the range

$$N \leq \frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} < M. \tag{13}$$

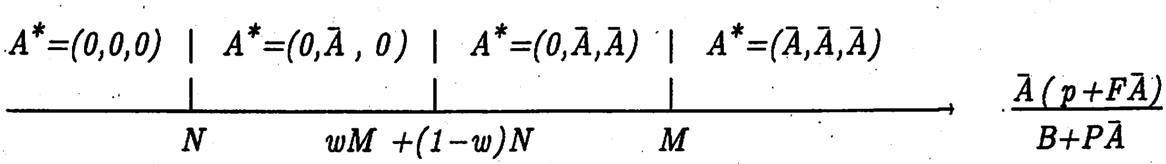

Figure 4.1: The optimal levels A^* of illegal waste disposal depending on $\dfrac{\bar{A}(p+F\bar{A})}{B+P\bar{A}}$

The decision problem which has not been considered so far is firm *f's* choice between I and $\bar{I}$. In case of $A^* = (0,\bar{A},0)$ the decision I is optimal whenever inequality (3.11) holds. In case of $A^* = (0,\bar{A},\bar{A})$ the decision is optimal if inequality (3.13) holds. With the help of the inequalities

$$N < \frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} < wM + (1-w)N, \tag{14}$$

$$wM + (1-w)N < \frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} < M. \tag{15}$$

we now summarise our results by

Lemma 4.3: *For 1–L > M and N > 1–H the only uniformly perfect pure signaling equilibrium is in case of (14):*

- $s^s_{0,1} := ((1, 0, \bar{A}, 0), 1, 0)$, *if (3.11) holds;*
- $s^s_{0,0} := ((0, 0, \bar{A}, 0), 1, 0)$, *if the reversed strict inequality holds;*

and in case of (15):

- $s^s_{\bar{A},1} := (1, 0, \bar{A}, \bar{A}), 1, 0)$, *if (3.13) holds;*
- $s^s_{\bar{A},0} := ((0, 0, \bar{A}, \bar{A}), 1, 0)$, *if the reversed strict inequality holds.* □

All four strategy vectors in Lemma 4.3 prescribe a *type differentiating behavior* in the sense of $\delta^e_T = 0$ and $\delta^n_T = 1$. Strictly speaking, signaling only takes place in case of $s^s_{0,1}$ and $s^s_{\bar{A},1}$ when firm f chooses its move I. But we will also refer to the strategy vectors $s^s_{0,0}$ and $s^s_{\bar{A},0}$ as signaling equilibria; In these strategies the controller would have signaled his type if an exploratory accident would have occurred.

Lemmata 4.1, 4.2, and 4.3 provide a complete overview over all uniformly perfect pure equilibria. Thus, in the range (14) there exist two uniformly perfect pure equilibria, namely s^p_0 and $[s^s_{0,1}$ *or* $s^s_{0,0}]$, whereas the range (15) contains $s^p_{\bar{A}}$ and $[s^s_{\bar{A},1}$ *or* $s^s_{\bar{A},0}]$. In all other more degenerate parameter regions there exists exactly one uniformly perfect pure strategy equilibrium point.

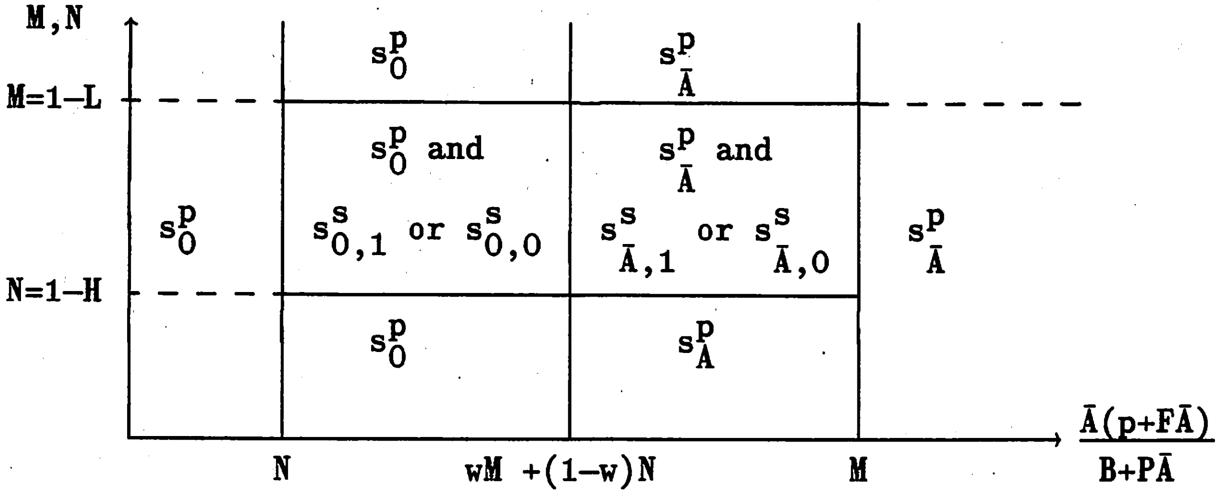

Figure 4.2: All uniformly perfect pure strategy equilibria in the *$[\bar{A}(p+F\bar{A})/(B+P\bar{A}), (M,N)]$*-plane except for border cases

The possible cases of multiple uniformly perfect pure equilibria are graphically illustrated by Figure 4.2 from which it follows that the coexistence of signaling and pooling equilibria is not resolved by using the more refined concept of uniformly perfect equilibria. Like other refinement concepts the uniformly perfect equilibrium also does not generally yield a unique solution. Consequently strategic uncertainty cannot be completely resolved by relying on this refinement concept.

5. Comparison of signaling and pooling equilibria

As shown in Figure 4.2 we have to distinguish two generic regions with more than one uniformly perfect pure strategy equilibrium, namely the range defined by "$1-L > M > N > 1-H$ and (4.14)" and the range defined by "$1-L > M > N > 1-H$ and (4.15)". Whenever the uniformly perfect pure strategy equilibrium is not unique, we have exactly two such equilibria, namely a signaling and a pooling one. Since the number of uniformly perfect equilibria is generically odd, in all these cases there exists also a uniformly perfect equilibrium in mixed strategies which we chose to ignore. In the terminology of equilibrium selection theory the mixed strategy equilibrium is no initial candidate for the solution of the game (Harsanyi and Selten, 1988; Güth and Kalkofen, 1989).

In the following we will try to *derive a unique solution* for all generic subregions. Whenever there is only one uniformly perfect pure strategy equilibrium, we take this equilibrium to be the solution of the game. In case of more than one such equilibrium we will apply *equilibrium selection theory* in order to decide which of them is the solution of the game.

