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Optimal Monetary Policy in an Optimizing Stochastic Dynamic Model with Sticky Prices^{*}

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Abstract

Recently macroeconomic researchers have begun studying models of optimal monetary policy within the Real Business Cycle (RBC) framework. A standard RBC model is augmented by New Keynesian elements like sticky prices and monopolistically competitive firms. The monetary authority acts as a social planner maximizing the utility of a representative agent while at the same time taking care of the optimal price setting behavior of the firms via an implementation constraint. King/Wolman (1999) analyze the outcome of such a model with respect to the appropriate monetary policy of the central bank. They conclude that the central bank achieves a complete stabilization of the price level. Inflation is not only constant at the steady state but also through time. It is shown that this very special result does not hold under alternative preference specifications that allow for a richer set of substitution effects between consumption and labor.

JEL classification: E52

Keywords: Monetary Policy Rules, New Neoclassical Synthesis, Sticky Prices, Real Business Cycle

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1 Introduction

In the last two or three years macroeconomists have intensified their interest in analyzing monetary policy. This is mainly due to the adoption of inflation targeting in several countries in the world, among them the United Kingdom and Sweden. These countries have been particular successful in driving down their inflation rates in the 1990s. [21, Svensson (1999)] gives an excellent overview of the literature on that topic.

The task of the monetary authority in these models is to regulate aggregate demand to stabilize output and inflation. Output stabilization is necessary because sticky prices deteriorate aggregate demand causing "Okun gaps". High and variable inflation is generally viewed as resulting in increased relative price volatility and in other costs of production or exchange and thus has to be avoided. In order to determine how the central bank will balance the "Okun gaps" against the costs of inflation a loss function in these two arguments is assumed. The specific optimal monetary policy rule depends on the specific form of this loss function and on the detailed structure of the economy. In general the policy cannot completely eliminate fluctuations in output and inflation.

In the model analyzed here the central bank focuses just on the stabilization of the price level. This policy is optimal although the macroeconomic equilibrium is inefficient because firms have market power and "Okun gaps" can arise through price stickiness. The model combines two strands of research: the public finance approach to policy analysis and features of the "New Keynesian" macroeconomics. This combination is quite new to the macroeconomic literature.

In general the public finance approach concentrates on identifying distortions and on measuring the resulting costs to individuals, sometimes called "Harberger triangles". So far "Okun gaps" have not been analyzed using the public finance approach because they were considered not to be caused by microeconomic distortions.¹ This is fundamentally different here. Making use of two central New Keynesian features, namely the optimizing approach to sticky prices, as e.g. in [3, Calvo(1983)], and the modeling of firms in an imperfect competition environment, as e.g. in [19, Rotemberg (1987)], and embedding this into a dynamic general equilibrium model of the form used in the Real Business Cycle literature Okun gaps in fact arise from microe-

¹See [23, Tobin (1977)]: "It takes a heap of 'Harberger triangles' to fill an 'Okun gap'."

conomic distortions. This "New Neoclassical Synthesis" makes it possible to use Harberger-type analysis to identify distortions and to characterize optimal policy.

The model at hand cannot (yet) be used to answer questions like "What is the trade-off between inflation variability and output variability under alternative specifications of an interest rate rule?". For this purpose the structure of the model has to be improved upon. So far the only exogenous disturbances are productivity shocks. To produce a reasonable outcome some other shocks as energy or government spending shocks have to be included. What the model can answer are questions concerning the response of the optimal policy to a productivity disturbance and its influence on output, inflation and interest rates.

The result of [17, King/Wolman (1999)] that the central bank can achieve a complete stabilization of the price level does not hold in this version of the model. Their result is mainly due to the assumed utility function which implies the absence of any substitution between consumption and leisure.

The paper proceeds as follows: Section 2 describes the model and its underlying structure in detail. Section 3 discusses the policy problem as a social planner exercise of the central bank. Section 4 demonstrates on a theoretical basis why prices will not be constant here. The model is calibrated and impulse response functions will be analyzed focusing on the their optimal character. Section 5 concludes and gives some suggestions for future research.

2 The Model

In the model monopolistically competitive firms are assumed to set final product prices optimally. Supply satisfies demand at theses prices. Firms do so in a staggered manner: each firm sets its price for two periods with half of the firms adjusting each period.² So far the model is in line with [22, Taylor (1980)]. Stickiness in individual prices causes stickiness in the price level and therefore there is room for monetary policy to combat this nonneutrality.

The model can be viewed as representative for a class of models in the spirit of the "New Neoclassical Synthesis" (see also [8, Goodfriend/King (1997)] for a detailed description of this new approach). It combines the above mentioned New-Keynesian-style price stickiness with an otherwise neoclassical business cycle model in the tradition of the RBC literature. To

²The analysis can be easily extended to multi-period price setting.

facilitate the analysis the model abstracts from capital accumulation considerations. The production functions will therefore all be constant returns to scale in the single production factor labor. There will be no money demand distortions caused by positive nominal interest rates in order to focus the analysis completely on the effects of monetary policy (money supply side) that operate through sticky prices. This is justified here since empirically money nearly bears an interest equal to other assets so that there is no distortion from holding money for the representative household. The model does not consider fiscal policy. Changes in the money supply are thus offset by transfers to or lump-sum taxes from the household.

2.1 Consumers

Consumers are assumed to have preferences over consumption (c_t) and leisure $(1 - n_t)$ given by the utility function

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t, a_t\right) \tag{1}$$

The momentary utility function used by [17, King/Wolman (1999)] is given by

$$u(c_t, n_t, a_t) = \ln\left(c_t - \frac{a_t\theta}{1+\gamma}n_t^{1+\gamma}\right)$$
(2)

Here a_t is a preference shock that also acts like a productivity shock. θ and γ are positive parameters, β is the discount factor. This function is familiar from the analysis of [9, Greenwood/Hercowitz/Huffman (1988)] and accordingly labeled GHH-preferences. It has the special property that hours worked only depend upon the real wage and not upon consumption.

The utility function analyzed in this paper is the standard CRRA function used in many Real Business Cycle models. σ governs the degree of risk aversion and ζ measures the relative weight of consumption for the representative agent.

$$u(c_t, n_t, a_t) = \frac{\left[a_t c_t^{\zeta} \left(1 - n_t\right)^{1 - \zeta}\right]^{1 - \sigma} - 1}{1 - \sigma}$$
(3)

It should be noted that in contrast to the standard use of this utility function there is a disturbance a_t acting like a preference shock.³

There is a micro structure for consumption as well as production of differentiated products that is derived explicitly in [1, Blanchard/Kiyotaki (1987)]. Every producer faces a downward-sloping demand curve with elasticity ϵ . When there is a continuum of firms the consumption aggregate c_t is an integral of differentiated products $c(\omega), \omega \in [0, 1]$

$$c_t = \left(\int_0^1 c\left(\omega\right)^{(\epsilon-1)/\epsilon} d\omega\right)^{\epsilon/(\epsilon-1)} \tag{4}$$

as in [4, Dixit/Stiglitz (1977)]. In case that prices are fixed for just two periods and assuming that all price adjusting producers in a given period choose the same price the consumption aggregate can be written as

$$c_t = c\left(c_{0,t}, c_{1,t}\right) = \left(\frac{1}{2}c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2}c_{1,t}^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)}$$
(5)

where $c_{j,t}$ is the quantity of a good consumed in period t whose price was set in period t - j. It can be shown that for this Dixit/Stiglitz-aggregator the constant elasticity demands for each good take the form

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} c_t \tag{6}$$

 $P_{j,t}$ is the nominal price at time t of any good whose price was set j periods ago and P_t is the price index at time t and is given by

$$P_t = \left(\int_0^1 P(\omega)^{(1-\epsilon)} d\omega\right)^{1/(1-\epsilon)}$$
(7)

Here firms set prices for two periods implying that half of them adjust their price in period t and half do not. Moreover all adjusting firms choose the

³[17, King/Wolman (1999)] argue that it is necessary in (2) to have a_t affecting equally production and preferences in order to achieve balanced growth. This is doubt-ful because the model does not explicitly account for growth aspects as, e.g., in [12, King/Plosser/Rebelo (1988)]. The use of a_t in (3) affecting preferences is a new feature not analyzed in the literature in the context of optimal monetary policy before.

same price. So the price index can be written as

$$P_t = \left(\frac{1}{2}P_{0,t}^{1-\epsilon} + \frac{1}{2}P_{1,t}^{1-\epsilon}\right)^{1/(1-\epsilon)}$$
(8)

