Does Money Demand Matter for Business Cycle Persistence?

Michael Gail*

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Abstract

Macroeconomists have devoted much effort to the setup of models that are able to generate persistent reactions of real macroeconomic aggregates to money growth shocks in stochastic dynamic general equilibrium (DGE) models with nominal rigidities. This has turned out to be quite difficult in models with price staggering as the only nominal rigidity as stressed by Chari, Kehoe and McGrattan (2000). Most papers show that output is above the steady state only as long as prices are fixed for the firms. In this article particular attention is given to the role of money demand and to its interaction with the labor supply elasticity. To this end a cash-in-advance- (CIA) as well as a money-in-the-utility-function- (MIU) model will be considered using a Greenwood-Hercowitz-Huffman (GHH) utility function to analyze the ability of the models to generate persistence. It turns out that persistent reactions emerge only with a high Frisch elasticity *and* a money demand function that depends on the interest rate. The results highlight the importance of the way money is introduced in a New Keynesian DGE model.

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*Universität Siegen, Fachbereich Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht, D-57068 Siegen/Germany
Phone: ++49 271 740 3217, Fax: ++49 271 740 3240, Email: Michael.Gail@uni-siegen.de

1 Introduction

Can monetary shocks generate persistent responses of inflation and output? This question has been addressed in a number of papers in the last few years. The most prominent paper is the one of Chari, Kehoe and McGrattan (2000). They conclude that standard models with staggered prices generate a positive output reaction only for the time of exogenous price stickiness. Several attempts have been made to challenge this result.

Christiano, Eichenbaum and Evans (2005) proposed a DGE model that can generate the observed persistence of monetary shocks in US data. With an average duration of two to three quarters wage contracts are the critical nominal friction, not price contracts. If the model is expected to display inertia in inflation and output variable capital utilization is most important. In order to explain the reaction of all variables they include habit persistence in consumption as well as adjustment costs in investment. Since these authors use a limited information econometric strategy that is not yet common in the literature the results are difficult to compare to other studies.

Dotsey and King (2006) stress the importance of variable capital utilization as well. They demonstrate that persistence is possible even in a sticky price model that incorporates labor supply variability through changes in employment and produced inputs as intermediate goods. All these ingredients together produce a flat reaction of real marginal costs to a money growth shock. This reduces the extent of price adjustments of the firms. Unfortunately, this gradual adjustment of the price level is responsible for the rise in the nominal interest rate: the model does not display the liquidity effect.

Bergin and Feenstra (2000) use a modified DGE model with intermediate goods and 'translog' preferences. These preferences are given by a non-CES aggregator for intermediate goods which is a substitute for the Dixit and Stiglitz (1977) aggregator. They show that intermediates in production are very important to generate persistent output responses but they also find that translog preferences play an important role: The higher the share of intermediates in production the higher the persistence.

Intermediates are also important in the work of Huang, Liu and Phaneuf (2001). They evaluate the performance of staggered wage models in relation to staggered price models. They show that only a model with intermediates, staggered price and staggered wage setting can explain persistent responses of output and, depending on the share of intermediates in production, a weak but slightly positive response of the real wage to a monetary shock as it is observed empirically in the postwar period in the US.

Huang and Liu (2001*a*) demonstrate the importance of such an input-output structure in a two-country model to explain the significant cross-country correlations in aggregate output and the persistent deviations of real exchange rates from purchasing power parity.

In a model with a vertical input-output structure and price staggering Huang and Liu (2001*b*) show that the higher the number of stages of production the more persistent the output response. With a sufficient number of stages the response can even be arbitrarily large, given that the share of intermediates is one at all stages of production.

Dib and Phaneuf (2001) discuss a model with price staggering instead of wage staggering. In a variant of the model with a nominal rigidity given by costly price adjustment and a real rigidity which emerges by adjusting the labor input output, hours and real wages show a persistent reaction to a monetary shock. Moreover, the model can explain the decline in hours worked after a productivity shock as it is observed in US postwar data.

In this paper special attention is given to the way money is introduced in a DGE model. To this end a CIA- as well as a MIU-model is analyzed. The importance of the way money demand is modeled in a DGE model has not yet been recognized in the literature. The results obtained here speak in favor of the setup. First, persistent output and inflation responses depend only in part on the value of the Frisch elasticity, as claimed by Andersen (1998) as well as by Chari, Kehoe and McGrattan (2000). Second, persistence depends also crucially on the implied money demand function. Persistent output reactions emerge only in a MIU-model with GHH preferences *and* a high value of the Frisch elasticity. In a CIA-model this result does not hold.