5.1 Cell and truncation consistency

According to equilibrium selection theory as developed in the pioneering approach of Harsanyi and Selten (1988) one does not solve the game directly but determines its solution g by deriving a unique solution g^{ε} of its ε–uniformly perturbed games and solves the unperturbed game via the limit

$$g = \lim_{\varepsilon \to 0} g^{\varepsilon} \tag{1}$$

of the sequence of equilibria g^{ε}. Since g^{ε} is an equilibrium point of the ε–uniformly perturbed game, the limit solution g, if it exists, is obviously a uniformly perfect equilibrium point of the original game. Thus for an equilibrium to become the solution of the unperturbed game it must be uniformly perfect according to equilibrium selection theory.

To determine the unique solution g^{ε} of an ε–uniformly perturbed game one first has to decompose the game if possible. A game can be *decomposed*, for instance, if it has a proper subgame in the sense of an *informationally closed subtree*. A subtree of the game tree is said to be informationally closed if all information sets containing a decision node of the subtree contain only decision nodes of the subtree. As many games with incomplete information our game model has no proper subgames (see Figure 2.1).

A substructure generalising the notion of the subgame is the *cell game* (Harsanyi and Selten, 1988). Consider the agent normal form of an extensive game which is defined as the normal form game with all agents as players whose strategies are the moves in the respective information sets and whose payoffs are those of the original player (Selten, 1975). Consequently, a player has as many agents as he has information sets. A subset of players in the agent normal form is a *cell* if for all cell players it only depends on the choices of the other cell players whether a certain strategy is a best reply or not. In other words, a cell is a subset of agents which is closed with respect to the best reply correspondence. The cell game has only the cell players as players whereas all other players are assumed to use all their pure strategies with the same probability.

Our basic game as illustrated in Figure 2.1 has no proper cells since in the unperturbed game the agents who make their move after I have all their moves as best replies if $\bar{I}$ is chosen. In the terminology of Harsanyi and Selten (1988, p. 95) the unperturbed game has only a *semi–cell game*. In the ε–uniformly perturbed games with $\varepsilon > 0$ the set of all agents who have to decide after I, is a proper cell. In such a game the move I is chosen with positive probability so that $\bar{I}$ cannot be chosen with certainty. All agents after I have therefore to react optimally to I and to the choices of the other agents after I. Thus all ε–uniformly perturbed games have a proper cell game with all agents after I as players. Also firm *f's* agent choosing A after $\bar{I}$ is a cell. The residual game has only one player – the agent deciding between I and $\bar{I}$ – who can only react optimally to the two cell game solutions. For details see the decomposition procedure of Harsanyi and Selten (1988) from which we deviate only by using the agent normal form.

Cell consistency requires to solve a cell game as if it were an independent game: The solution of the cell game should not depend on how it is embedded in a larger game context. Given the solution of the cell games the residual game is the game with the non–cell members as players where all cell players are fixed at their strategies according to the cell game solutions. *Truncation consistency* requires to solve the residual game as if it were independent.

When solving an ε–uniformly perturbed game one first looks for the smallest cell games not containing proper subcells, then one looks for the residual games of the second smallest cell games resulting from anticipating the solution of the smallest cell games, etc. This procedure is closely related to the backward induction procedure of dynamic programming. In the case at hand we only have two smallest cell games of our basic model, namely the one with the agents after I as players and the cell game with firm *f's* agent after $\bar{I}$ as the only player. What is left is only one residual game with just one player, namely firm *f's* agent choosing between I and $\bar{I}$.

In the following we will distinguish two selection principles, payoff dominance and risk dominance. Cell and truncation consistency requires to apply these principles first to the two cell games and then to the residual game of the ε–uniformly perturbed games. The solution of the original game is then determined by the limit of the combination of the cell game solutions and the residual game solution for the ε–uniformly perturbed games when ε approaches zero.

5.2 Payoff dominance

Let $\hat{s}$ and $\tilde{s}$ be two different strategy vectors and denote by $u_i(\hat{s})$ and $u_i(\tilde{s})$ player *i's* payoffs implied by $\hat{s}$ and $\tilde{s}$, respectively. The strategy vector $\hat{s}$ is said to *payoff dominate* the strategy vector $\tilde{s}$ if

$$u_i(\hat{s}) > u_i(\tilde{s}) \quad \text{for all players with } \hat{s}_i \neq \tilde{s}_i . \tag{2}$$

Suppose that a game has only two uniformly perfect equilibria in pure strategies, namely $\hat{s}$ and $\tilde{s}$, and let $\hat{s}$ payoff dominate $\tilde{s}$. Equilibrium selection theory then assumes $\hat{s}$ to be selected as the solution (for instance, Harsanyi and Selten, 1988).

The interpretation of payoff dominance as an equilibrium selection concept relies on the hypothetical expectation that all players think that either $\hat{s}$ or $\tilde{s}$ will be the solution. Clearly, players i with $\hat{s}_i = \tilde{s}_i$ will then know what to do, and he or she can therefore be neglected. Players i with $\hat{s}_i \neq \tilde{s}_i$ will be called the *active players* for comparing $\hat{s}_i$ and $\tilde{s}_i$. If (2) holds all remaining players are interested in $\hat{s}$ to become the solution. Payoff dominance assumes that expectations will concentrate on this commonly desired solution (Harsanyi and Selten, 1988, p. 81 and p. 223).

Payoff dominance as a selection principle is not totally convincing since other considerations, e.g. how risky a strategy vector is, might suggest a different result. In a sense payoff dominance is a way to avoid the more appropriate way of formally representing preplay communication which may or may not yield a payoff dominant solution.

For $1-L > M > N > 1-H$ in addition to (4.15) and (3.11) Lemma 4.1 and 4.3 imply that s_0^p and $s_{0,1}^s$ are uniformly perfect equilibria. According to s_0^p and $s_{0,1}^s$ only type e of controller c and firm f after I and $\bar{T}$ use different strategies in the cell game after I, i.e. are active players. In an ε–uniformly perturbed game with $\varepsilon > 0$ type e receives the payoff

$$\varepsilon(1-\varepsilon) + (1-\varepsilon)^2 + M[\varepsilon^2 + (1-\varepsilon)\varepsilon] - \varepsilon L \tag{3}$$

given that $t = e$, that firm f has chosen I, and that the choices according to s_0^p are realised with maximal probability. For $s_{0,1}^s$ this payoff is

$$(1-\varepsilon)^2 + \varepsilon^2 + 2\varepsilon(1-\varepsilon)M - (1-\varepsilon)L. \tag{4}$$

Due to $1 > M$ and $L > 0$ the payoff (3) is greater than payoff (4) for $\varepsilon < 1/2$.