The *intertemporal optimization* problem for the household is to maximize lifetime utility over aggregate consumption and leisure subject to an intertemporal budget constraint. The household is assumed to have access to a bond market and to hold money. Its budget constraint is therefore given by

$$c_{t} + \frac{M_{t}}{P_{t}} + \frac{B_{t}}{P_{t}} + w_{t}h\left(\frac{M_{t}}{P_{t}c_{t}}\right) = w_{t}n_{t} + \left(1 + R_{t-1}^{M}\right)\frac{M_{t-1}}{P_{t}} + \left(1 + R_{t-1}\right)\frac{B_{t-1}}{P_{t}}(9)$$

The uses of wealth are real consumption c_t , holdings of real money balances M_t/P_t and real bonds B_t/P_t . $h\left(\frac{M_t}{P_tc_t}\right)$ is the time spent on transactions activity, i.e. for purchasing goods while the real wage w_t is considered to be the opportunity cost of a unit of time spent shopping.⁴ The household has several sources of his wealth. It earns money working in the market at the real wage rate (w_tn_t) . As money is assumed to be interest bearing it can spend its money holdings carried over from the previous period augmented by the interest on these real money balances $(1 + R_{t-1}^M) \frac{M_{t-1}}{P_t}$. Finally there are previous period bond holdings including the interest on them $(1 + R_{t-1}) \frac{B_{t-1}}{P_t}$. The interest rate on bonds is endogenous while the one on money R_{t-1}^M is set exogenously by the monetary authority and is slightly below the bond rate.

The Lagrangian for the household (index H) can be written as follows:

$$L_{H} = \sum_{t=0}^{\infty} \beta^{t} u(c_{t}, n_{t}, a_{t}) + \sum_{t=0}^{\infty} \beta^{t} \lambda_{t} \bigg[w_{t} n_{t} + (1 + R_{t-1}^{M}) \frac{P_{t-1}}{P_{t}} m_{t-1} + (1 + R_{t-1}) \frac{P_{t-1}}{P_{t}} b_{t-1} - c_{t} - m_{t} - b_{t} - w_{t} h\left(\frac{m_{t}}{c_{t}}\right) \bigg]$$

$$(10)$$

Here small variables indicate real quantities, i.e. for example $b_t = B_t/P_t$. This function is maximized over c_t, n_t, m_t and b_t . The first order conditions

 $^{^{4}}$ For a detailed discussion of the shopping-time approach see [16, King/Wolman (1996)].

will be important for the optimal policy of the central bank so they are reported below.

$$\frac{\partial L_H}{\partial c_t} = \beta^t \frac{\partial u\left(c_t, n_t, a_t\right)}{\partial c_t} - \beta^t \lambda_t + \beta^t \lambda_t w_t \frac{m_t}{c_t^2} h'\left(\frac{m_t}{c_t}\right) = 0$$
(11)

$$\frac{\partial L_H}{\partial n_t} = \beta^t \frac{\partial u\left(c_t, n_t, a_t\right)}{\partial n_t} + \beta^t \lambda_t w_t = 0$$
(12)

$$\frac{\partial L_H}{\partial m_t} = -\beta^t \lambda_t \left[1 + \frac{w_t}{c_t} h'\left(\frac{m_t}{c_t}\right) \right] + E_t \left[\beta^{t+1} \lambda_{t+1} \left(1 + R_t^M \right) \frac{P_t}{P_{t+1}} \right] = 0 \quad (13)$$

$$\frac{\partial L_H}{\partial b_t} = -\beta^t \lambda_t + E_t \left[\beta^{t+1} \lambda_{t+1} \left(1 + R_t \right) \frac{P_t}{P_{t+1}} \right] = 0 \tag{14}$$

The third condition defines implicitly the money demand function. $h'(\cdot)$ is the derivative of h with respect to m/c. Combining this equation with (14) allows to analyze the nature of money demand in this model.

$$\lambda_t \left[\frac{w_t}{c_t} h'\left(\frac{m_t}{c_t}\right) \right] = \beta E_t \left[\lambda_{t+1} \left(R_t^M - R_t \right) \frac{P_t}{P_{t+1}} \right]$$
(15)

When the rate on money approaches the rate on bonds $(R_t^M \cong R_t)$ real costs of holding money go to zero. This implies that $h'(\cdot)$ is zero. Since only the derivative of a constant is zero real money holdings per unit of consumption must be constant: $m_t/c_t = k$. Hence money demand is given by

$$m_t = kc_t \Leftrightarrow M_t = kP_tc_t \tag{16}$$

k represents the satiation level of cash balances. Money supply always satisfies the demand for cash. As there are lump-sum taxes and transfers available for the household they can be used to offset changes in the money supply. The efficiency condition for bond holdings establishes a relation between the nominal interest rate (on bonds) and the price level. Rearranging terms yields

$$(1+R_t) = E_t \left[\frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \right]$$
(17)

Supposed the Fisher equation is valid the real interest rate r_t is implicitly defined as

$$(1+r_t) = E_t \left[\frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \right]$$
(18)

because $E_t [P_{t+1}/P_t]$ equals one plus the rate of expected inflation. Combining the first two efficiency conditions and remembering that $h'(\cdot) = 0$ reveals that the marginal rate of substitution between consumption and labor is equal to the real wage.

$$-\frac{\partial u\left(c_t, n_t, a_t\right)/\partial n_t}{\partial u\left(c_t, n_t, a_t\right)/\partial c_t} = \frac{dc_t}{dn_t} = w_t$$
(19)

2.2 Firms

Firms produce with a technology that is linear in labor $n_{j,t}$ and subject to random productivity shocks a_t .

$$y_{j,t} = c_{j,t} = a_t n_{j,t} (20)$$

Here $n_{j,t}$ is the labor input employed in period t by a firm who set the price in period t - j. Firms always meet the demand for their product, that is $y_{j,t} = c_{j,t}$. Those who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally.

Firms set their prices to maximize the present discounted value of their profits.⁵ With a relative price defined by $p_{j,t} = P_{j,t}/P_t$ real profit $z_{j,t}$ for a firm of type j is given by

$$z_{j,t} = p_{j,t}a_t n_{j,t} - w_t n_{j,t} (21)$$

 w_t is the real wage rate. Firms minimize their costs which are given in this environment solely by wages. Thus minimizing $P_t w_t n_{j,t}$ subject to the production function implies for the total cost function $TC_{j,t}^{6}$

$$TC_{j,t} = \frac{P_t w_t c_{j,t}}{a_t} \tag{22}$$

⁵The model deviates in this respect from the standard textbook model in which profits are maximized over the quantity.

⁶It should be noticed that the wage is perfectly flexible in a competitive input market. So there is no index j for w_t and P_t which means that they are not firm-specific.

It is useful for further calculations to define real marginal cost as ψ_t which is equal to $(\partial TC_{j,t}/\partial c_{j,t})/P_t = w_t/a_t$. So the profit function can be rewritten as

$$z_{j,t} = z \left(p_{j,t}, c_t, \psi_t \right) = p_{j,t}^{-\epsilon} c_t \left(p_{j,t} - \psi_t \right)$$
(23)

making use of the fact that demand equals output $(c_{j,t} = p_{j,t}^{-\epsilon}c_t = a_t n_{j,t}).$

In the case in which prices are not sticky the firm can just set prices on a period by period basis optimizing the profit function (23) with respect to $p_{j,t}$. The result of this exercise would be that relative prices will have to be set according to

$$p_{j,t} = p_t^* = \frac{\epsilon}{\epsilon - 1} \psi_t \tag{24}$$

But when prices are fixed for two periods the firm has to take into account the effect of the price chosen in period t on current and future profits. The price in period t + 1 will be affected by the gross inflation rate Π_{t+1} between t and t + 1 ($\Pi_{t+1} = P_{t+1}/P_t$).

$$p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \tag{25}$$

If there is positive inflation, $p_{1,t+1}$ will fall because nominal prices are fixed for two periods. As the nominal price in period t is defined by $P_{0,t}$ and in period t + 1 by $P_{1,t+1}$, one has $P_{0,t} = P_{1,t+1}$, so that $p_{0,t} = P_{0,t}/P_t$ and $p_{1,t+1} = P_{1,t+1}/P_{t+1} = (P_{0,t}/P_t) (P_t/P_{t+1})$ which is what is stated in (25). So the optimal relative price has to balance the effects due to inflation between profits today and tomorrow. This intertemporal maximization problem is formally given by

$$\max_{p_{0,t}} E_t \left[z \left(p_{0,t}, c_t, \psi_t \right) + \beta \frac{\lambda_{t+1}}{\lambda_t} z \left(p_{1,t+1}, c_{t+1}, \psi_{t+1} \right) \right]$$

s.t. $p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}}$ (26)