These results make clear that it matters a lot how money is introduced. The equivalence result for CIA- and MIU-models in Feenstra (1986) cannot be generalized to a broader setup where utility depends also on leisure and where prices are set in a staggered way. In addition, the paper shows that the results in Chari, Kehoe and McGrattan (2000) have to be interpreted more carefully as these authors only analyze a MIU-model.

The paper is organized as follows: Section 2 describes in detail the different models and the calibration. In Section 3 impulse responses are discussed for the CIA- and the MIU-model. Section 4 concludes and gives some suggestions for future research.

2 The Models

2.1 The Household

Two different setups will be considered. In the first setup, a CIA-model is analyzed while in the second a MIU-model will be evaluated.

Preferences of the representative household depend on consumption (c_t) and leisure ($1 - n_t$). The momentary utility function in the CIA-setup is the one used by King and Wolman (1999) and it is given by

$$u\left(c_t, n_t\right) = \frac{\left[c_t - \frac{\theta}{1+\gamma} n_t^{1+\gamma}\right]^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

θ and γ are positive parameters, σ determines the degree of risk aversion. This function is familiar from the analysis of Greenwood, Hercowitz and Huffman (1988) and has been labeled GHH preferences. In standard real business cycle models it implies that hours worked only depend on the real wage and not on consumption; there are no wealth effects. We will analyze whether this result changes for a CIA-model.

Under a MIU-specification the corresponding GHH function to (1) is given by

$$u\left(c_t, \frac{M_t}{P_t}, n_t\right) = \frac{\left[\left(\eta c_t^{\nu} + (1-\eta)\left(\frac{M_t}{P_t}\right)^{\nu}\right)^{\frac{1}{\nu}} - \frac{\theta}{1+\gamma} n_t^{1+\gamma}\right]^{1-\sigma} - 1}{1-\sigma} \tag{2}$$

The MIU-specification was - among others - proposed by Sidrauski (1967). Real money balances M_t/P_t are included in the utility function since they facilitate transactions. They are embedded into a CES function with consumption. η is a share parameter and ν determines the interest elasticity of the implied money demand function. Note that for $\nu = \eta = 1$ the MIU-specification is identical to the CIA-setup. The nonseparability allows us to consider the influence of money demand distortions on the dynamics of consumption and labor.

The intertemporal optimization problem for the household is given by maximizing lifetime utility subject to an intertemporal budget constraint. In the case of utility function (1) the household also faces a CIA-constraint. It has access to a bond market and it can hold money. Its budget constraint is therefore given by

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1 + R_{t-1}) B_{t-1} + \Xi_t + M_t^s \tag{3}$$

where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \tag{4}$$

is the sum of the nominal profits $\Xi_{j,t}$ of the intermediate goods producing firms. The household decides to use its wealth for nominal consumption expenditures $P_t c_t$ during period t and for money balances M_t and bonds B_t at the end of period t. The household has several sources of its wealth. It receives a labor income $P_t w_t n_t$ working in the market at the real wage rate w_t and can spend its money holdings carried over from the previous period (M_{t-1}) in addition to its bonds B_{t-1} including the interest payments $(1 + R_{t-1}) (B_{t-1})$ where R_{t-1} is the nominal interest rate. Finally, the household receives a monetary transfer M_t^s from the monetary authority and profits from the intermediate goods firms Ξ_t, respectively. The transfer is equal to the change in money balances, i.e.

$$M_t^s = M_t - M_{t-1} \tag{5}$$

where M_{t-1} is money at the end of period $t-1$. In the CIA-model consumption of the household can only be financed by cash balances left over from the previous period and by the monetary transfer. The CIA-constraint is therefore given by[1]

$$P_t c_t \leq M_{t-1} + M_t^s \tag{6}$$

The familiar result that the efficiency conditions for consumption and labor imply the equality of the marginal rate of substitution between consumption and labor and the real wage does not hold here. This is due to the presence of the CIA-constraint. Instead the condition reads as follows:

$$w_t = -\frac{\frac{\partial u(c_t,n_t)}{\partial n_t}}{\frac{\partial u(c_t,n_t)}{\partial c_t}} (1 + R_{t-1}) \tag{7}$$

First, there is an influence of the nominal interest rate R_{t-1}. Second, it is the lagged interest rate that matters so that the dynamics of the real wage