For firm f's agent after I and $\bar{T}$ the payoff expectation implied by s_0^p is

$$[\varepsilon^2 + (1-\varepsilon)\varepsilon]\{(p + F\bar{A})\bar{A} - [wM + (1-w)N](B + P\bar{A})\} - C - \varepsilon E \tag{5}$$

given that I has been realised. For $s^s_{0,1}$ this payoff is

$$(1-w)\{[\varepsilon^2 + (1-\varepsilon)^2][(p + F\bar{A})\bar{A} - N(B + P\bar{A})] - \varepsilon E\} - C +$$
$$+ w\{2\varepsilon(1-\varepsilon)[(p + F\bar{A})\bar{A} - M(B + P\bar{A})] - (1-\varepsilon)E\}. \tag{6}$$

For $\varepsilon \to 0$ the payoff (5) converges to $-C$ whereas (6) approaches

$$(1-w)[(p + F\bar{A})\bar{A} - N(B + P\bar{A})] - C - wE. \tag{6'}$$

Due to (4.14) the payoff (6') is smaller than $-C$ if

$$(p + F\bar{A})\bar{A} - N(B + P\bar{A}) < \frac{w}{1-w} E. \tag{6''}$$

Thus for (6") the equilibrium point s^p_0 payoff dominates $s^s_{0,1}$. In case that (6") is not satisfied, one cannot discriminate between s^p_0 and $s^s_{0,1}$ by payoff dominance. Since the behavior for the cell game after I does not differ between $s^s_{0,1}$ and $s^s_{0,0}$, the same result is true for the comparison of s^p_0 and $s^s_{0,0}$.

For the trivial cell game with firm f's agent after $\bar{I}$ as the only player the trivial solution is $A = 0$ in the range (4.14). The residual game with firm f's agent choosing between I and $\bar{I}$ is not defined if the cell game after I cannot be solved by payoff dominance.

Lemma 5.1: *Suppose that in the range (4.14) two uniformly perfect pure strategy equilibria coexist. Then the pooling equilibrium payoff dominates the signaling one in case of (6") whereas one cannot select one of them as the solution by cell and truncation consistent payoff dominance if (6") is not satisfied.*

For $1-L > M$ and $N > 1-H$ in addition to (4.15) and (3.13) Lemmata 4.2 and 4.3 imply that $s^p_{\bar{A}}$ and $s^s_{\bar{A},1}$ are uniformly perfect equilibria. In the range (4.15) the trivial cell game after $\bar{I}$ has the solution $A = \bar{A}$. When comparing $s^p_{\bar{A}}$ and $s^s_{\bar{A},1}$ the active players of the cell game after I are type e of controller c and firm f's agent after I and T. Given that I has been chosen, type e's conditional payoff expectation is

$$\varepsilon^2 + (1-\varepsilon)\varepsilon + M[\varepsilon(1-\varepsilon) + (1-\varepsilon^2)] - \varepsilon L \tag{7}$$

if the choices according to $s^p_{\bar{A}}$ are made with maximal probability in an ε–uniformly perturbed game with $\varepsilon > 0$. For $s^s_{\bar{A},1}$ the analogous payoff expectation is given by (4). Due to $1-L > M$ type e prefers (4) over (7) if ε is positive and sufficiently small. Therefore, in every ε–uniformly perturbed game with ε positive and sufficiently small type e of controller c prefers the cell game solution of $s^s_{\bar{A},1}$ over the one of $s^p_{\bar{A}}$.

For firm f's agent after I and T the payoff expectation for given I according to $s^p_{\bar{A}}$ is

$$[\varepsilon(1-\varepsilon) + (1-\varepsilon)^2]\{(p + F\bar{A})\bar{A} - [wM + (1-w)N](B + P\bar{A})\} - C - \varepsilon E. \tag{8}$$

The analogous payoff expectation for $s^s_{\bar{A},1}$ is given by (6). For $\varepsilon \to 0$ the payoff (8) converges to

$$(p + F\bar{A})\bar{A} - [wM + (1-w)N](B + F\bar{A}) - C, \tag{8'}$$

whereas (6) converges to (6'). The difference of (8') and (6') is

$$w[(p + F\bar{A})\bar{A} - M(B + P\bar{A})] + wE \tag{9}$$

which is negative in case of

$$M(B + P\bar{A}) - (p + F\bar{A})\bar{A} > E. \tag{8''}$$

For (8") firm f prefers the cell game equilibrium according to $s^p_{\bar{A},1}$ over the one induced by $s^p_{\bar{A}}$. Otherwise there exists no payoff dominance relationship.

Lemma 5.2: *Suppose that in the range (4.15) two uniformly perfect pure strategy equilibria coexist. Then the signaling equilibrium payoff dominates the pooling one in case of (8") whereas one cannot select one of them as the solution by cell and truncation consistent payoff dominance if (8") is not satisfied.*

According to the Lemmata 5.1 and 5.2 one cannot always rely on payoff dominance to discriminate among the uniformly perfect pure strategy signaling and pooling equilibria. It is interesting to observe that according to (6") payoff dominance of pooling over signaling behavior in the range (4.14) becomes more likely if w increases whereas in the range (4.15) the condition for payoff dominance does not depend at all on w. In parameter region (4.15) signaling is more likely to payoff dominate pooling behavior when costs E to firm f of being inspected become small. Conversely, in parameter region (4.14) pooling payoff dominates signaling behavior if E is sufficiently large. Thus according to the selection criterion of payoff dominance small inspection costs tend to support signaling behavior.

5.3 Risk dominance

Payoff dominance and considerations of strategic risk are mutually inconsistent solution requirements. Equilibrium selection theory (Harsanyi and Selten, 1988, and also Güth and Kalkofen, 1989) relies on both requirement and has avoided their mutual inconsistency by giving priority to payoff dominance. But this is a very premature decision and one that will very likely be reversed in the future. Indeed, earlier versions of the Harsanyi and Selten theory as in Güth (1978) did not rely on payoff dominance at all. Also the recent ad hoc selection concept of Carllson and van Damme (1989) does not use payoff dominance.

In the following we will therefore apply risk dominance as an *alternative* solution requirement even in those regions where the game can also be solved by payoff dominance.