The term λ_{t+1}/λ_t is equal to the ratio of future to current marginal utility of consumption (derived in the household's optimization problem) and considered to be - in conjunction with β - the appropriate discount factor for real profits. This is a consequence of the assumption that households own the production factor labor and rent it to the firms. They also own a diversified

portfolio of claims to the profits earned by the firms. Although there will be no asset accumulation in equilibrium λ_t can be used to determine the present value of profits.⁷ The efficiency condition for this problem is given by

$$0 = \frac{\partial z \left(p_{0,t}, c_t, \psi_t\right)}{\partial p_{0,t}} + \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial z \left(p_{1,t+1}, c_{t+1}, \psi_{t+1}\right)}{\partial p_{1,t+1}} \frac{1}{\Pi_{t+1}}\right)$$
(27)

Multiplying this equation by $p_{0,t}$ and λ_t produces a more symmetric form of the efficiency condition that will be more convenient to derive the optimal monetary policy later.

$$0 = \lambda_t p_{0,t} \frac{\partial z \left(p_{0,t}, c_t, \psi_t \right)}{\partial p_{0,t}} + \beta E_t \left(\lambda_{t+1} p_{1,t+1} \frac{\partial z \left(p_{1,t+1}, c_{t+1}, \psi_{t+1} \right)}{\partial p_{1,t+1}} \right)$$
(28)

Using (23) one can solve this condition for the optimal price to be set in period t which corresponds to the optimal price in case that prices are flexible derived before. This yields a forward-looking form of the price equation and is in that respect similar to the one in [22, Taylor (1980)].

$$p_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t c_t \psi_t + \beta E_t \lambda_{t+1} \left(P_{t+1} / P_t \right)^{\epsilon} c_{t+1} \psi_{t+1}}{\lambda_t c_t + \beta E_t \lambda_{t+1} \left(P_{t+1} / P_t \right)^{\epsilon - 1} c_{t+1}}$$
(29)

The optimal price $p_{0,t}$ depends upon the current and future real marginal costs, the gross inflation rate, current and future consumption as well as today's and tomorrow's interest rates (through the influence of the λ -terms). It is thus fundamentally different from the one derived under fully flexible prices on a period-by-period basis (see (24)).

2.3 Constraints of the Monetary Authority

The objective of the monetary authority is to maximize welfare which means here maximizing the utility of the representative agent. In the absence of any distortions any rate of inflation would coincide with an optimal policy. But in this setup there are monopolistic competition and sticky prices. So the authority has to offset - in principle - the effects of these two frictions. It is constraint by technology and resource conditions as well as the price setting behavior of the firms.

⁷More details on this can be found in [6, Dotsey/King/Wolman (1999)], p. 659-665 as well as in [5, Dotsey/King/Wolman (1997)], p. 9-13.

It is assumed that the central bank follows an optimal plan under commitment. Fiscal policy instruments are not available so a first best allocation cannot be achieved. The purpose is to isolate the characteristics of an optimal monetary policy without a discussion of fiscal issues.

Three resource conditions have to be considered. Consumption of a good whose price was set j periods ago cannot exceed production of that good.

$$c_{j,t} \le a_t n_{j,t} \quad \text{for } j = 0,1 \tag{30}$$

The consumption aggregator for a firm setting its price for two periods is given by (5) and repeated here.

$$c_t \le \left(\frac{1}{2}c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2}c_{1,t}^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)}$$
(31)

The agent's time endowment is n_t and can be used for production of goods whose prices were set in period t and t - 1.

$$n_t = \frac{1}{2}n_{0,t} + \frac{1}{2}n_{1,t} \le 1 \tag{32}$$

The household equally splits his time for producing goods whose prices were set in the actual period and the period before.⁸

The quantities the monetary authority chooses must be consistent with those of the monopolistic price setting firms. Formally this is achieved via an implementation constraint. The central bank must make sure that the firms will in fact set their quantities as the optimal plan implies. It has to induce the firms to choose those quantities which are consistent with an optimal monetary policy. So it takes into account the optimality condition in (28) which is repeated here for convenience.

$$\lambda_t p_{0,t} \frac{\partial z_{0,t}}{\partial p_{0,t}} + \beta E_t \lambda_{t+1} p_{1,t+1} \frac{\partial z_{1,t+1}}{\partial p_{1,t+1}} = 0$$
(33)

This condition can be expressed in a more compact way making use of the function x which only depends on real quantities.

$$x(c_{0,t}, c_t, n_t, a_t) + \beta E_t x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1}) = 0$$
(34)

⁸This seems to be an arbitrary assumption. It is used to make the outcome comparable to that of [17, King/Wolman (1999)].

Using the demand function $c_{j,t} = p_{j,t}^{-\epsilon}c_t$ one can eliminate relative prices. Real marginal costs are substituted by w_t/a_t and the real wage w_t is eliminated by use of the equality with the rate of substitution between consumption and leisure (see (19)). This yields:

$$x\left(c_{j,t}, c_t, n_t, a_t\right) = \lambda_t c_t \left[\left(1 - \epsilon\right) \left(\frac{c_{j,t}}{c_t}\right)^{1 - 1/\epsilon} + \epsilon \frac{1 - \zeta}{\zeta} \frac{1}{a_t} \frac{1}{1 - n_t} \left(\frac{c_{j,t}}{c_t}\right) c_t \right]$$
(35)

As λ_t is equal to the marginal utility of consumption in period t it is in that respect also a function of c_t , n_t and a_t . This formula deviates from the one in [17, King/Wolman (1999)] in that there is also a direct influence of a_t in the second term in the bracketed expression.

3 The Policy Problem

The determination of the optimal monetary policy is conducted in a two-step procedure. First, the optimal choices for the real variables in the model are derived by solving the policy problem of the monetary authority acting as a social planner. Second, the implications for the nominal variables such as prices and interest rates are determined by using these optimal decision functions and by combining them with those of the household's problem. While unusual in macroeconomics this practice is common in public finance and other areas of applied general equilibrium analysis and is getting more and more standard in dynamic macroeconomic models with distortions. Expectations are not considered here so that the solution is derived under a certainty equivalence perspective. The Lagrangian of the central bank (index C) can be written as follows:

$$L_{C} = \sum_{t=0}^{\infty} \beta^{t} u(c_{t}, n_{t}, a_{t}) + \sum_{t=0}^{\infty} \beta^{t} \phi_{t} [x(c_{0,t}, c_{t}, n_{t}, a_{t}) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})] + \sum_{t=0}^{\infty} \beta^{t} \Lambda_{t} [c(c_{0,t}, c_{1,t}) - c_{t}] + \sum_{t=0}^{\infty} \beta^{t} \Omega_{t} \left[n_{t} - \frac{1}{2}n_{0,t} - \frac{1}{2}n_{1,t} \right] + \sum_{t=0}^{\infty} \beta^{t} [\rho_{0,t} (a_{t}n_{0,t} - c_{0,t}) + \rho_{1,t} (a_{t}n_{1,t} - c_{1,t})]$$
(36)

This function is maximized over $c_{0,t}, c_{1,t}, c_t, n_{0,t}, n_{1,t}, n_t$ and of course with respect to the Lagrange multipliers $\phi_t, \Lambda_t, \Omega_t, \rho_{0,t}$ and $\rho_{1,t}$.

3.1 Optimality Conditions

Defining an artificial multiplier ϕ_{-1} at date t = 0 the optimality conditions can be written in the time-invariant form below. The multiplier will be discussed more thoroughly in the next section.