[1] The formulation of the CIA-constraint, the monetary transfer and the intertemporal budget constraint is consistent with the timing in Walsh (1998), pp. 100-102.

will change. This condition is crucial for understanding the implications of the CIA-setup.
The marginal utility of consumption is given by $(1 + R_{t-1}) \lambda_t$ so that the nominal interest rate acts like a tax on consumption. λ_t is the Lagrange multiplier of the budget constraint.
The efficiency condition for bond holdings implies a relation between the nominal interest rate and the price level. Rearranging terms results in

$$(1 + R_t) = E_t \left(\frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \right) \tag{8}$$

Assuming that the Fisher equation is valid the real interest rate r_t is implicitly defined as

$$(1 + r_t) = E_t \left(\frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \right) \tag{9}$$

since P_{t+1}/P_t is an approximation for expected inflation.

In case of the MIU-model the CIA-constraint is dropped since money demand will be determined endogenously through the derivative with respect to m_t. The marginal utility of consumption is then just equal to the shadow price λ_t, there is no consumption tax working through the nominal interest rate. But in the efficiency condition for money the marginal utility of real balances has to be considered. This derivative determines the endogenous money demand function. Combining the optimum conditions for consumption, bonds and money we can derive the following equation:

$$\frac{\partial u\left(c_t, m_t, n_t\right)}{\partial m_t} = \frac{\partial u\left(c_t, m_t, n_t\right)}{\partial c_t} \frac{R_t}{1 + R_t} \tag{10}$$

This specification can be estimated to derive the empirical money demand function. A detailed analysis will be presented in the calibration section.

There are two important implications that can be summarized: First, the real wage rate will be determined by the usual marginal rate of substitution between consumption and labor, in contrast to the additional dynamics in the CIA-model (see (7)).

$$w_t = -\frac{\frac{\partial u(c_t, n_t)}{\partial n_t}}{\frac{\partial u(c_t, n_t)}{\partial c_t}} \tag{11}$$

Second, the implied money demand function depends directly on the nominal interest rate (see (10)).

2.2 The Finished Goods Producing Firm

The firm producing the final good $c_t = y_t$ in the economy uses $c_{j,t}$ units of each intermediate good $j \in [0,1]$ purchased at price $P_{j,t}$ to produce c_t units of the finished good. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with $\epsilon > 1$.

$$c_t = \left(\int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \tag{12}$$

The firm maximizes profits choosing $c_{j,t}$ given the above production function and given the price P_t. The first order conditions for each good j imply

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} c_t \tag{13}$$

where $-\epsilon$ measures the constant price elasticity of demand for each good j. Because the firm operates under perfect competition profits will be zero. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price P_t that is consistent with this requirement is given by

$$P_t = \left(\int_0^1 P_{j,t}^{(1-\epsilon)} dj \right)^{1/(1-\epsilon)} \tag{14}$$

In the case that prices are fixed for just two periods and assuming that all price adjusting producers in a given period choose the same price the consumption aggregate can be written as

$$c_t = c\left(c_{0,t}, c_{1,t}\right) = \left(\frac{1}{2} c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2} c_{1,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \tag{15}$$

where $c_{j,t}$ can then be interpreted as the quantity of a good consumed in period t the price of which was set in period $t-j$. Similarly, in the two period price setting case to be explored in detail in the next section the price equation simplifies. With prices set for two periods half of the firms adjust their price in period t and half do not. Moreover, all adjusting firms choose the same price. Then $P_{j,t}$ is the nominal price at time t of any good the price of which was set j periods ago and P_t is the price index at time t and is given by

$$P_t = \left(\frac{1}{2}P_{0,t}^{1-\epsilon} + \frac{1}{2}P_{1,t}^{1-\epsilon}\right)^{1/(1-\epsilon)} \tag{16}$$

2.3 The Intermediate Goods Producing Firm

Intermediate good firms consist of a producing and a pricing unit. The producing unit operates under a technology that is linear in labor $n_{j,t}$ and subject to random productivity shocks a_t.[2]

$$y_{j,t} = c_{j,t} = a_t n_{j,t} \tag{17}$$

Here $n_{j,t}$ is the labor input employed in period t by a firm who set the price in period $t-j$. Firms always meet the demand for their product, that is $y_{j,t} = c_{j,t}$. Those firms who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally.