To compare pooling and signaling equilibria one first has to solve the smallest cell games. For the case at hand the only non–trivial game is the cell game after I. The cell game after $\bar{I}$ as well as the residual game are trivial games since they have only one player. In the cell game after I both type n of controller c and one of the firm's agents use the same strategy. Thus the cell game after I has only two active players: type e of controller c and one of the firm's agents. Furthermore, due to $A \in \{0, \bar{A}\}$ both active players have only two pure strategies and both cell game equilibria are strict whenever an ε–uniformly perturbed game with $\varepsilon > 0$ is considered. Here an *equilibrium is said to be strict* if a unilateral deviation by a player yields a lower payoff for the deviator. To solve the game at hand we therefore need a selection concept by which one can solve 2×2–bimatrix games with two strict equilibria. Fortunately, Harsanyi and Selten (1988, Chapter 3.9) have developed a rigorous and easily applicable concept to solve such games. This concept is axiomatically characterised by the following three very convincing requirements: independence of isomorphic transformations (IIT), best reply invariance (BRI), and monotonicity (MO).

Independence of isomorphic transformations requires the solution in isomorphically transformed games to be the same except for differences in strategically unessential details such as the names of players or strategies or positive affine transformations of utilities. Observe that IIT implies symmetry invariance, i.e. this axiom requires the solution of symmetric games to be symmetric.

Since the equilibrium concept implies all players to use mutually best replies, one can argue that it is only the best reply structure what matters for equilibrium analysis – thus contradicting payoff dominance. *Best reply invariance* requires the solution of games with the same best reply structure to be the same.

To explain *monotonicity* consider a pure strategy equilibrium point s of a given game. The game which results from this game by increasing an active player's payoff for s is called the game resulting from strengthening the equilibrium s. If no other pure strategy equilibrium except s is the solution of the original game, payoff monotonicity requires s to be the solution of the game resulting from strengthening s. The stronger incentive for s should then make s the solution.

Harsanyi and Selten (1988, p. 87) show that in the class of 2×2–bimatrix games with two strict equilibria there is only one solution of each such game satisfying the axioms IIT, BRI, and MO. They call this solution the *risk dominant solution.* Expressed in terms of a dominance relation they say that this solution risk dominates the other strict equilibrium.

When determining the solution of the cell game after I we can refer directly to the axioms IIT, BRI, and MO which will allow us to transform the original game into a more appropriate one.

5.4 Solutions in the range (4.14)

In this section it will be generally assumed that condition (4.14) is valid. Recall from Figure 4.2 that if the inequalities $1-L > M > N > 1-H$ do not hold along with (4.14), the pooling equilibrium s_0^p is the solution. But in case of $1-L > M > N > 1-H$ both a pooling and a signaling equilibrium exist. In the following we will therefore presuppose $1-L > M > N > 1-H$ in addition to (4.14).

Since the crucial problem is to solve the cell game after I, we consider the strategic situation after I in an ε–uniformly perturbed game with $\varepsilon > 0$. According to s_0^p and $[s_{0,1}^s$ or $s_{0,0}^s]$ the only active players of this cell game are type e of controller c and firm f's agent after I and $\bar{T}$. Type e's strategies are clearly T and $\bar{T}$ whereas firm f's agent after I and $\bar{T}$ can choose $A = 0$ and $A = \bar{A}$. The 2×2–bimatrix presentation of the cell game is given in Figure 5.1 in which firm f's payoff is the upper left entry and type e's payoff is the lower right entry.

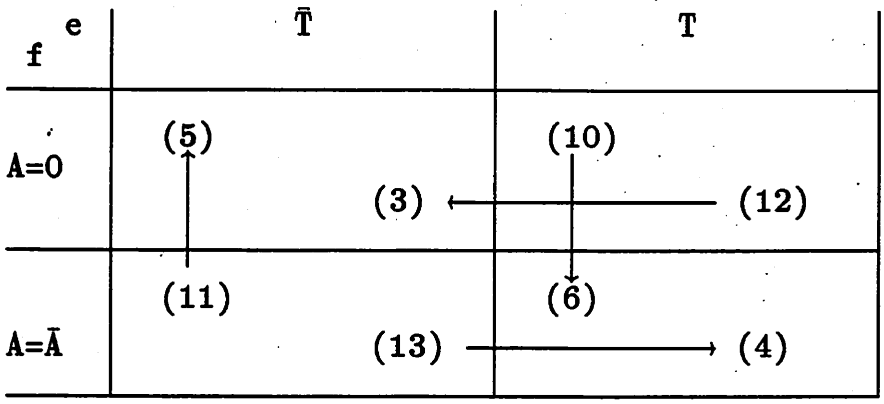

Figure 5.1: The restricted cell game after *I* for the comparison of s_0^p with $[s^s_{0,1}$ *or* $s^s_{0,0}]$.

The payoff entries in Figure 5.1 are – as far as they have not yet been defined above:

$$(1-w)[(\varepsilon^2+(1-\varepsilon)\varepsilon][(p+F\bar{A})\bar{A}-N(B+P\bar{A})]-\varepsilon E] - C + \\ + w\{[\varepsilon(1-\varepsilon) + \varepsilon^2][(p+F\bar{A})\bar{A}-M(B+P\bar{A})]-(1-\varepsilon)E\}, \tag{10}$$

$$[\varepsilon^2 + (1-\varepsilon)^2]\{(p+F\bar{A})\bar{A}-[wM+(1-w)N](B+P\bar{A})\} -C-\varepsilon E, \tag{11}$$

$$(1-\varepsilon)^2 + \varepsilon(1-\varepsilon) + M[\varepsilon(1-\varepsilon)+\varepsilon^2]-(1-\varepsilon)L, \tag{12}$$

$$2\varepsilon(1-\varepsilon) + M[\varepsilon^2 + (1-\varepsilon)^2]-\varepsilon L, \tag{13}$$

where, of course, all payoff expectations are conditional payoffs given the choice of *I*.

In the range (4.18) the expression in curved brackets in (5) and (11) is negative so that (5) is greater than (11). This explains the upward pointing arrow of firm *f*'s agent in the left column of Figure 5.1. Similarly, one can see that (6) is greater than (10) which explains the downward pointing deviation arrow of firm *f*'s agent in the right column.

For $L > 0$ one has, furthermore, (3) > (12) and due to $1 - L > M$ also (4) > (13) if ε is positive and sufficiently small. This explains the direction of the two horizontal deviation arrows and proves that $(A = 0, \bar{T})$ as well as $(A = \bar{A}, T)$ are strict equilibria of the restricted cell game after *I*. Here "restricted" indicates that only those agents are considered as active players who use different strategies in $s^p_{0,1}$ or $s^s_{0,0}$. Expressed in terms of deviation arrows an equilibrium is strict, if all deviation arrows are pointing to it. Thus the game in Figure 5.1 has two strict equilibria.