The first order condition with respect to each firm's labor input is given by

$$\frac{\partial L_C}{\partial n_{j,t}} = \beta^t \left(-\frac{1}{2} \Omega_t + \rho_{j,t} a_t \right) = 0 \quad \text{for } j = 0, 1 \tag{37}$$

The optimal choice of consumption levels from each type of firm is determined by

$$\frac{\partial L_C}{\partial c_{j,t}} = \beta^t \left(\phi_{t-j} \frac{\partial x \left(c_{j,t}, c_t, n_t, a_t \right)}{\partial c_{j,t}} + \Lambda_t \frac{\partial c \left(c_{0,t}, c_{1,t} \right)}{\partial c_{j,t}} - \rho_{j,t} \right) = 0$$

for $j = 0, 1$ (38)

Aggregate consumption must obey

$$\frac{\partial L_C}{\partial c_t} = \beta^t \left(\frac{\partial u \left(c_t, n_t, a_t \right)}{\partial c_t} + \phi_t \frac{\partial x \left(c_{0,t}, c_t, n_t, a_t \right)}{\partial c_t} \right) \\
+ \beta^t \left(\phi_{t-1} \frac{\partial x \left(c_{1,t}, c_t, n_t, a_t \right)}{\partial c_t} - \Lambda_t \right) = 0$$
(39)

whereas for aggregate labor the condition is

$$\frac{\partial L_C}{\partial n_t} = \beta^t \left(\frac{\partial u \left(c_t, n_t, a_t \right)}{\partial n_t} + \phi_t \frac{\partial x \left(c_{0,t}, c_t, n_t, a_t \right)}{\partial n_t} \right) + \beta^t \left(\phi_{t-1} \frac{\partial x \left(c_{1,t}, c_t, n_t, a_t \right)}{\partial n_t} + \Omega_t \right) = 0$$
(40)

In addition the constraints have to hold with equality, that is the derivatives of the Lagrangian with respect to the multipliers.

$$\frac{\partial L_C}{\partial \rho_{j,t}} = \beta^t \left(a_t n_{j,t} - c_{j,t} \right) = 0 \quad \text{for } j = 0, 1 \tag{41}$$

$$\frac{\partial L_C}{\partial \Lambda_t} = \beta^t \left[c\left(c_{0,t}, c_{1,t} \right) - c_t \right] = 0$$
(42)

$$\frac{\partial L_C}{\partial \phi_t} = \beta^t \left[x \left(c_{0,t}, c_t, n_t, a_t \right) + \beta x \left(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1} \right) \right] = 0$$
(43)

$$\frac{\partial L_C}{\partial \Omega_t} = \beta^t \left[n_t - \frac{1}{2} n_{0,t} - \frac{1}{2} n_{1,t} \right] = 0 \tag{44}$$

It should be noted that the multiplier ϕ_{-1} is not present in (36). It is introduced to have a simple representation of optimal policy in a world of commitment of the central bank. The multiplier guarantees that the efficiency conditions take the same form irrespective of the period the monetary authority is optimizing.

3.2 General Implications of the Optimality Conditions

In most cases the optimality conditions differ from those of an unrestricted representative agent. This is due to the effect of the implementation constraint on the social planner's behavior.

The conditions for the firms' labor input $n_{j,t}$ just equate the utilitydenominated price of a unit of each type of good $(\rho_{j,t})$ to the utility-denominated value of labor (Ω_t) divided by productivity (a_t) which is the same under purely flexible prices. But the efficiency conditions for aggregate consumption and labor differ from those of an unrestricted planner. ϕ_t is the shadow price of decreasing a price-settings firm's marginal present discounted profits with respect to relative price and it is negative here because the planner wants the firms to have positive marginal profits.⁹ In comparison to the decentralized problem the central bank values a marginal unit of consumption, measured by Λ_t , higher. This is because the derivative of $x(c_{j,t}, c_t, n_t, a_t)$ with respect to aggregate consumption c_t is negative so that Λ_t is higher than marginal utility of consumption $(\partial u (c_t, n_t, a_t) / \partial c_t)$. For similar reasons a marginal unit of labor Ω_t is valued higher here than under perfect competition.

The first-order conditions for $c_{j,t}$ do not have such a straightforward analogue in the competitive model. But it can be shown that the monetary authority equates appropriately chosen marginal rates of substitution and transformation (see [17, King/Wolman (1999)], p. 376).

The multiplier ϕ_t appears not only in the current period t, but also in lagged form ϕ_{t-1} as can be seen in the conditions for aggregate labor and consumption. This is a consequence of the fact that changes in future consumption affect the price-setting behavior of firms in period t - 1. Recall that from (29) $p_{0,t}$ depends not only on current period consumption but also on future consumption c_{t+1} . (29) is a forward-looking constraint. It must be clear that the efficiency conditions are valid for all periods, including t = 0, whereas in the Lagrangian ϕ_{-1} is not present. This lagged multiplier in t = 0is introduced to make sure that an optimal plan is feasible. It allows the use of standard fixed-coefficient linear rational expectations solution methods.

If the monetary authority is allowed to reformulate its policy on a periodby-period basis, then there will be the problem of time-inconsistency of the

$$-\sum_{t=0} \beta^t \phi_t \left[0 - x \left(c_{0,t}, c_t, n_t, a_t \right) - \beta x \left(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1} \right) \right].$$

⁹This can be shown when introducing a zero bound on marginal profits which means rewriting the implementation constraint as

optimal plan as in [18, Kydland/Prescott (1977)] and [2, Barro/Gordon (1983)]. In that case the optimal policy problem cannot be formulated in the way just described. But here it is assumed that the central bank is required to commit to the state-contingent plan in the initial period and to stick to it through time. The introduction of ϕ_{-1} prohibits the study of an optimal policy dependent on the effects of an initial start-up period. Therefore ϕ_{-1} is set to the steady state value of ϕ and not to zero. The use of the steady state of ϕ reflects that the central bank has been following an optimal plan for a long time.

3.3 The Steady State

Looking at (37) in the steady state reveals that $\rho_0 a = \rho_1 a = (1/2)\Omega$ so that $\rho_1 = \rho_0 = (1/2)(\Omega/a)$. A conjecture for the steady state values of consumption is that they are all equal which means $c_0 = c_1 = c$. In order to determine whether this is possible one has to check whether the other optimality conditions are consistent with the conjecture. For this end it is helpful to look at first at equations (38). Dividing (38) for j = 0 by (38) for j = 1 results in the following term:

$$\frac{\phi \frac{\partial x(c_0,c,n,a)}{\partial c_0} + \Lambda \frac{\partial c(c_0,c_1)}{\partial c_0}}{\phi \frac{\partial x(c_1,c,n,a)}{\partial c_1} + \Lambda \frac{\partial c(c_0,c_1)}{\partial c_1}} = \frac{\rho_0}{\rho_1}$$
(45)

As just shown the right hand side of this expression is unity in the steady state. For (45) to be satisfied the left hand side must also be equal to unity. Calculating the derivative of the consumption aggregator and imposing $c_0 = c_1$ reveals that $\partial c (c_0, c_1) / \partial c_0 = 1/2 = \partial c (c_0, c_1) / \partial c_1$. Moreover, the effect of consumption related to today's price-setting firms on today's implementation constraint $\partial x (c_0, c, n, a) / \partial c_0$ is just the same as the effect of consumption related to yesterday's price-setting firms on yesterday's implementation constraint $\partial x (c_1, c, n, a) / \partial c_1$. As $\phi_t = \phi_{t-1} = \phi$ in the steady state and as Λ is constant the left hand side is equal to unity. Along a similar line of argument the derivatives of $x (c_0, c, n, a)$ and $x (c_1, c, n, a)$ with respect to c are identical so that (39) can be written as

$$\frac{\partial u(c,n,a)}{\partial c} + 2\phi \frac{\partial x(c_0,c,n,a)}{\partial c} - \Lambda = 0$$
(46)

The condition for aggregate labor (40) reduces to

$$\frac{\partial u(c,n,a)}{\partial n} + 2\phi \frac{\partial x(c_0,c,n,a)}{\partial n} + \Omega = 0$$
(47)

Using the results for (38) it suffices to use one of the conditions of (37) to get

$$\phi \frac{\partial x \left(c_0, c, n, a\right)}{\partial c_0} + \frac{1}{2}\Lambda - \frac{1}{2}\frac{\Omega}{a} = 0$$
(48)

These three equations form a linear system in the three remaining Lagrange multipliers. The solution determines uniquely steady state values for Λ , Ω and ϕ .

Closer inspection of (41) - (44) reveals that all labor inputs are equal $(n_0 = n_1 = n = c/a)$, and that marginal profits should be zero in any period $(x (c_0, c, n, a) = x (c_1, c, n, a) = 0)$.

The optimal steady state inflation rate is equal to zero. This can be seen by calculating (6) at the steady state. Since $c_j = c$ one gets $P_j = P$. The price of a firm setting the price in t is just equal to the overall price level and equal to the price firms set in period t - 1. Accordingly the gross inflation rate $\Pi_t = P_t/P_{t-1}$ is equal to unity and inflation is zero.