The pricing unit sets prices to maximize the present discounted value of profits whereas the producing unit chooses labor to minimize costs. Real marginal costs are then given by $\psi_t = w_t/a_t$.[3] With a relative price defined by $p_{j,t} = P_{j,t}/P_t$ real profits $\xi_{j,t} = \Xi_{j,t}/P_t$ for a firm of type j are equal to

$$\xi_{j,t} = \xi\left(p_{j,t}, c_t, \psi_t\right) = p_{j,t}^{-\epsilon} c_t \left(p_{j,t} - \psi_t\right) \tag{18}$$

This equation is derived by inserting the demand function for the intermediate goods and real marginal costs. When prices are fixed for two periods the firm has to take care for the effect of the price chosen in period t on current and future profits. The price in period $t+1$ will be affected by the gross inflation rate Π_{t+1} between t and $t+1$ ($\Pi_{t+1} = P_{t+1}/P_t$). The optimal relative price has to balance the effects due to inflation between profits today and tomorrow. Thus, the intertemporal maximization problem is formally given by

$$\begin{aligned} &\max_{p_{0,t}} E_t \left[\xi\left(p_{0,t}, c_t, \psi_t\right) + \beta \frac{\lambda_{t+1}}{\lambda_t} \xi\left(p_{1,t+1}, c_{t+1}, \psi_{t+1}\right)\right] \\ &\text{s.t.} \quad p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \end{aligned} \tag{19}$$

[2] There are no diminishing returns to labor.

[3] Note that the wage rate is perfectly flexible in a competitive input market. So there is no index j for w_t and P_t which means that these variables are not firm-specific.

The term λ_{t+1}/λ_t is the pricing kernel.[4] The efficiency condition for the optimal price $P_{0,t}$ implies a forward-looking price setting equation which is similar to that in Taylor (1980).

$$P_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t P_t^{\epsilon} c_t \psi_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon} c_{t+1} \psi_{t+1}}{\lambda_t P_t^{\epsilon-1} c_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon-1} c_{t+1}} \tag{20}$$

The optimal price $P_{0,t}$ depends on the current and future real marginal costs, the current and future price level, current and future consumption as well as today's and tomorrow's interest rate which operate through λ_t, λ_{t+1}.[5] Finally, aggregate labor demand must be equal to the aggregate labor supply of the household.[6]

$$n_t = \frac{1}{2} n_{0,t} + \frac{1}{2} n_{1,t} \tag{21}$$

2.4 Market Clearing Conditions and Other Equations

It is well known that models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. To escape this problem the household budget constraint is dropped and bonds are set to zero: $b_t = 0$ for all t.[7] Note that due to Walras' law the intertemporal budget constraint will also hold in equilibrium.

In the CIA-model the implicit money demand function is derived by substituting M_t^s in the CIA-constraint - holding with equality. This implies:

$$M_t = P_t c_t \tag{22}$$

It is essentially a quantity theoretic type of money demand. Note that in this case money demand does not depend on the interest rate.

In the MIU-model the efficiency condition for money determines the money demand function (see the discussion of (10)).

The markup μ_t is just the reciprocal of real marginal cost so that

$$\mu_t = \frac{1}{\psi_t} \tag{23}$$

[4]See Dotsey, King and Wolman (1999), p. 659-665.

[5]This equation is exactly equal to that in Walsh (1998), p. 197, when using (8) for the nominal interest rate factor.

[6]The factor 0.5 shows up because $n_{j,t}$ is labor hired per j-type firm and half the firms are of each type.

[7]See Flodén (2000), p. 1413. He argues that bonds are introduced to determine the nominal interest rate.

2.5 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent but non-permanent effects the level of money follows an AR(2)-process. Assume that money grows at a factor g_t:

$$M_t = g_t M_{t-1} \tag{24}$$

Suppose that $\widehat{g}_t$ follows an AR(1)-process $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_{g_t}$. Then money will follow an AR(2)-process.[8] Since inflation is zero at the steady state money growth will also be zero ($g = 1$).

To simplify the exposition the productivity shock a_t will not be considered. So a_t is constant and equal to a.

2.6 Calibration

To compute impulse response functions the parameters of the model have to be calibrated. It is possible to specify either β or r exogenously. Here β will be set to 0.99 which implies a value of $r = 0.0101$ per quarter. This is in line with other values used for the real interest rate in the literature. ψ and μ can be determined by fixing a value for the elasticity of the demand functions for the differentiated products. This elasticity is set equal to 6 so that the static markup is given by $\mu = \epsilon/(\epsilon - 1) = 1.2$ which is the mean value found for μ in the study of Linnemann (1999) about average markups. a is set to 1. Either n or c have to be set exogenously to calculate $c = an$. Because more information is available about hours worked, n is specified to be equal to 0.25 implying that agents work 25 % of their time.[9]

σ, the parameter governing the degree of risk aversion, is set to 2. In the benchmark case, γ will be set to 2. In the sensitivity analysis this value is changed to 0.1.