The game in Figure 5.2 results from the game in Figure 5.1 by subtracting in each column (row) firm *f's* (type *e's*) non–equilibrium payoff. This transformation preserves the best reply structure since the mixed strategy equilibrium and therefore the stability sets of the two games are identical (Harsanyi and Selten, 1988). The *stability set* of a strategy combination is defined as the set of all mixed strategy combinations to which this strategy combination is a best reply. Due to axiom BRI it is therefore possible to solve the game in Figure 5.1 by solving the game in Figure 5.2.

f \ e	$\bar{T}$	T
A=0	(5)–(11) ; (3)–(12)	0 ; 0
A=$\bar{A}$	0 ; 0	(6)–(10) ; (4)–(13)

Figure 5.2: Best reply preserving transformation of the game in Figure 5.1

In Figure 5.3 the letters X and Y are defined as

$$X = \frac{(5) - (11)}{(6) - (10)} > 0 \quad \text{and} \tag{14}$$

$$Y = \frac{(4) - (13)}{(3) - (12)} > 0. \tag{15}$$

f \ e	$\bar{T}$	T
A=0	X ; 1	0 ; 0
A=$\bar{A}$	0 ; 0	1 ; Y

Figure 5.3: Isomorphic transformation of the game in Figure 5.2

With this notation it is clear that the game in Figure 5.3 is derived from that in Figure 5.2 by dividing firm f's payoff by the positive constant (6) – (10) and type e's payoff by the positive constant (3) – (12). These positive affine transformations of utilities are covered by axiom IIT implying that we can solve the game of Figure 5.1 by solving the game of Figure 5.3.

With the help of (14) and (15) it is easy to see that the problem of solving any 2×2–bimatrix game with two strict equilibria can be reduced to solving the class of games described in Figure 5.3.

Now for $X = Y$ the game of Figure 5.3 is completely symmetric. Axiom IIT therefore forbids to select $(A = 0, \bar{T})$ or $(A = \bar{A}, T)$ as the solution. Symmetry invariance implies that the mixed strategy equilibrium is chosen as the solution which is, of course, also uniformly perfect. Thus due to axiom MO the solution is $(A = 0, \bar{T})$ for $X > Y$ and $(A = \bar{A}, T)$ for $X < Y$. Neglecting again the degenerate case $X = Y$ our results are summarised by

Lemma 5.4: *The solution of the restricted cell game after I, as described by Figure 5.1, is*

– $(A = 0, \bar{T})$, if (14) is greater than (15), and

– $(A = \bar{A}, T)$, if (14) is smaller than (15). □

If $(A = 0, \bar{T})$ is the cell game solution, it does not pay for firm f to invest into an exploratory accident. The solution is the pooling equilibrium s_0^p with no illegal waste disposal. In case of $(A = \bar{A}, T)$ as the cell game solution, the result depends on whether (3.11) or the opposite strict inequality holds: $s_{0,1}^s$ is the solution for (3.11) and $s_{0,0}^s$ for the reversed strict inequality.

Corollary 5.5: *In the range (4.14) the solution of the ε–uniformly perturbed game with $\varepsilon \in (0, 1/2)$ is*

– s_0^p, *if (i) $1–L < M$ or $N < 1–H$,*
or if (ii) $1–L > M > N > 1–H$ and $(14) > (15)$;

– $s_{0,1}^s$, *if $1–L > M > N > 1–H$, $(14) < (15)$ and (3.11);*

– $s_{0,0}^s$, *if $1–L > M > N > 1–H$, $(14) < (15)$ and if the reversed strict inequality (3.11) holds.* □

In an ε–uniformly perturbed game a solution s means, of course, that the choices according to s are realised with maximal probability. Substitution of (3) to (6) and (10) to (13) into (14) and (15) yields

$$X = -\frac{(1-\varepsilon)\{(p+F\bar{A})\bar{A}-[wM+(1-w)N](B+P\bar{A})\}}{(1-w)(1-\varepsilon)[(p+F\bar{A})\bar{A}-N(B+P\bar{A})]+w\varepsilon[(p+F\bar{A})\bar{A}-M(B+P\bar{A})]} \tag{14'}$$

and

$$Y = \frac{(1-2\varepsilon)(1-M) - L}{L}. \tag{15'}$$

Using the notation

$$X^0 := \lim_{\varepsilon\to 0} X = -\frac{(p + F\bar{A})\bar{A} - [wM + (1-w)N](B + P\bar{A})}{(1-w)[(p + F\bar{A})\bar{A} - N(B + P\bar{A})]}, \text{ and} \tag{14''}$$

$$Y^0 := \lim_{\varepsilon\to 0} Y = \frac{1 - M - L}{L} \tag{15''}$$

the limit solution of the unperturbed game can be described as follows:

Theorem 5.6: *In the range (4.14) the solution of the game with $A \in \{0, \bar{A}\}$ is*

– s^p_0, *if (i) $1-L < M$ or $N < 1-H$,*
or if (ii) $1-L > M > N > 1-H$ and $X^0 > Y^0$;

– $s^s_{0,1}$, *if $1-L > M > N > 1-H$, $X^0 < Y^0$ and (3.11),*

– $s^s_{0,0}$, *if $1-L > M > N > 1-H$, $X^0 < Y^0$ and (3.11) reversed.* □

Observe that neither the case $X^0 > Y^0$ nor $Y^0 < X^0$ can be excluded since X^0 and Y^0 are both positive.

With Lemma 5.1 and Theorem 5.6 one would have determined a unique solution for all generic subregions of the multi–dimensional parameter space Ω satisfying (4.14). Either there exists only one uniformly perfect pure strategy equilibrium – the solution is then always a pooling equilibrium – or one can select between the pooling and the signaling equilibrium by risk dominance or by payoff dominance. Actually this would be the result according to the theory of Harsanyi and Selten which defines a dominance relation by payoff dominance or by risk dominance, in case that payoff dominance does not apply. Here we investigate risk dominance also in situations where payoff dominance applies.

5.5. The solution in the range (4.15)

If either $1-L < M$ or $N < 1-H$ in the range (4.15), then the pooling equilibrium $s^p_{\bar{A}}$ is the solution. For $1-L > M$ and $N > 1-H$ the game has also a signaling equilibrium, namely $s^s_{\bar{A},1}$ in case of (3.13) and $s^s_{\bar{A},0}$ in case of the reversed strict inequality. We therefore rely on (4.15), $1-L > M$ and $N > 1-H$ when selecting the solution in what follows.