4 Optimal Monetary Policy

In this section the solution to the policy problem is analyzed in detail. Having determined the steady state of all endogenous variables one can start taking linear approximations of the efficiency conditions around it. These equations are given in detail in the appendix. The model has still to be augmented by equations for the nominal variables (nominal interest rate, price level, inflation rate and money demand) to analyze the implications of optimal monetary policy.

The solution is conducted using an extended version of the algorithm of [13, King/Plosser/Rebelo (1990)] which allows for singularities in the system matrix of the reduced model. The theoretical background of this algorithm is developed in [15, King/Watson (1999)] whereas computational aspects and the implementation are discussed in [14, King/Watson (1997)].

4.1 Implications of the Model Solution

One can use the resulting decision functions to calculate optimal responses to a productivity shock (impulse responses). In contrast to [17, King/Wolman (1999)] prices and labor inputs fluctuate and are not constant. The equality result for all relative prices even in the dynamic context is a very special one and due to the specific utility function used by these authors which has very strong implications for the substitution effects at work. It will be demonstrated which mechanisms are at work to stimulate fluctuating prices.

According to [17, King/Wolman (1999)] the labor input must not fluctuate in response to a productivity shock in order to make sure that marginal profits do not deviate form zero, that is $dx_{j,t}(\cdot) = 0$. This constancy is necessary to guarantee that a productivity shock does not induce optimal price variation. Under their preference specification labor does indeed not respond to a technology shock as long as the markup is constant. The markup μ_t is the reciprocal of real marginal cost and is given by $\mu_t = a_t/w_t = 1/\psi_t$. They prove this to be possible and consistent with the linearized first order conditions with the help of the conjecture that $\hat{\phi}_t$ does not respond to productivity either and show that this is in fact the case and that it is compatible with the solution of the model.

This result is very special and does not hold in the model considered here. The utility function used in this model specification features a much richer set of substitution effects between consumption and labor. Refer to (3) and analyze the relationship between c and n in the household's optimization problem. The first order conditions of this problem imply - as shown above - the equality of the real wage with the rate of substitution between consumption and leisure. In the steady state this relationship is given by

$$n = 1 - \frac{c}{w} \frac{1 - \zeta}{\zeta} \tag{49}$$

Labor increases in the real wage as in case of utility function (2). But there is also a direct influence of consumption. With a positive technology shock, consumption will be increased while at the same time labor will be decreased. The impact of this favorable shock will be used completely to reduce working effort and to raise consumption. So the dynamics of consumption influence the dynamics of labor. Note that a does not appear in (49) as opposed to [17, King/Wolman (1999)]. The respective steady state relation in their model is

$$n = \left(\frac{w}{a\theta}\right)^{1/\gamma} \tag{50}$$

This is the reason why they have to show that the markup $\mu = a/w$ is constant. Constancy of the markup implies constancy of real marginal cost. Using this in (29) one can show that firms will not adjust their relative prices $p_{0,t}$ if the price level will be constant, i.e. $P_t = P_{t-1}$. This is because one can factor out ψ of the numerator. Then the remaining expression and the denominator cancel. To demonstrate that a constant price level is really a consequence of the optimal monetary policy they need zero response of the shadow price of real marginal profits $\hat{\phi}_t$ to the technological shock and responses of consumption levels $\hat{c}_t, \hat{c}_{j,t}$ exactly equal to productivity dynamics. This is proved by just imposing the conjecture of constancy on the linearized equations. But here $\hat{\phi}_t$ does respond to \hat{a}_t as will be shown in the next subsection with the help of impulse responses.¹⁰ In addition the symmetry of the responses of the consumption levels of the firms adjusting prices in t and t-1 vanishes. They also cease to be exactly the same as productivity dynamics.

4.2 Impulse Response Functions and Optimal Monetary Policy

To compute impulse responses the parameters of the model have to be calibrated. Going back to section 3.3 one can see that a and either n or c have to be set exogenously. Because more information is available about hours worked, n is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time. The steady state value of the productivity shock is arbitrarily chosen to be 10.¹¹ It is assumed that productivity follows an AR(1)-process with $\rho_a = 0.8$. The discount factor β will be 0.99 and σ , the parameter governing the degree of risk aversion, is set to 2.¹² The elasticity

¹⁰It is not clear whether it is possible to show this analytically.

¹¹In contrast to the well known basic neoclassical model of [12, King/Plosser/Rebelo (1988)] there is no escape from specifying parameters such as a at the steady state. The system cannot be reduced until only deep parameters remain to be calibrated.

¹²Model results are qualitatively *not* sensitive to this value. Especially the result of a fluctuating price level also holds for log-linear preferences with $\sigma = 1$. Quantitative results for other values of σ will be discussed below.

of demand ϵ is 4 causing the average static markup $\mu = \epsilon/(\epsilon - 1)$ to be 1.33.¹³ All remaining parameters can be calculated with the help of these specified values: the steady state consumption levels c = an, the real wage $w = a/\mu$, the real and nominal interest rates $r = R = (1 - \beta)/\beta$, and the preference parameter $\zeta = c/(w + c - wn)$. For ζ this implies 0.3077, a value that is reasonably in line with other studies.

Figure 1 shows the impulse responses of consumption and labor caused by a one percent productivity shock. As mentioned above the response of \hat{c} does not exactly track the reaction of \hat{a} as in [17, King/Wolman (1999)]. Aggregate consumption reacts a bit weaker than productivity. Moreover it is weakly influenced by $\hat{\phi}$ which is itself hit by \hat{a} . The shadow price of marginal profits as well as aggregate labor input fall but the fall in ϕ is a persistent one whereas labor \hat{n} shows an interesting cyclical movement that is not very long-lasting. Figure 2 gives some more detailed insight in the mechanisms at work. Here one can see that labor used by firms setting their price in period t (\hat{n}_0) rises slightly while at the same time firms who set prices one period earlier will reduce labor input \hat{n}_1 . This reduction is more pronounced than the expansion so that overall aggregate labor decreases. In the following period the picture changes. Now \hat{n}_1 rises and \hat{n}_0 falls. The effect is due to the shadow price of real marginal profits. Its relatively strong decline in the initial period of the shock drops to about only half of this magnitude. It should be noted that the fluctuations in labor are small compared with the respective values for consumption and ϕ . In addition, there is also a different reaction of \hat{c}_1 and \hat{c}_0 . Because the impulse responses look quite similar the differences of the respective functions are plotted. This reveals the stronger reaction of \hat{c}_0 so that the difference is negative. \hat{c}_1 responds a bit weaker causing a positive difference. Notice that both graphs are mirror images of each other.¹⁴

Figure 3 shows the reaction of prices. As the shadow price of real marginal profits falls the central bank as the social planner has to optimally induce firms to reduce their prices. The reaction of \hat{P}_0 resembles much the behavior of $\hat{\phi}$ but not that strong. Note that \hat{P}_0 falls only by -0.005 % whereas $\hat{\phi}$ drops by nearly -0.11 %. Firms who have set their prices one period before react with a lag of one period so that the impulse response for \hat{P}_1 is the same

¹³This formula can be deduced by combining (29) with the price index (8) evaluated at the steady state, i.e. for zero inflation.

¹⁴The respective coefficients in the decision functions for \hat{c}_0 , \hat{c}_1 and \hat{c} are 1.0052, 0.9851 and 0.9951.

as the one for \hat{P}_0 just shifted one period ahead. This results in an overall variation in the price level. The central bank optimally induces a disinflation in order reach its goals: to maximize the utility of the representative agent in an environment of monopolistically competitive firms fixing nominal prices for two periods. Due to this two period price setting behavior there is a kink in \hat{P} that translates into a small inflationary period beginning two periods after the initial productivity shock but lasting only for just ten quarters. Although there are substitution effects at work (see (49)) these are small and cause only a weak inflationary bias. But nevertheless the monetary authority does not succeed in stabilizing the price level completely.