In the MIU-model the parameters ν and η are calibrated by estimating an empirical money demand function. The general form of this function is implied by the efficiency conditions of the household. This functional form

[8] A hat ($\widehat{\ }$) represents the relative deviation of the respective variable from its steady state. ρ_g lies between 0 and 1 and ϵ_{g_t} is white noise.

[9] Cooley and Hansen (1995) use $n = 0.31$.

is obtained by solving (10) for m_t and taking logarithms:

$$\ln m_t = \frac{1}{\nu - 1} \ln \frac{\eta}{1-\eta} + \frac{1}{\nu - 1} \ln \left(\frac{R_t}{1+R_t} \right) + \ln c_t \tag{25}$$

Estimates of Chari, Kehoe and McGrattan (2000) reveal that $\eta = 0.94$ and $\nu = -1.56$. They use US data from Citibase covering the period 1960:1-1995:4. They run a regression where the log of the consumption velocity $\ln(m_t/c_t)$ depends on the log of the interest rate $\ln(R_t/(1+R_t))$ and a constant. The parameter ρ_g of the exogenous money growth process is set to 0.5. The same value is used by Cooley and Hansen (1995).

3 Impulse Response Functions

The models are solved using an extended version of the algorithm in King, Plosser and Rebelo (2002) that allows for singularities in the system matrix of the reduced model. This algorithm builds upon the Blanchard and Kahn (1980) approach for solving a system of linear stochastic difference equations. The theoretical background is explored in King and Watson (1999) whereas computational aspects and the implementation are discussed in King and Watson (2002).

In the next two subsections impulse responses of the MIU- and CIA-model variables to a 1% shock to the money growth rate will be discussed. Figures 1 and 2 present the reaction of selected variables to this shock. In the benchmark case the Frisch elasticity is equal to 0.5 while it is equal to 10 in the sensitivity analysis. The benchmark impulses are given by the solid lines.

3.1 MIU-Model

Figure 1 displays the impulse responses for the MIU-model. On the one hand we can see that there are cyclical responses of aggregate consumption c_t and real marginal costs ψ_t in the benchmark calibration. On the other hand inflation and the nominal interest rate show a higher degree of persistence. We use the ratio of the period $t+1$ reaction of a variable to the period t reaction as a metric of persistence as proposed by Andersen (2004) for two period contracts and defined as the contract multiplier in Huang and Liu (2002). This implies a value of 0.17 for R_t and of 0.67 for Π_t. The value

for inflation is quite high compared to Andersen's results.[10] Unfortunately, the nominal rate rises so that the model does not explain the liquidity effect. There is also no inertia in inflation beyond the second period. The responses of c_t and ψ_t are not persistent at all.
In the literature several authors argue in favor of models generating flat marginal cost curves because then there is little incentive for firms to raise prices. Hence, money growth shocks can have persistent effects on output. In the case of a GHH utility function the static steady state elasticity of real marginal cost with respect to output is constant and equal to γ.

$$\frac{\partial \psi}{\partial c}\frac{c}{\psi} = \gamma \tag{26}$$

In the benchmark case γ was calibrated to be equal to 2. Changing this value to 0.1 would considerably reduce this elasticity and would probably enhance the persistence effects of money growth shocks in the model. But a low value for this elasticity implies at the same time a high Frisch elasticity which is given by $1/\gamma$ and which is thus equal to 10.[11] Does the model give any support for this reasoning? The dashed lines in Figure 1 give the answer. Now all variables display very strong persistence after a money growth shock. The contract multiplier of c_t is now equal to 0.55 while that of ψ_t is given by 0.53. Real marginal costs are flat showing only a 0.14% initial deviation from the steady state value. Note that this is very close to $\gamma = 0.1$ which highlights the influence of the output elasticity of ψ_t. Inflation displays a hump as can be found in the data and the contract multiplier is now 1.54. The nominal interest rate counterfactually rises again but the initial response is weaker and the contract multiplier rises to 0.46.

Is there some intuition behind this result? To this end it is useful to look at the real wage rate. As real marginal costs are proportional to the real wage and the response of the optimal price of price setting firms is determined largely by the reaction of real marginal costs it is useful to examine (11)

[10]His values for output range between 0.55 and 0.87. A variable that is cyclical is not persistent at all. Note that Chari, Kehoe and McGrattan (2000) use a different definition of the contract multiplier.