The restricted cell game after I which we consider for the comparison of $s^p_{\bar{A}}$ with $[s^s_{\bar{A},1}$ or $s^s_{\bar{A},0}]$ is shown in Figure 5.4. The not yet defined payoff entries in Figure 5.4 are

$$[(1-\varepsilon)^2 + \varepsilon^2]\{(p + F\bar{A})\bar{A} - [wM + (1-w)N](B + P\bar{A})\} - \varepsilon E - C, \tag{16}$$

$$\left.\begin{aligned}&(1-w)\{[(1-\varepsilon)^2 + (1-\varepsilon)\varepsilon][(p + F\bar{A})\bar{A} - NB + P\bar{A})] - \varepsilon E\} - C + \\ &+ w\{[\varepsilon(1-\varepsilon) + (1-\varepsilon)^2][(p + F\bar{A})\bar{A} - M(B + P\bar{A})] - (1-\varepsilon)E]\},\end{aligned}\right\} \tag{17}$$

$$\varepsilon^2 + \varepsilon(1-\varepsilon) + M[\varepsilon(1-\varepsilon) + (1-\varepsilon)^2] - (1-\varepsilon)L, \tag{18}$$

$$2\varepsilon(1-\varepsilon) + M(\varepsilon^2 + (1-\varepsilon)^2) - \varepsilon L. \tag{19}$$

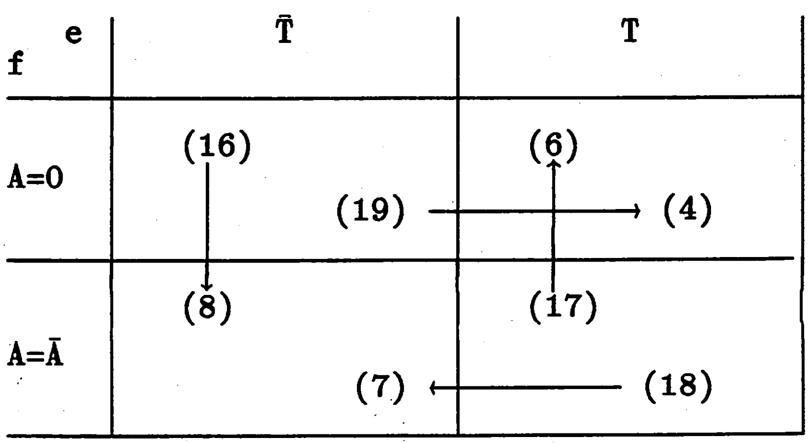

Figure 5.4: The restricted cell game after I for the comparison of $s^p_{\bar{A}}$ with $[s^s_{\bar{A},1}$ or $s^s_{\bar{A},0}]$

For $\varepsilon \in (0, 1/2)$ the payoff (8) is greater than (16) which explains the downward pointing deviation arrow in the left column. Similarly, for the range (4.15) it follows that (6) is greater than (17). Then, in the right column, the deviation arrow has to point upward. For $1-L > M$ and $L > 0$ the horizontal deviation arrows have to be as indicated in Figure 5.4. This proves that the game of Figure 5.4 has two strict equilibria, namely $(A = 0, T)$ and $(A = \bar{A}, \bar{T})$.

The next step is to transform the game resulting from Figure 5.4 by a procedure (covered by the axioms BRI and IIT) similar to that used to transform Figure 5.1 into 5.3. For that purpose define

$$U = \frac{(8) - (16)}{(6) - (17)} \quad \text{and} \tag{20}$$

$$V = \frac{(4) - (19)}{(7) - (18)} \tag{21}$$

to obtain the 2×2–bimatrix of Figure 5.5.

f \ e	$\bar{T}$	T
A=0	0 ; 0	1 ; V
$A=\bar{A}$	U ; 1	0 ; 0

Figure 5.5: The transformed restricted cell game after I of Figure 5.4

Now the same arguments as used for proving Lemma 5.4 yield

Lemma 5.7: *The solution of the restricted cell game after I — as described by Figure 5.4 — is*

– $(A = \bar{A}, \bar{T})$, if (20) is greater than (21);

– $(A = 0, T)$, if (20) is smaller than (21). □

In the range (4.15) firm f's agent after $\bar{I}$ uses $A = \bar{A}$ with maximal probability in every ε–uniformly perturbed game. If the pooling behavior $(A = \bar{A}, \bar{T})$ is therefore the solution of the cell game after I, firm f will choose $\bar{I}$, i.e. $s^p_{\bar{A}}$ is the solution of the game. For the signaling solution $(A = 0, T)$ of the cell game after I the solution depends on whether (3.13) holds or not. In case of (3.13) it is $s^s_{\bar{A},1}$ whereas it is $s^s_{\bar{A},0}$ in case of the reversed inequality.

Corollary V.8: *In the range (4.15) the solution of the ε–uniformly perturbed game with* $\varepsilon \in (0, 1/2)$ *is*

- $s^p_{\bar{A}}$ — *if (i)* $1-L < M$ *or* $N < 1-H$ *holds, or if (ii)* $1-L > M > N > 1-H$ *and (20) > (21) holds;*
- $s^s_{\bar{A},1}$ — *if* $1-L > M > N > 1-H$, *(20) < (21) and (3.13) holds;*
- $s^s_{\bar{A},0}$ — *if* $1-L > M > N > 1-H$, *(20) < (21) and if the reversed strict inequality of (3.13) holds.* □

Since

$$U = \frac{\varepsilon\{(p+F\bar{A})\bar{A}-[wM+(1-w)N](B+P\bar{A})\}}{(1-w)\varepsilon[(p+F\bar{A})\bar{A}-N(B+P\bar{A})]+w(1-\varepsilon)[(p+F\bar{A})\bar{A}-M(B+P\bar{A})]} \tag{20'}$$

and

$$V = \frac{(1-2\varepsilon)(1-M)-L}{L}, \tag{21'}$$

one has $V > U$ for positive and sufficiently small ε. This proves .

Theorem 5.9: *In the range (4.15) the solution of the game with* $A \in \{0, \bar{A}\}$ *is*

- $s^p_{\bar{A}}$, — *if* $1-L < M$ *or* $N < 1-H$ *holds;*
- $s^s_{\bar{A},1}$, — *if* $1-L > M > N > 1-H$ *and (3.13) holds;*
- $s^s_{\bar{A},0}$, — *if* $1-L > M > N > 1-H$, *and (3.13) reversed holds.* □

Observe that Lemma 5.2 and Theorem 5.9 determine identical solutions for the region $1-L > M > N > 1-H$, (8") and (4.15). Thus the signaling solution for this parameter region is very convincing since it is supported both by payoff dominance and by risk dominance. For such games it does not matter whether we rely on payoff dominance or not.

5.6. Discussion of the solution

Except for degenerate cases reflecting the boundaries of parameter regions we have solved all games of the basic game model. We ignore degenerate games since they rely on highly special assumptions for the game parameters and have therefore no practical relevance. A small change of one of the relevant parameters will usually imply that the game falls into one of the generic regions for which the solution has been derived above. Of course, one can also solve the degenerate games uniquely by applying equilibrium selection theory. But this would be a purely game theoretic exercise with hardly any economic relevance.