Figure 4 gives a graphical impression of the nominal and real interest rate. They are no longer equal. Since the price level is not constant there is inflation resulting in a slightly stronger reaction of the nominal rate to a productivity shock. The fall in the interest rate is quite strong: about 1.8 % on an annual basis. It is more than four times the reaction in the [17, King/Wolman (1999)] model version with preference specification (2). Because the coefficients are even closer in value than those of the consumption levels in figure 2 again the difference between the nominal and real rate is plotted. The nominal rate initially decreases stronger so that the difference is negative.¹⁵ Note the interesting "cyclical" character of this curve. There is a highly nonlinear relation between the response of the two rates due to the richer internal dynamics of the model. The response of money demand is nearly equal to that of consumption. This is due to the fact that the price level - which is the difference between the cyclical components of money and consumption - does not react very strong to a productivity shock. (See the respective graphs in figure 4 and refer to (71).)

The result that real and nominal rate differ can also be demonstrated analytically. A typical Taylor-rule would link the real and nominal rate according to^{16}

$$R_t = r_t + f\left(lnP_t - ln\overline{P}\right) \tag{51}$$

where f would be a positive coefficient and \overline{P} the target price level. r_t would be determined from the real interest rate \hat{r}_t of the model solution. In [17, King/Wolman (1999)] the price level is constant under optimal policy so they

¹⁵This is because the negative reaction of the nominal rate is stronger than that of the real rate so that the sign gets negative.

¹⁶This formulation ignores a term for the output gap.

reach the strong result that the central bank should just set the nominal rate equal to the real rate: $R_t = r_t$, which also implies the equality of the cyclical components: $\hat{R}_t = \hat{r}_t$. Here P fluctuates so that the term in parentheses ceases to be zero. Hence f would be different from zero. In addition it is no longer possible to write down the policy rule as in (51) because the fact that \hat{R}_t is different from the real rate makes it depend upon several state variables.

Varying the degree of risk aversion has no qualitative consequences for the model results regarding the fact that prices cannot be completely stabilized. But there are interesting quantitative effects. For small values of σ (smaller than 2) the reactions of prices, inflation, labor and consumption decline while for higher values the cyclical variation gets stronger. The higher the degree of relative risk aversion which corresponds to a lower degree of the elasticity of intertemporal substitution the stronger the effects of productivity shocks. The strengthening of the cyclical character is most probably due to the two-period-price setting of the firms. Because very risk averse households care more for today than for tomorrow they frequently change their demand for the differentiated consumption goods and their labor supply. So firms react stronger in setting their prices so that the central bank is less successful in stabilizing the price level and inflation. Figure 5 illustrates this result for prices and inflation.

A second important factor for the success of the central bank to stabilize prices is the length of the price setting period. To explore the consequences the model is solved assuming that every period a constant fraction of 20% of the firms can change their price with all adjusting firms choosing the same price (5-period price setting). As figure 6 depicts this has the consequence of strengthening the persistence effects of the productivity shock. Optimal pricing now implies a very smooth development of $P_{0,t}$ which translates into an even smoother curve for the price level. Note that the intensity of the disinflation gets smaller compared to the case with a high degree of relative risk aversion (see figure 5). The pronounced peak of the gross inflation rate in the sixth period is due to the assumption of equal fractions of firms adjusting every period. This causes the marginal adjustment probability to be zero all the time and one in every fifth period. Relaxing that assumption and using a vector of declining fractions gives the central bank the opportunity to smooth the productivity shock even more. Figure 7 is an example for the vector of fractions given by $[0.30\ 0.25\ 0.20\ 0.15\ 0.10]$ meaning that in the first period 30% of the firms do adjust prices, in the second 25% and so on. This causes the inflationary period to start earlier but it also allows the monetary authority to dampen the peak and therefore to stabilize prices more efficiently.

The success of the central bank in stabilizing the price level depends also to a great degree upon the specific productivity process at work. So far an AR(1)-process has been assumed. The decision functions of the model change sharply when considering an AR(2)-process for productivity. In order to be able to compare results to those of [17, King/Wolman (1999)] the structure of the process is assumed to be

$$\widehat{a}_t = \rho_{a_1} \widehat{a}_{t-1} + \rho_{a_2} \widehat{a}_{t-2} + \epsilon_{\widehat{a}_t} \tag{52}$$

Using $\rho_{a_1} = 1.3$ and $\rho_{a_2} = -0.3$ one in fact has an ARIMA(1,1,0)-process for productivity. With ρ_{a_1} unchanged and $\rho_{a_2} = -0.4$ it follows a stationary AR(2)-process. Figure 8 shows the impulse responses under this AR(2) specification. The graph for the nominal interest rate looks very much like that of King/Wolman, but the central bank has to raise this rate by more than twice the value of their study. Note that \hat{R}_t rises initially as opposed to the decline under the AR(1)-process for the productivity shock.¹⁷ This also implies a longer lasting inflationary bias. For difference stationary productivity figure 9 gives a graphical impression of the impact of a technology shock. Again the reaction of the nominal rate is more than twice as large as in King and Wolman's study. Surprisingly the central bank can nearly completely eliminate the inflationary bias but not the initial disinflation. Prices decline permanently due the permanent character of the shock.

As has been shown prices are not constant in this model. Accordingly there is some room for the analysis of other policy rules. Even the optimal policy does not produce a constant price level. It may be that some other type of rule performs better. To evaluate the performance of alternative rules one has to add the rule under consideration to the model and compare the resulting life-time-utility (1) to the optimal one. Technically the rule would be a function for the nominal interest rate replacing the equation that derives R_t in the optimal policy from the household's optimality conditions (see (17)). Such calculations are conducted in [10, Henderson/Kim (1999)]. They analyze models with *one-period* wage and price contracts where exact solutions can be obtained so that it is also possible to derive exact welfare

 $^{^{17}\}mathrm{It}$ should be mentioned that the nominal rate also rises for very small degrees of risk aversion.

levels. The Pareto optimal welfare level can be reached in any situation so that all other policies focusing only on the stabilization of prices, the output gap or nominal income are suboptimal. This special result is due to the oneperiod contract structure. [7, Erceg/Henderson/Levin (2000)] extend the framework to *staggered* price and wage contracts. They show that under these circumstances the Pareto optimal welfare level cannot be achieved so that the policymaker always faces a tradeoff between wage inflation, price inflation and the output gap. However their claim that an optimal stabilization of prices is feasible if only prices are staggered is not supported by the results in this paper. Their expectational Phillips curve depends on the utility function used which is additively separable in consumption and leisure. It would probably have a different form under CRRA preferences implying similar results to those obtained here.

5 Conclusions

This paper has considered a version of the [17, King/Wolman (1999)] model of optimal monetary policy in a "New Neoclassical Synthesis" environment. It has turned out that their result of complete price level stabilization is a very special one that depends - at least to a great extent - on the specific preference specification with zero substitution between consumption and labor. Prices fluctuate optimally under a more general utility function so that inflation will not be constant through time. Nevertheless these fluctuations are quite small.

Future research should focus on a richer production structure including capital accumulation considerations. Also the pricing structure can be extended to allow for state-dependent pricing, as opposed to time-dependent pricing assumed here. [6, Dotsey/King/Wolman (1999)] have begun studying the implications.

It would also be interesting to consider welfare losses associated with the inflationary bias. This could be done by comparing the approximated expected life-time-utility under CRRA preferences with that under King and Wolman's GHH specification. The approximation method proposed by [7, Erceg/Henderson/Levin (2000)] can be used to answer that question. Yet in a recent article [11, Kim/Kim (2000)] have shown that sometimes the method of log-linearization around the steady state can lead to spurious welfare re-

versals.¹⁸[24, Woodford (2001)] derives some conditions for the validity of this method in models like the one considered here. Whether the model at hand satisfies these conditions remains an open question since the price level fluctuates and is not constant through time.

Finally the model could also be used to analyze the business cycle implications of optimal monetary policy. It can answer, for example, questions about the variability of output and consumption as well as inflation and money. Further it implies certain correlation patterns between real and nominal variables. This line of research has not been pursued in this kind of literature on monetary policy.