[11]Compared to empirical estimates of the Frisch elasticity this value is too high. But as it is the purpose of the paper to analyze the interaction of the money demand and the labor supply elasticity this choice is justified.

carefully. It is repeated here for convenience.[12]

$$w_t = \psi_t = -\frac{\frac{\partial u(c_t,n_t)}{\partial n_t}}{\frac{\partial u(c_t,n_t)}{\partial c_t}} \tag{27}$$

Suppose there is an expansionary money growth shock. This leads to an increase in aggregate demand because prices are sticky. Those firms who cannot adjust prices face relatively higher demand and thus hire more workers. The household has to work harder so that n_t goes up. This results in an increase in $-\partial u/\partial n_t$ because working more means a higher disutility of work.[13] Meanwhile c_t rises as well leading to a fall in $\partial u/\partial c_t$. Thus $(-\partial u/\partial n_t) / (\partial u/\partial c_t)$ goes up and the real wage will rise. This in turn means higher real marginal costs and firms who can adjust prices will choose to increase their prices fully. At the end of the contract duration of two periods all firms have had the chance to adjust prices. Thus the maximum increase in the price level occurs in the second period. Accordingly inflation is hump-shaped.
For the GHH utility function the above condition simplifies considerably:

$$w_t = \psi_t = \theta n_t^{\gamma} \tag{28}$$

We see that the reaction of labor is proportionately translated into the reaction of w_t which is additionally determined by the elasticity of real marginal costs with respect to output γ. In the benchmark case γ is equal to 2 so that the real wage response is a multiple of the reaction of labor. This can explain the strong initial deviation of ψ_t from steady state (2.6%). So it comes at no surprise that for a low value of γ real marginal costs react moderately. This gives rise to persistent reactions of consumption and inflation as explained above.

3.2 CIA-Model

Figure 2 visualizes the impulse responses for the CIA-model in the benchmark case and for the sensitivity analysis. Again the solid lines represent the benchmark results.

We can see that in the benchmark case c_t and ψ_t are again cyclical and not persistent. But their initial reaction is much weaker compared to the

[12] Note that $a = 1$.
[13] Note that $\partial u/\partial n_t$ is negative.

MIU-setup. The nominal interest rate and inflation are more persistent in comparison to the MIU-model. The contract multiplier for Π_t is 1.00 and for R_t it is equal to 0.73. There is also a bit more inertia in inflation. Note that the initial response of R_t is much stronger than in the MIU-setup. But the nominal rate rises again so that there is no liquidity effect here either.

The dashed lines in Figure 2 are the results for a low output elasticity of real marginal costs. Can real persistence also be enhanced in the CIA-setup? The answer is no. There is now a slightly stronger and a smoother reaction of aggregate consumption but it is again cyclical approaching the new steady state from below. Real marginal costs are no longer cyclical, instead they display a reduced initial reaction and have a contract multiplier equal to 0.48. The nominal rate shows a reduced reaction in the second quarter which lowers the multiplier a bit to 0.67. But overall the reaction is more persistent because it takes more time for the nominal rate to approach the new steady state. Inflation is now hump-shaped implying a considerable increase in the contract multiplier (1.42).

What is the reason for this result? Why does a high Frisch elasticity not enhance the persistence as in the MIU-model? Again it is useful to examine (7) carefully. It is repeated here for convenience.[14]

$$w_t = \psi_t = -\frac{\frac{\partial u(c_t,n_t)}{\partial n_t}}{\frac{\partial u(c_t,n_t)}{\partial c_t}} \left(1 + R_{t-1}\right) \tag{29}$$

We see that the dynamics in period t are identical to those of the MIU-model since the nominal rate R_{t-1} is still on the steady state path. In period $t+1$ however the rise in R in period t will further increase the response of the real wage. At the same time the household will reduce work effort so n_{t+1} goes down leading to a decrease in the marginal disutility of work $\partial u/\partial n_{t+1}$. Consumption will also fall leading to a rise in the marginal utility of consumption. This will cause $\left(-\partial u/\partial n_{t+1}\right)/\left(\partial u/\partial c_{t+1}\right)$ to decrease so that we observe downward pressure on the real wage.
For the GHH utility function (29) has a very simple form which is given by

$$w_t = \psi_t = \theta n_t^{\gamma} \left(1 + R_{t-1}\right) \tag{30}$$

The initial rise in labor n_t is again proportionately translated into the reaction of w_t depending further on the Frisch elasticity γ. But we also observe wealth

[14]Note again that $a = 1$.

effects which are captured by the change in the nominal interest rate. The rise in R_t tends to increase the real wage while the fall in n_{t+1} causes w_{t+1} to fall. The overall effect is a stronger reaction of real marginal costs even for low values of γ. The initial response of ψ_t is 0.29 which is more than twice as high as in the MIU-model. In turn this leads to a stronger increase in the price of firms who can adjust and hence less real persistence in consumption.