Let us first look at the ε–uniformly perturbed games with $\varepsilon \in (0, 1/2)$. To illustrate how the solution of such a game depends on the parameters, we use a graphical presentation of results similar to Figure 4.2.

In Figure 5.6 the major dividing line is

$$\frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} = wM + (1-w)N. \tag{22}$$

With some modifications this dividing line played a major role throughout the paper, e.g. in (3.5'), (3.10'), (3.15), (3.28). The right hand side of (22) is the ratio of what firm f can win and what it can lose from (maximal) illegal waste disposal. This ratio can be interpreted as the firm's "chance to gain" from violating the law as a percentage of fine in case of detection. The right hand side is the a priori probability for being detected as an illegal polluter ($A = \bar{A}$). If the firm's chance to gain is less than the detection probability (i.e. on the left hand side of (22)) firm f will choose $A = 0$ with maximal probability provided that it has to rely on its a priori belief. On the right hand side of (22) the choice of $A = \bar{A}$ is optimal according to the a priori belief. The solution of the game reflects this behavior by the appropriate choice of A after $\bar{I}$ with maximal probability as well as by the pooling solution of the cell game after I.

Other essential dividing lines are $M = 1 - L$ and $N = 1 - H$. For $M > 1 - L$ even type e of controller c prefers not to thoroughly investigate an exploratory accident $(\bar{T})$ since his

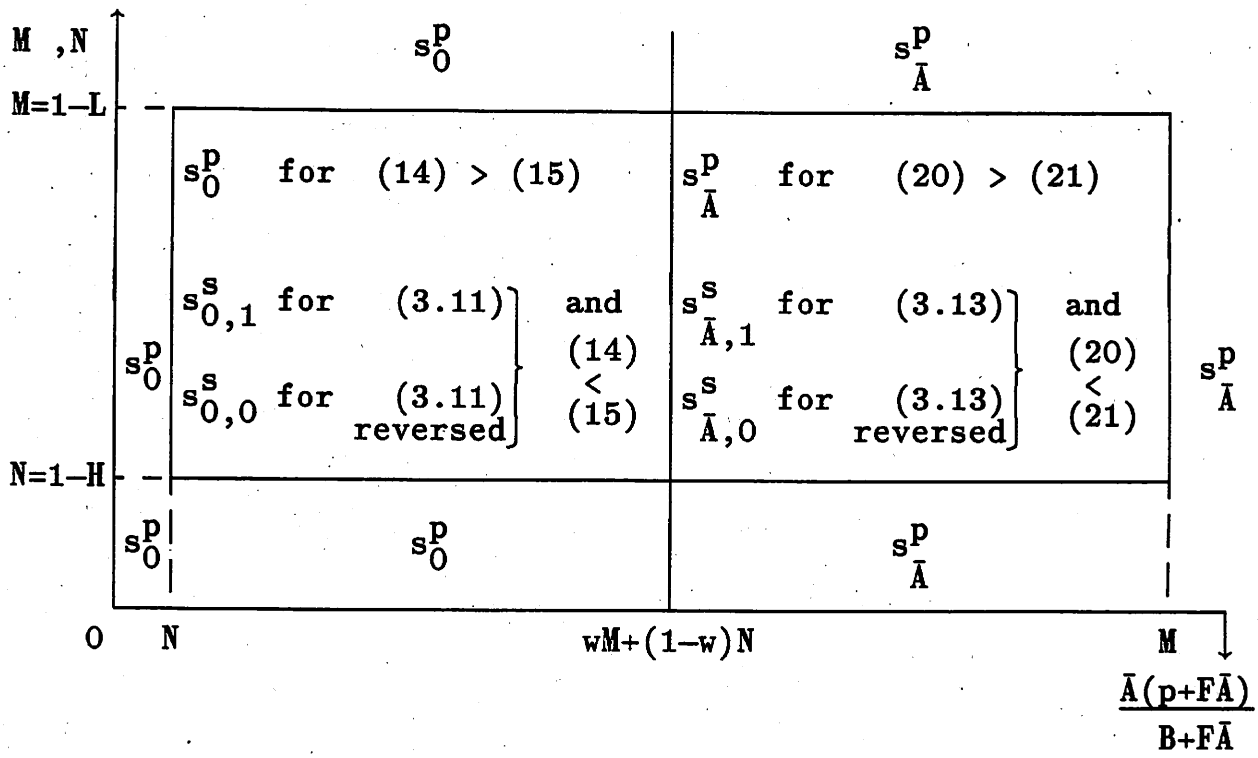

Figure 5.6: Solution of an ε–uniformly perturbed game with $\varepsilon \in (0, \frac{1}{2})$ in the $\left[\frac{\bar{A}(p+F\bar{A})}{B+P\bar{A}}, (M,N)\right]$ – plane except for border cases.

detection probability M for $A = \bar{A}$ exceeds his payoff $1 - L$ in case of $A = 0$ and a thorough exploration (T). Thus for $M > 1 - L$ only a pooling equilibrium is possible. For $N < 1 - H$ on the other hand even type n of controller c prefers T over $\bar{T}$ since his detection probability N for $A = \bar{A}$ is too low compared to his payoff $1 - H$ for T if T induces the choice of $A = 0$ by firm f. Below $N = 1 - H$ one can therefore have only a pooling solution.

The region where the solution of an ε–uniformly perturbed game is most reactive to the game parameters is the rectangular area determined by

$$1\text{–}L > M > N > 1\text{–}H \quad \text{and} \tag{23}$$

$$N < \frac{A(p + F\bar{A})}{B + P\bar{A}} < M. \tag{24}$$

The latter range is again subdivided by (22). On the left hand side of (22) it depends on the relation of X and Y whether the solution is a pooling or a signaling equilibrium. Only if the latter is true, condition (3.11) or its reverse determine firm f's initial choice between I and $\bar{I}$. Inequality (3.11) simply says that firm f prefers I over $\bar{I}$ if the solution of the cell game after I is of the signaling type. Similarly, on the right hand side of (22) the variables U and V determine whether the solution is a pooling or signaling equilibrium. Here inequality (3.13) or its reverse matter only in case that the pooling equilibrium is not selected. Like (3.11) the condition (3.13) requires that I is a best reply given that the solution of the cell game after I is the signaling equilibrium.