A The linearized Equations

A.1 The Real Variables

The linearized equations for the firms' labor inputs are given by

$$0 = -\widehat{\Omega}_t + \widehat{\rho}_{0,t} + \widehat{a}_t \tag{53}$$

$$0 = -\widehat{\Omega}_t + \widehat{\rho}_{1,t} + \widehat{a}_t \tag{54}$$

A hat (^) represents the relative deviation of the respective variable from its steady state $(\hat{a}_t = (a_t - a)/a)$. For the consumption levels one gets

$$0 = \phi n D_{13} x (c_0, c, n, a) \,\widehat{n}_t + [\phi c_0 D_{11} x (c_0, c, n, a) + \Lambda c_0 D_{11} c (c_0, c_1)] \,\widehat{c}_{0,t} + \Lambda c_1 D_{12} c (c_0, c_1) \,\widehat{c}_{1,t} + \phi c D_{12} x (c_0, c, n, a) \,\widehat{c}_t + \Lambda D_1 c (c_0, c_1) \,\widehat{\Lambda}_t - \rho_0 \widehat{\rho}_{0,t} + \phi D_1 x (c_0, c, n, a) \,\widehat{\phi}_t + \phi a D_{14} x (c_0, c, n, a) \,\widehat{a}_t$$
(55)

$$0 = \phi n D_{13} x (c_1, c, n, a) \, \hat{n}_t + \Lambda c_0 D_{21} c (c_0, c_1) \, \hat{c}_{0,t} + \left[\phi c_1 D_{11} x (c_1, c, n, a) + \Lambda c_1 D_{22} c (c_0, c_1) \right] \hat{c}_{1,t} + \phi c D_{12} x (c_1, c, n, a) \, \hat{c}_t + \Lambda D_2 c (c_0, c_1) \, \hat{\Lambda}_t - \rho_1 \hat{\rho}_{1,t} + \phi D_1 x (c_1, c, n, a) \, \hat{\phi}_{t-1} + \phi a D_{14} x (c_1, c, n, a) \, \hat{a}_t$$
(56)

It should be noted that the equality of c_0, c_1 and c is not yet considered here in order to make clear the different derivatives of the x-function with respect

¹⁸They propose a quite complicated method very recently developed by [20, Sims (2000)].

to c_0, c_1 and c.¹⁹ The condition for aggregate consumption results in

$$0 = [nD_{12}u(c, n, a) + \phi nD_{23}x(c_0, c, n, a) + \phi nD_{23}x(c_1, c, n, a)] \hat{n}_t + \phi c_0 D_{21}x(c_0, c, n, a) \hat{c}_{0,t} + \phi c_1 D_{21}x(c_1, c, n, a) \hat{c}_{1,t} + [cD_{11}u(c, n, a) + \phi cD_{22}x(c_0, c, n, a) + \phi cD_{22}x(c_1, c, n, a)] \hat{c}_t -\Lambda \hat{\Lambda}_t + \phi D_2x(c_0, c, n, a) \hat{\phi}_t + \phi D_2x(c_1, c, n, a) \hat{\phi}_{t-1}$$
(57)
+ $[aD_{13}u(c, n, a) + \phi aD_{24}x(c_0, c, n, a) + \phi aD_{24}x(c_1, c, n, a)] \hat{a}_t$

whereas for aggregate labor the linearized equation is

$$0 = [nD_{22}u(c, n, a) + \phi nD_{33}x(c_0, c, n, a) + \phi nD_{33}x(c_1, c, n, a)] \hat{n}_t + \phi c_0 D_{31}x(c_0, c, n, a) \hat{c}_{0,t} + \phi c_1 D_{31}x(c_1, c, n, a) \hat{c}_{1,t} + [cD_{21}u(c, n, a) + \phi cD_{32}x(c_0, c, n, a) + \phi cD_{32}x(c_1, c, n, a)] \hat{c}_t + \Omega \hat{\Omega}_t + \phi D_3x(c_0, c, n, a) \hat{\phi}_t + \phi D_3x(c_1, c, n, a) \hat{\phi}_{t-1}$$
(58)
+ $[aD_{23}u(c, n, a) + \phi aD_{34}x(c_0, c, n, a) + \phi aD_{34}x(c_1, c, n, a)] \hat{a}_t$

The production functions (resource constraints) must obey

$$0 = \hat{n}_{0,t} - \hat{c}_{0,t} + \hat{a}_t \tag{59}$$

$$0 = \hat{n}_{1,t} - \hat{c}_{1,t} + \hat{a}_t \tag{60}$$

and for the consumption aggregator one arrives at

$$0 = \frac{1}{2}\hat{c}_{0,t} + \frac{1}{2}\hat{c}_{1,t} - \hat{c}_t \tag{61}$$

A crucial condition for the dynamic responses is the linearized implementation constraint:

$$\beta n D_3 x (c_1, c, n, a) \,\widehat{n}_{t+1} + \beta c_1 D_1 x (c_1, c, n, a) \,\widehat{c}_{1,t+1} + \beta c D_2 x (c_1, c, n, a) \,\widehat{c}_{t+1} = -n D_3 x (c_0, c, n, a) \,\widehat{n}_t - c_0 D_1 x (c_0, c, n, a) \,\widehat{c}_{0,t} - c D_2 x (c_0, c, n, a) \,\widehat{c}_t -a D_4 x (c_0, c, n, a) \,\widehat{a}_t - \beta a D_4 x (c_1, c, n, a) \,\widehat{a}_{t+1}$$

$$(62)$$

The labor resource constraint results in

$$0 = \frac{1}{2}\hat{n}_{0,t} + \frac{1}{2}\hat{n}_{1,t} - \hat{n}_t \tag{63}$$

 $¹⁹D_ix(\cdot)$ denotes the first partial derivative of the *x*-function with respect to the *i*-th argument. Similarly $D_{ij}x(\cdot)$ denotes the partial derivative of $D_ix(\cdot)$ with respect to the *j*-th argument.

So far the system contains eleven equations and twelve variables.²⁰ To close the system one has to add one extra equation. This is the tautology $\phi = \phi$ which is given in linearized form by

$$\widehat{\phi}_t = \widehat{\phi}_t \tag{64}$$

A.2 The Nominal Variables

As is known from the main text the nominal interest rate R_t is determined from the efficiency condition for bond holdings in the household's optimization plan (14). The real rate r_t was deduced via the Fisher equation (see (18)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r}\widehat{r}_t + \widehat{\lambda}_t \tag{65}$$

where λ_t is equal to the marginal utility of consumption $\partial u(c_t, n_t, a_t)/\partial c_t$. This implies for the Taylor approximation

$$0 = nD_{12}u(c, n, a) \,\widehat{n}_t + cD_{11}u(c, n, a) \,\widehat{c}_t - D_1u(c, n, a) \,\widehat{\lambda}_t + aD_{13}u(c, n, a) \,\widehat{a}_t$$
(66)

The nominal interest rate follows, according to (17),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R}\widehat{R}_t + \widehat{\lambda}_t \tag{67}$$

in the approximated form, with R (respective r for the real rate) as the steady state values. As discussed earlier firms are unable to change their prices for two periods so $P_{0,t-1} = P_{1,t}$. The Taylor approximation for this condition is given by

$$0 = -\hat{P}_{0,t-1} + \hat{P}_{1,t} \tag{68}$$

Using the demand function (6) allows to determine the relation between $P_{0,t}, P_{1,t}$ and consumption.

$$\widehat{P}_{0,t} = -\frac{1}{\epsilon}\widehat{c}_{0,t} + \frac{1}{\epsilon}\widehat{c}_{1,t} + \widehat{P}_{1,t}$$
(69)

²⁰The twelfth variable is $\hat{\phi}_{t-1}$.

The price level is uniquely determined since $P_{1,t}$ is predetermined and $P_{0,t}$ is given by (69). Using (8) one gets

$$0 = \frac{1}{2}\widehat{P}_{0,t} + \frac{1}{2}\widehat{P}_{1,t} - \widehat{P}_t$$
(70)

The cyclical behavior of money demand can be deduced from (16).

$$0 = \widehat{c}_t + \widehat{P}_t - \widehat{M}_t \tag{71}$$

Since there are now eight new variables $(\hat{\lambda}_t, \hat{r}_t, \hat{R}_t, \hat{P}_t, \hat{P}_{0,t-1}, \hat{P}_{1,t}, \hat{M}_t)$ but only seven equations another tautology must be added to the model. This is

$$\widehat{P}_{0,t} = \widehat{P}_{0,t} \tag{72}$$

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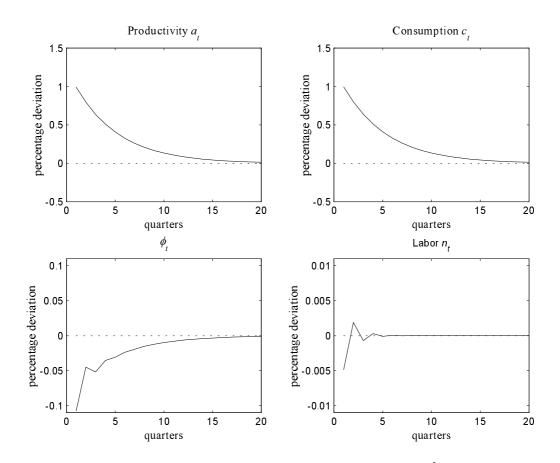