This dynamic response is due to the CIA-setup and the implied money demand function. Obviously, it does not suffice to have a low output elasticity of real marginal costs to explain persistent output responses to a money growth shock. The results obtained here suggest that the reason is the implied quantity theoretic money demand function since the literature focuses exclusively on the MIU-setup. The inclusion of a CIA-constraint alters significantly the dynamics of the model which is very obvious from (30). This leads to more complicated dynamics of real marginal costs and of consumption.

This leads to the conclusion that two conditions have to be fulfilled in order to enable a DGE model with Taylor price staggering to generate persistent output and inflation responses: First, the Frisch elasticity must be high, and second, the money demand function must depend on the interest rate. Only one of these ingredients is not enough to generate persistence. This refines results in the literature, for example in Ascari (2003). Ascari investigates only MIU-specifications and concludes that a high Frisch elasticity is crucial for persistent output reactions in a price staggering model. Similarly, Chari, Kehoe and McGrattan (2000) study a MIU-model and use a utility function that is separable in all arguments in their sensitivity analysis. They also point out the role of a high Frisch elasticity for a persistent output reaction.

4 Conclusions

In light of the main question of this paper it can be concluded that persistent reactions of output and prices to a money growth shock can only be explained in a MIU-model with a high Frisch elasticity. In a CIA-economy a high value of this elasticity cannot generate persistence in macroeconomic aggregates. It is thus of fundamental importance how money is introduced in DGE models of the business cycle.

An interesting future direction of research is to study models that include capital accumulation. Chari, Kehoe and McGrattan (2000) do not find any persistence at all in models with capital. As they consider only MIU-models it would be of special interest whether their results change in a CIA-model.

It is worthwhile to analyze different price staggering mechanisms. Kiley (2002) finds that persistence in models where prices are fixed for a specified period of time (Taylor staggering) is different from persistence in models where there is a constant probability that firms are able to adjust their prices (Calvo staggering). We could explore the implications in a CIA-model.

Another promising line of research is to analyze open economy models. Ghironi (2002) has shown that once openness is taken into account a sticky price model can generate endogenous output persistence.[15] This depends crucially on incomplete asset markets. We could check whether these results are robust in the CIA-setup.

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[15] See also Cavallo and Ghironi (2002).

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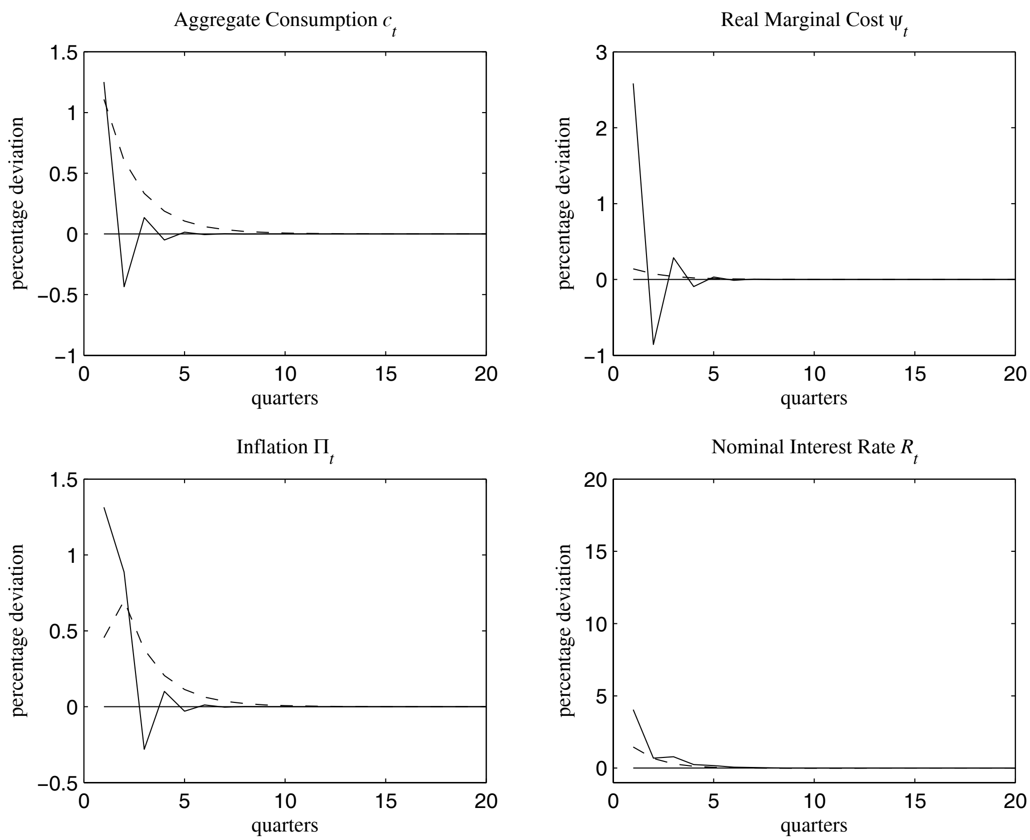