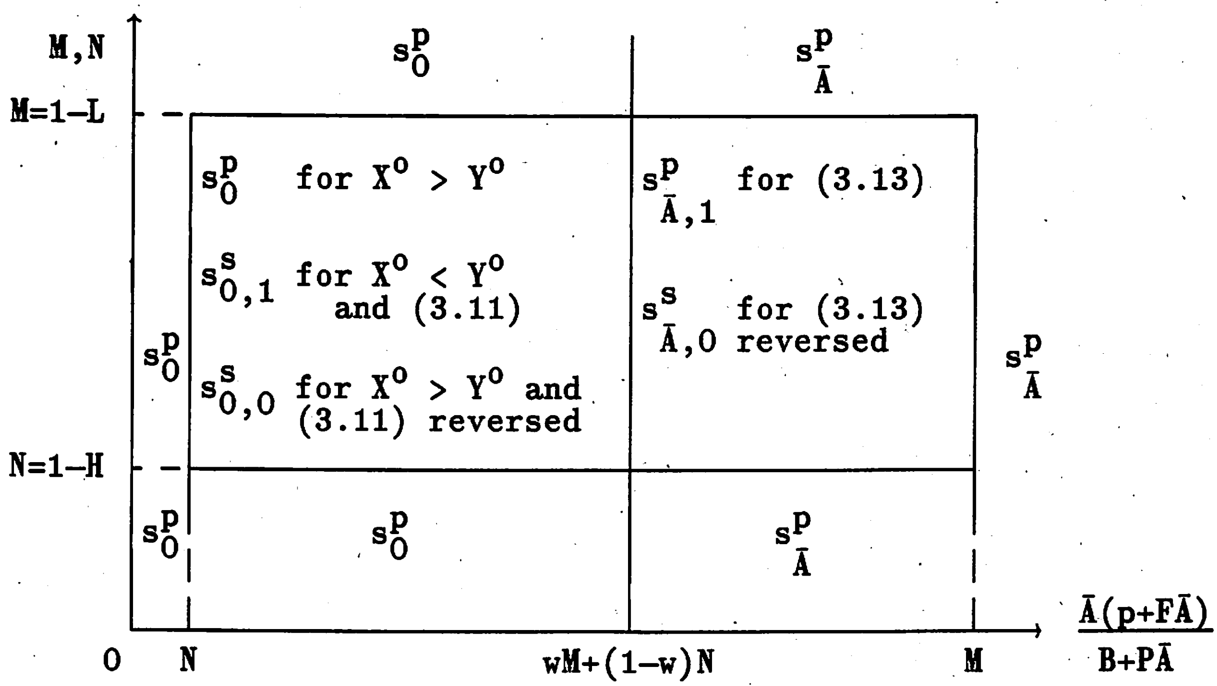

Figure 5.7: Limit solution for the unperturbed game in the $\left[\frac{\bar{A}(p+F\bar{A})}{B+P\bar{A}}, (M;N)\right]$ – plane

The limit solution of the unperturbed game, i.e. the solution of our game model with $A \in \{0, \bar{A}\}$, is illustrated in Figure 5.7 in the same way as in Figure 5.6 for its uniformly perturbed games. Compared to the uniformly perturbed games there is a surprising asymmetry between the left and right hand side of the rectangular area, described by (23) and (24). While to the right of (22) a signaling solution prevails whenever such behavior is a uniformly perfect equilibrium, on the left hand side of (22) the solution is of the signaling type only for $X^0 < Y^0$. The variable X^0, defined by (14"), can be described as the relation of the a priori expected loss

$$[wM + (1-w)N](B + P\bar{A}) - (p + F\bar{A})\bar{A} \tag{25}$$

of $A = \bar{A}$ and the expected profit

$$(1 - w)[(p + F\bar{A})\bar{A} - N(B + P\bar{A})] \tag{26}$$

due to unqualified monitoring by the non–expert type n of controller c. Similarly, Y^0, defined by (15"), relates the difference $(1 - L) - M$ to L. Here $(1 - L) - M$ is type e's incentive to choose T if firm f reacts to T by $A = 0$ and to $\bar{T}$ by $A = \bar{A}$ whereas L is the incentive to choose $\bar{T}$ if firm f reacts to T and $\bar{T}$ in the same way. This shows that there are intuitive interpretations for the variables X^0 and Y^0 which determine whether the solution on the left hand side of (22) in the rectangular area (23) and (24) is of the signaling or the pooling type.

It is interesting to observe that such different game parameters as p and F, determining the cost advantage of illegal waste disposal as well as the parameters B and P defining the fine in case of detected illegal waste disposal, influence the solution only via the term

$$\frac{\bar{A}(p + F\bar{A})}{B + P\bar{A}} \tag{27}$$

which also captures the impact of the level $\bar{A}$ of illegal waste disposal. One can very well imagine compensating changes of the various parameters constituting (27) which nevertheless leave the value of (27) and thereby the solution unchanged. This indicates that there exist trade off relationships which could be economically and politically very important. So, for example, our solution allows to calculate the necessary increases for the fine parameters B and/or P to offset a higher profitability of illegal waste disposal due to higher values of p and F or due to a larger waste amount $\bar{A}$.

6. Conclusions

A major motivation for our study was to demonstrate the applicability of non–cooperative game theory for analysing illegal pollution, especially the strategic problems involved in enforcing environmental control. That is why we have used quite different methods ranging from equilibrium scenarios to concepts yielding unique solutions. The approach taken here is by no means confined to illegal pollution but easily lends itself to analysing other strategic and institutional problems of environmental management such as strategic interaction between private polluters and pollutees.

In our view, it is very important to take into account that environmental policy has to be determined in situations of uncertainty and private information. For example, important aspects of pollution damages can only be evaluated by the pollutees themselves and are therefore private information. Another information deficit on which our model focuses is that polluters are poorly informed about the effectiveness of environmental control by public authorities. Unlike in conventional principal–agent–relationships (for instance, Hart and Holmström, 1987) neither the firm nor the inspector can be viewed as a subordinate of the other player. Nevertheless the monitoring of private employment contracts can easily be analysed by appropriately reinterpreting the assumptions of our model.

Private information as such does not always lead to signaling. For signaling to occur it is necessary, in addition, to have a sequential decision process which allows a less informed player to elicit the superior information of other players by inferences from their earlier activities. Signaling aspects of environmental problems therefore always require an extensive game form like that in the preceding analysis.

To demonstrate that environmental policy might be improved by institutional arrangements related to signaling assume that the government considers a new policy instrument with only poor information about the polluter's abatement costs. If these costs are low, the new measure would lead to welfare improvement, whereas high abatement costs, as probably claimed by self–interested polluters, would suggest to refrain from such a measure. In order to elicit cost information, the government might introduce the envisaged measure either on a very small scale or only locally. One can easily imagine market institutions where it does not pay for a polluter to pretend high abatement costs even though they are actually low. Thus the reactions to the exploratory move, closely resembling the move I in Figure 2.1, may allow for updating the government's cost estimates, thus changing its subjective probability for high or low abatement cost and increase its chances for implementing welfare improving environmental management.

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