Figure 1: Impulse Response Functions for $\hat{a}_t, \hat{c}_t, \hat{\phi}_t, \hat{n}_t$

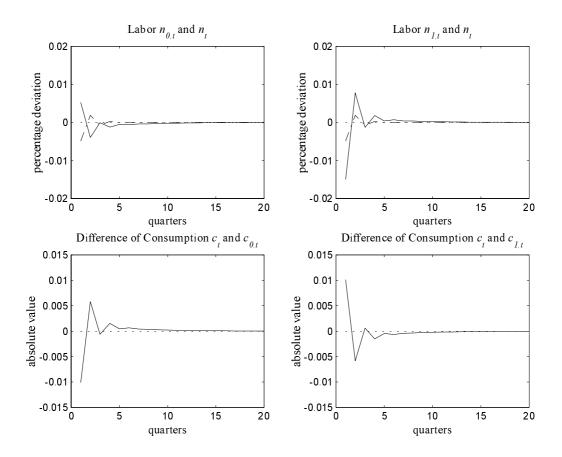


Figure 2: Impulse Response Functions for $\widehat{n}_{0,t},\widehat{n}_{1,t},\widehat{c}_{0,t},\widehat{c}_{1,t}$

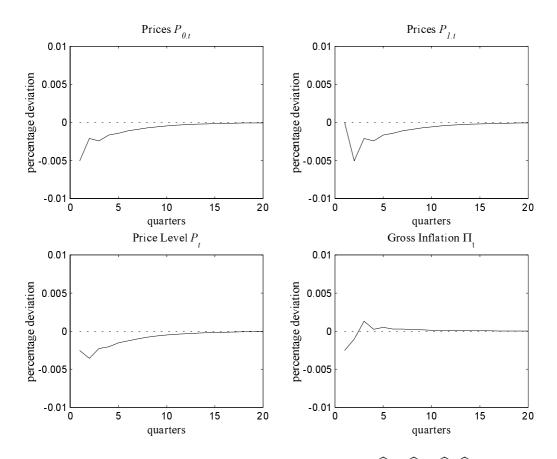


Figure 3: Impulse Response Functions for $\widehat{P}_{0,t}, \widehat{P}_{1,t}, \widehat{P}_t, \widehat{\Pi}_t$

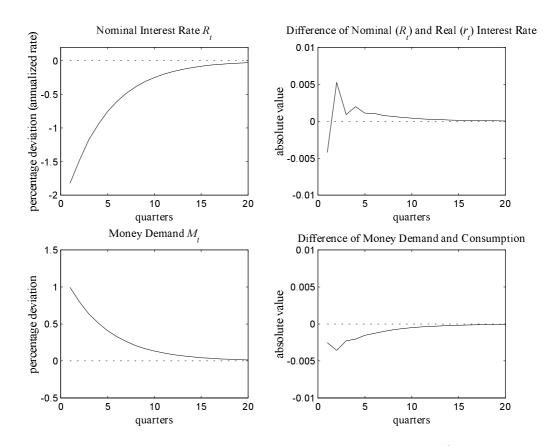


Figure 4: Impulse Response Functions for $\widehat{R}_t, \widehat{r}_t, \widehat{M}_t$

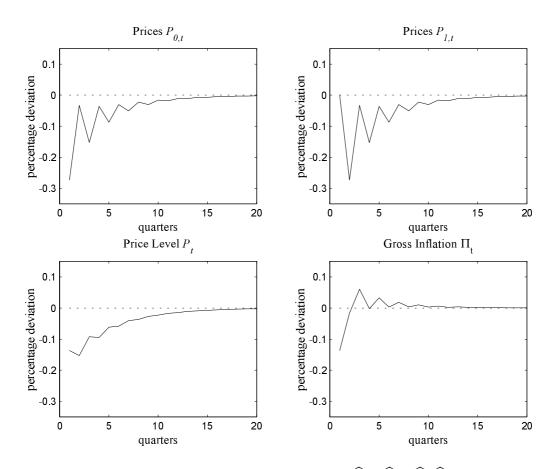


Figure 5: Impulse Response Functions for $\hat{P}_{0,t}, \hat{P}_{1,t}, \hat{P}_t, \hat{\Pi}_t, \sigma = 10$

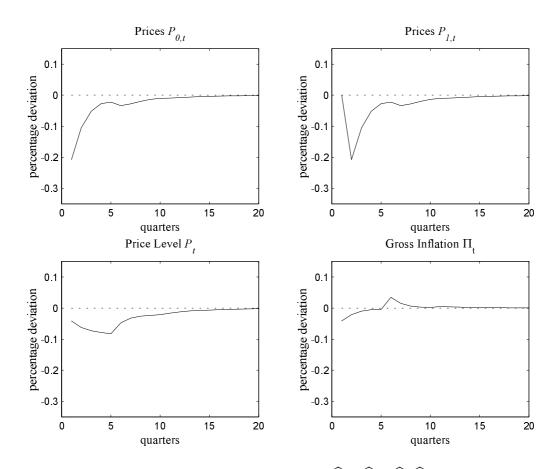


Figure 6: Impulse Response Functions for $\hat{P}_{0,t}, \hat{P}_{1,t}, \hat{P}_t, \hat{\Pi}_t$, 5-period price setting, equal fractions

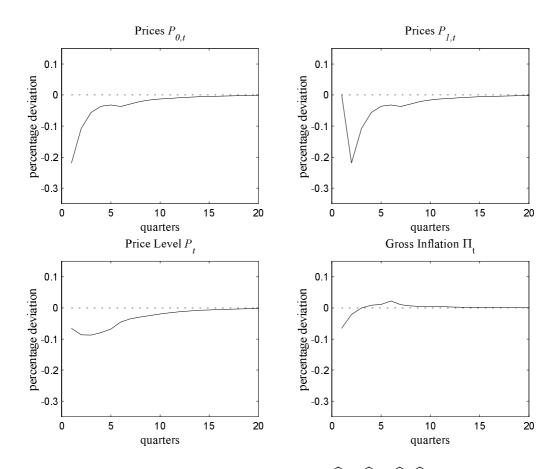


Figure 7: Impulse Response Functions for $\hat{P}_{0,t}, \hat{P}_{1,t}, \hat{P}_t, \hat{\Pi}_t$, 5-period price setting, different fractions

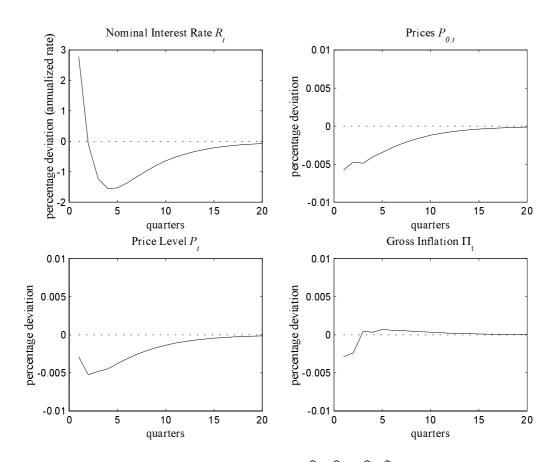


Figure 8: Impulse Response Functions for $\widehat{R}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{\Pi}_t, AR(2)$ productivity shock

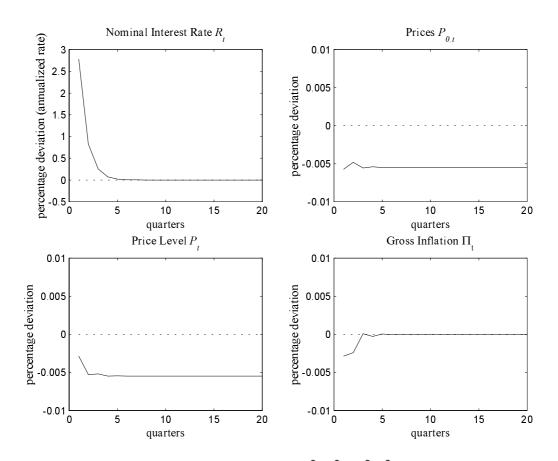


Figure 9: Impulse Response Functions for $\widehat{R}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{\Pi}_t$, ARIMA(1,1,0) productivity shock

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