Figure 1: Impulse Response Functions for $\widehat{c}_t, \widehat{\psi}_t, \widehat{\Pi}_t, \widehat{R}_t$,
MIU-Model, benchmark case (solid) and high Frisch elasticity (dashed)

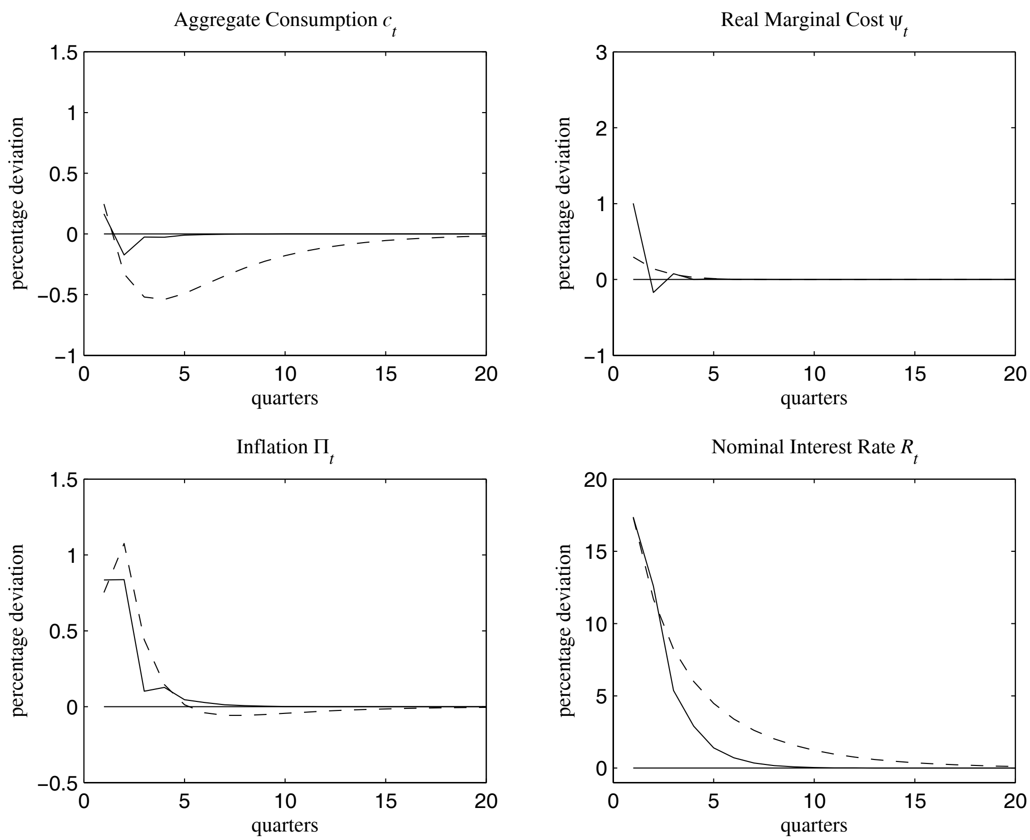

Figure 2: Impulse Response Functions for $\widehat{c}_t, \widehat{\psi}_t, \widehat{\Pi}_t, \widehat{R}_t$,
CIA-Model, benchmark case (solid) and high Frisch elasticity (dashed)